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Levy Stability for Anisotropic Dynamical Fluctuation in High Energy Multiparticle Production

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ABSTRACT

The behaviour of Levy index μ in various phase space dimensions is studied. The reason for the violation of Levy stable condition at three-dimension observed in experiment is investigated. It is shown that the apparent violation of Levy stable condition is due to the wrong isotropic analysis for the highly anisotropic phase space of multiparticle production.

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The idea of fractality, or intermittency, used in the investigation of strange attractors and of turbulent phenomena since the late seventies^[1], has recently found promising application in the area of high energy physics^[2]. The manifestation of fractality (intermittency) in high energy multiparticle production is the anomalous scaling^[3]

$$F_q(\delta x) = F_q(\Delta x) \left(\frac{\Delta x}{\delta x} \right)^{\varphi_q} \quad \text{as } M \rightarrow \infty \delta x \rightarrow 0 \quad (1)$$

of factorial moment (FM) F_q , defined as

$$F_q(\delta x) = \frac{1}{M} \sum_{m=1}^M \frac{\langle n_m(n_m - 1) \dots (n_m - q + 1) \rangle}{\langle n_m \rangle^q}, \quad (2)$$

where x is some phase space variable, e.g. rapidity, $\delta x = \Delta x/M$ is the bin width for a M -partition of the region Δx in consideration, n_m is the multiplicity in the m th bin. The exponent φ_q in Eq.(1), called intermittency index, characterizing the strength of dynamical fluctuation, is connected with the anomalous fractal dimension d_q of rank q , $d_q = \varphi_q/(q - 1)$ ^[4].

A possible mechanism for the intermittency phenomena is the random cascading process^[3]. An important parameter, characterizing the width of probability distribution in the elementary partition of random cascade, is the Levy index μ , which takes value in the range $[0, 2]$ according to the requirement of Levy stability^[5]. There is, however, experimental evidence that in three-dimensional phase space of multiparticle final states the measured value of μ goes out of the allowed range and becomes greater than 2^[6]. The aim of this letter is to study the behaviour of Levy index μ in various phase space dimensions and, especially, the violation of Levy stable condition at three-dimension. It will be shown that the later is connected with the anisotropy of phase space, i.e. the apparent violation of Levy stable condition is due to the wrong isotropic analysis for the highly anisotropic phase space of multiparticle production.

Let us first briefly recall the definition of Levy index μ and the method for its experimental measurement in both one- and higher- dimensional phase space.

Let w be the elementary probability in each step of phase space partition of the random cascading and $r(w)$ its probability distribution. In the log-normal approximation the anomalous dimensions of different order satisfy the relation^[3]

$$\frac{d_q}{d_2} = \frac{\varphi_q}{\varphi_2} \frac{1}{q - 1} = \frac{q}{2}. \quad (3)$$

However, as is well known^[7], the use of Central Limit Theorem for a cascading process is a very crude approximation particularly in the tails. When the dynamical fluctuation is large, the relative variance of the distribution $r(w)$ is large, and the central limit theorem can not describe correctly the tail of the distribution

of $\sum_k^\infty \ln w(k)$, which means that the distribution of $\sum_k^\infty \ln w(k)$ is not a simple Gaussian, but should be a Levy stable distribution^[6]. In this case, Eq.(3) is no longer valid, and should be replaced by the following relation^[8]:

$$\frac{\varphi_q}{\varphi_2} = \frac{q^\mu - q}{2^\mu - 2}, \quad (4)$$

or

$$\frac{d_q}{d_2} = \frac{1}{q-1} \frac{q^\mu - q}{2^\mu - 2}, \quad (5)$$

where μ is a continuous parameter between 0 and 2, characterizing the limiting behavior of Levy stable distribution^[6]. The value of μ determines the degree of fluctuation of the random variable w . When $\mu = 0$, the fluctuation of w is the largest. In this case, we have $d_q = d_2$, the system reduces to a simple fractal, and the value of d_q can be related to a critical exponent for the phase transition as demonstrated explicitly for the Ising model^[8]. When $\mu = 2$, the fluctuation of w tends to zero, and Eq.(4) goes back to Eq.(3) of central limit theorem.

