Towards a Unified Description of the Baryon Spectrum and the Baryon-Baryon Interaction within a Potential Model Scheme

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Abstract. We study the low energy part of the nucleon and \Delta spectra by solving the Schrödinger equation for the three-quark system in the hyperspherical harmonic approach. The quark-quark hamiltonian considered includes, besides the usual one-gluon exchange, pion and sigma exchanges generated by the chiral symmetry breaking. This quark-quark potential reproduces, in a Resonating Group Method calculation, the nucleon-nucleon scattering phase shifts and the deuteron properties. The baryonic spectrum obtained is quite reasonable and the resulting wave function is consistent with the ansatz used in the two baryon system.

1 Introduction

Models have been widely used to study the properties of the hadron spectrum due to the impossibility to solve Quantum Chromodynamics (QCD) at the current moment. In particular, the quark potential models incorporate the perturbative one-gluon exchange quark-quark (qq) potential (\(V_{0GB}\)) derived from QCD as well as a parametrization of some nonperturbative effects through a qq confining potential (\(V_{con}\)) [1]. Such an effective theory [the quark-gluon coupling constant is taken as an effective one and the constituent quark masses (\(m_q\)) are parameters fitted from the baryon magnetic moments] provides a reasonable understanding of the baryon spectrum and the hadron static properties [2].

The idea of an interquark potential has been also used to study baryon-baryon interactions. More explicitly, the repulsive core of the nucleon-nucleon (NN) force has been shown to arise from the color-spin structure of the \(V_{0GB}\) [3]. Nevertheless, the same scheme has been proven incapable of describing the long range part of the NN interaction unless pion exchange between quarks is
introduced. From the basic theory, the origin of the one-pion exchange potential \( V_{\text{PE}} \) is associated with the spontaneous chiral symmetry breaking of QCD. Moreover, the inclusion of the \( qq \) sigma potential \( V_{\text{SE}} \) consistently with its chiral partner (the pion) allows to reproduce the intermediate range \( NN \) interaction as well as the deuteron properties [4].

In this paper we examine the consistency of the two scenarios. We determine a \( qq \) interaction by studying two nucleon properties and charge exchange reactions and proceed to analyze the baryon spectrum for this interaction. As we shall discuss the predictions for the baryon masses and the baryon wave functions, it is not only a stringent test of the potential but also a consistency test of the formalism used to generate the NN interaction [resonating group method (RGM)].

2 The Quark-Quark Potential

The starting point of our description is a quark-quark interaction of the following form:

\[
V_{qq}(r_{ij}) = V_{\text{con}}(r_{ij}) + V_{\text{GE}}(r_{ij}) + V_{\text{PE}}(r_{ij}) + V_{\text{SE}}(r_{ij}) ,
\]

where \( r_{ij} \) is the interquark distance.

The confinement is chosen to be linear as suggested by the meson spectrum and lattice calculations:

\[
V_{\text{con}}(r_{ij}) = -a_c (\lambda_i \cdot \lambda_j) r_{ij} ,
\]

where \( \lambda' \)'s are the SU(3) color matrices.

\( V_{\text{GE}}, V_{\text{PE}} \) and \( V_{\text{SE}} \) have been derived in detail elsewhere [1, 4] and we shall limit here to write the final expressions:

\[
V_{\text{GE}}(r_{ij}) = \frac{1}{4} \alpha_s \lambda_i \cdot \lambda_j \left\{ \frac{1}{r_{ij}} - \frac{1}{4m_q^2} \left[ 1 + \frac{2}{3} \sigma_i \cdot \sigma_j \right] \frac{e^{-r_{ij}/r_0}}{r_{ij}^3} - \frac{1}{4m_q^2 r_{ij}^3} S_{ij} \right\} ,
\]

where \( \alpha_s \) is the effective quark-quark-gluon coupling constant, \( r_0 \) is the range of a smeared \( \delta \) function in order to avoid and unbound spectrum [5], the \( \sigma' \)'s stand for the spin Pauli matrices and \( S_{ij} \) is the quark tensor operator \( S_{ij} = 3(\sigma_i \cdot \hat{r}_{ij})(\sigma_j \cdot \hat{r}_{ij}) - \sigma_i \cdot \sigma_j \).