Eq.(4) is the basis for extracting Levy index μ from experimental data, but some remarks have to be made. In fact, the power law Eq.(1) of FM has not been strictly observed experimentally up to now. For the 3D experimental data there is a general tendency of upward bending^[9] for the log-log plot of FM versus M , while in a lower dimensional projection the dynamical fluctuation will be reduced^[10] by the averaging process and the FM saturates for $\delta \rightarrow 0$. So it is unfeasible to extract φ_q by fitting Eq.(1) directly to experimental data. In the first paper of Ref.[10], a modified power law assumption was proposed, i.e.

$$F_q \propto [g(\delta)]^{\varphi_q}, \quad (6)$$

where $g(\delta)$ is a general function of phase space interval δ . Expressing $g(\delta)$ in terms of $F_2(\delta)$, one can find the linear relation

$$\ln F_q = C_q + \frac{\varphi_q}{\varphi_2} \ln F_2. \quad (7)$$

From Eq.(7) the ratio of intermittency index can be obtained and the value of μ follows from Eq.(4).

Let us turn now to the discussion of the physical origin of the apparent violation of Levy stable condition in the 3D analysis of experimental data.

In this respect, it is worthwhile noticing that for higher dimensional case, in discussing fractal (or intermittency), the way of subdivision of phase space must be defined^[11]. In particular, in an anisotropic phase space, when the phase space is divided into λ_a pieces in direction a , it should be divided into $\lambda_b = \lambda_a^{1/H}$ pieces in direction b , where $0 \leq H \leq 1$ is a parameter, characterizing the anisotropy of phase space, and

is called Hurst exponent^[12]. A method has been proposed for extracting the value of Hurst exponent from experimental data.

As is well known, the phase space of multiparticle production is highly anisotropic^[13]. Therefore, the right way for measuring Levy index should be, first extracting the values of Hurst exponents H 's and then calculating the FM's with different shrinking ratios in different directions according to the values of H 's. The Levy index μ can then be derived from Eq.(4) using the obtained FM's. This procedure is called self-affine analysis^[11]. However, in the presently available experimental results the higher-dimensional Levy indices were obtained in a self-similar analysis, i.e. the shrinking ratios were taken to be the same in different phase space directions. Therefore, the apparent violation of Levy stable condition may be due to the wrong way of phase space shrinkage in FM analysis.

In order to show the above argument explicitly, let us construct a 3D self-affine random cascading model^[14]. We take, as an example, the numbers of subdivision in the directions a , b and c as $\lambda_a = 2$, $\lambda_b = 3$, $\lambda_c = 4$ with $\lambda_a < \lambda_b < \lambda_c$ respectively. Consider a three-dimensional phase space region $\Delta x_a \Delta x_b \Delta x_c$ and divide it subsequently with the subdividing ratio $\lambda_a \lambda_b \lambda_c$. In order to keep the conservation of probability in each step of splitting, the probability for a subdivision is taken to be

$$\omega_{ijk} = \frac{1 + \alpha r_{ijk}}{\sum_{i=1}^{\lambda_a} \sum_{j=1}^{\lambda_b} \sum_{k=1}^{\lambda_c} (1 + \alpha r_{ijk})}, \quad (8)$$

where α is the parameter of fluctuation strength, $0 \leq \alpha \leq 1$; r_{ijk} is a random number in the interval $[-1, 1]$. After ν steps, the probability in a subcell is

$$p_{i_\nu j_\nu k_\nu}^{(\nu)} = \omega_{i_1 j_1 k_1}^{(1)} \omega_{i_2 j_2 k_2}^{(2)} \cdots \omega_{i_\nu j_\nu k_\nu}^{(\nu)}, \quad (9)$$

where $i_\nu = 1, \dots, \lambda_a$, $j_\nu = 1, \dots, \lambda_b$, $k_\nu = 1, \dots, \lambda_c$.