\[
V_{\text{PE}}(r_{ij}) = \frac{1}{3} \alpha_c \frac{A^2}{A^2 - m_\rho^2} m_\rho \left\{ Y(m_\rho r_{ij}) - \frac{A^3}{m_\rho^3} Y(A r_{ij}) \right\} \sigma_i \cdot \sigma_j + \left[ H(m_\rho r_{ij}) - \frac{A^2}{m_\rho^2} H(A r_{ij}) \right] S_{ij} \tau_i \cdot \tau_j ,
\]

\[
V_{\text{SE}}(r_{ij}) = -\alpha\chi \frac{4m_\sigma^2}{m_\rho^2} \frac{A^2}{A^2 - m_\sigma^2} m_\sigma \left[ Y(m_\sigma r_{ij}) - \frac{A}{m_\sigma} Y(A r_{ij}) \right] , \]

where \( \alpha \) is the 

Figure 1. Relative energy $N$ and $\Delta$ spectrum up to 0.7 GeV excitation energy. The solid line corresponds to the results of our model. The boxes represent the experimental data with the corresponding uncertainties.

where $m_\pi$ ($m_\sigma$) is the pion (sigma) mass, $\alpha_{ch}$ is the chiral coupling constant (related to the $\pi NN$ coupling constant), $\Lambda$ is a cutoff parameter and $Y(x)$, $H(x)$ are the Yukawa functions defined as:

$$Y(x) = \frac{e^{-x}}{x}, \quad H(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right)Y(x), \quad (6)$$

With this interaction, the two baryon system has been studied in the RGM framework assuming for the spatial part of the wave function of the quarks a harmonic oscillator ground state,

$$\eta_{os}(r_i - R) = \left(\frac{1}{\pi b^2}\right)^{3/4} e^{-\left(r_i - R\right)^2/2b^2}, \quad (7)$$

where the parameter $R$ determines the position of the baryon and $b$ is the harmonic oscillator constant.

Table 1. Value of the potential parameters

<table>
<thead>
<tr>
<th>$m_\pi$</th>
<th>$b$</th>
<th>$\alpha_s$</th>
<th>$\alpha_{ch}$</th>
<th>$a_c$</th>
<th>$m_\pi$</th>
<th>$m_\sigma$</th>
<th>$\Lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MeV</td>
<td>fm</td>
<td></td>
<td></td>
<td>MeV · fm$^{-1}$</td>
<td>fm$^{-1}$</td>
<td>fm$^{-1}$</td>
<td></td>
</tr>
<tr>
<td>313</td>
<td>0.518</td>
<td>0.485</td>
<td>0.027</td>
<td>91.488</td>
<td>0.7</td>
<td>3.42</td>
<td>4.2</td>
</tr>
</tbody>
</table>

Using the same set of parameters given in Table 1, the $NN$ scattering phase
Figure 2. Adiabatic \( N \bar{N} \) potential for different nucleon wave functions for the channel \((S,T)=(1,0)\). The solid line corresponds to the solution of the Schrödinger equation. The others to a gaussian ansatz with the values of \( b \) given in the figure.

shifts, the static and electromagnetic properties of the deuteron [4] and reactions that take place with the excitation of the \( \Delta \) resonance [6] are reasonably reproduced.

3 Results

To study the baryonic spectrum with the potential just described (parameters as in Table 1) we solve the Schrödinger equation in the hyperspherical harmonic approach [7]. The low energy \( N \) and \( \Delta \) spectrum obtained, a part of which is shown in Fig. 1, is quite reasonable though some small discrepancy remains concerning the relative energy position of the Roper resonance \([N^- (1440)]\) and the first negative parity state excitation \([N^- (1535)]\), as it is common in two body potential models.

Regarding the wave function and in order to check the internal RGM consistency, it makes sense to compare the \( N \bar{N} \) potential one obtains from the wave function solution of the Schrödinger equation, with the potential obtained from the ansatz RGM wave function for different values of the parameter \( b \). For the sake of technical simplicity we do this in the Born-Oppenheimer approximation. The results appear in Fig. 2, where we can see that it is precisely the value of \( b \) that gives the best overall fit to the \( N \bar{N} \) data with RGM, the one which provides a better approximation. This confers to \( b \) a self-consistent character and solves the controversy about its possible values.

Certainly, more work and further refinements are needed along this direc-
tion. Meanwhile, the proposed model points out the plausibility of a unified description of the baryon structure and the baryon-baryon interaction.

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References