Using this model, we have made a 20000-events Monte Carlo simulation up to $\nu = 6$. The data sample thus obtained is then used to calculate the FM of different dimensions in self-similar analysis way. The Levy indices obtained by fitting the MC data to Eq.'s (4) and (7) for different α 's are shown in Fig.1a.

It is noticeable that the Levy index of 3-dimensional phase space obtained in this (self-similarly analyzed) way can be greater than 2 for small dynamical fluctuation parameter α . When $\alpha = 0.15$ the Levy index μ and the ratio φ_4/φ_2 obtained from the model fit the results from NA22 data, cf. Table I and Fig.1a.

On the contrary, when we analyse the Levy stability of the system in a self-affine way according to the values of Hurst exponents H 's of the original self-affine cascading process the value of μ obtained never go beyond the theoretically allowed region $[0, 2]$, cf. Fig.1b.

It can be seen from the figure that Levy indices decrease with the increasing of fluctuation strength. This can be understood as the following. When the fluctuation parameter α is very small, the distribution $p(\ln \omega)$ of logarithmic subdivision probability will be very narrow, belonging to the attraction basin of normal law^[6], and the distribution $p(\zeta)$ of $\zeta = \sum_{i=1}^N \ln \omega_i$ for large N is a Gaussian so that the corresponding Levy index μ tends to 2. In contrast, when the fluctuation parameter α becomes large enough, the distribution $p(\ln \omega)$ will decay in the region of large $\ln \omega$ slower than $(\ln \omega)^{-3}$, and go outside of the basins of attraction of normal law. Therefore the Levy index μ decreases when α becomes large.

In order to compare the MC simulation with the experimental results of lower dimensional phase space, we have also projected three-dimension self-affine cascading model into two- and one-dimensional phase space and then calculate Levy index in a self-similar way. It is found that the Levy index μ after projecting depends upon the dynamic fluctuation parameter α in the same way as that of the three-dimensional ones (see Fig.1a). On the other hand, the Levy index μ of lower dimensional phase space does not go beyond 2 even for self-similar analysis, just like the results of experimental data^[6,15]

In this letter we have discussed the Levy stable law of self-affine cascading process and found that the apparent violation of Levy stability may be attributed to the wrong way of phase space partition in measuring the FM's in the highly anisotropic phase space. Using MC simulation, we have shown that if the FM's are calculated for a self-affine fractal system in self-similar way, the Levy index μ can go beyond the theoretically allowed boundary and the data of NA22 about Levy index μ can be recovered. In contrast, if we calculate the FM's in a correct way of phase space shrinkage, the meaningful Levy index of cascading process will be obtained, keeping always inside the region $[0,2]$.

It is also shown by MC simulation that the Levy index of cascading process depends upon the strength of dynamic fluctuation, i.e. the stronger the fluctuation is, the smaller the Levy index is.

Thus it is urged for the experimentalist to extract the Hurst exponents of multiparticle final states and find out the meaningful Levy index. If this is accomplished successfully, then the results together with the observation of the strict scaling behaviour of FM's^[14] will be strong evidence for the self-affinity of the dynamical fluctuations.

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Table I, The comparison between the MC results of 3D self-affine random cascading model with $\alpha = 0.15$ and the results of the NA22 data from self-similar analysis.

	φ_3/φ_2	φ_4/φ_2	φ_5/φ_2	φ_6/φ_2	μ
data of NA22	3.4 ± 0.2	7.8 ± 0.5	14 ± 1		2.53 ± 0.08
result of MC	3.473	7.768	14.208	23.04	2.529

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Figure Captions:

Fig.1, The Levy index μ in different phase space dimensions versus the fluctuation parameter α for 3D self-affine random cascading model:

- (a). with self-similar analysis;
- (b). with self-affine analysis.