Squeezed Vacuum Interferometry

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Abstract

A high-sensitive interferometric scheme is presented. It is based on homodyne detection and squeezed vacuum phase properties. The resulting phase sensitivity scales as $\delta \phi \simeq \frac{1}{4} n^{-1}$ with respect to input photons number.

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The task of any interferometric measurement is to monitor fluctuations in some environmental parameter through the detection of the induced phase shift on the electromagnetic field. The pressure of the impinging radiation, as a general rule, disturbs the monitored parameter itself and therefore the problem is that of optimizing the instrumental precision introducing into the apparatus as little energy as possible. Interferometric measurements are widely utilized in many fields of physics and generally represent an accurate detection, even when classical light is placed at the input. However, in detecting gravitational waves [1] or for monitoring minute variations in the refractive index of a medium very accurate measurements are required. Making use of nonclassical light is expected to lead to the necessary improvement in phase sensitivity, beating shot noise limit $\delta \phi \sim \bar{n}^{-1/2}$ with respect to the input photons number $\bar{n}$. In the last decade [2] and especially in the recent years [3, 4, 5, 6] this possibility has been investigated and some detection schemes have been presented. Here I suggest a high-sensitive interferometric detection scheme which is based on highly nonclassical properties of squeezed vacuum state of radiation field.

A generic interferometric scheme is outlined in Fig. 1. The stable configuration corresponds to a fixed phase shift $\phi_0$ between the input signal and a stable reference (local oscillator), for example an intense laser beam. Without loss of generality I consider $\phi_0 = 0$ throughout the paper. A fluctuation in the considered parameter induces a phase shift on the input signal thus leading to a different probability distribution of the detector outcomes. The phase sensitivity of the whole apparatus, namely the smallest shift which can be resolved, depends both on the apparatus itself and on the quantum state of radiation at the input. Once the detection scheme has been chosen sensitivity depends only on the behaviour of input signal under phase shift evolution, in other words as it alters by the action of the operator $\hat{U} = \exp \{i\bar{n}\phi\}$.

Squeezed vacuum $|0, \zeta\rangle \equiv \hat{S}(\zeta)|0\rangle$ can be obtained from vacuum state by the action of the squeezing operator $\hat{S}(\zeta) = 1/2(\zeta a^\dagger - a^\dagger \zeta) = \bar{\zeta} a^\dagger - \bar{\zeta} a$ with $\zeta = re^{\imath \phi}$. Fluctuations of the field are increased in the direction individuated by $\zeta$ and correspondingly decreased in the orthogonal direction. Squeezed states $|\alpha, \zeta\rangle$ are then obtained by acting with the displacement operator $\hat{D}(\alpha) = \alpha a^\dagger - \bar{\alpha} a$ on squeezed vacuum. Squeezing a state increases its energy, in particular squeezed vacuum possesses a mean photon numbers given by $\bar{n} = \sinh^2 r$. 

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The property of squeezed vacuum I deal with is expressed by the formula
\[ e^{i\phi} |0, \zeta\rangle = |0, \zeta e^{i2\phi}\rangle. \tag{1} \]

From Eq. (1) two crucial informations can be extracted: first squeezed vacuum is invariant under phase evolution and moreover it enhances the occurred phase shift. It is worth noticing that this is a peculiar feature of squeezed vacuum which is lost for non-zero amplitude squeezed states. In this case, in fact, signal and squeezing phases differently evolve
\[ e^{i\phi} |\alpha, \zeta\rangle = |\alpha e^{i\phi}, \zeta e^{i2\phi}\rangle, \tag{2} \]
leading to a state with broad phase distribution. The situation is illustrated in Fig. 2 where I report the Wigner distribution function
\[ W(\alpha, \tilde{\alpha}) = \int \frac{d^2 \lambda}{\pi^2} e^{\alpha \lambda - \tilde{\alpha} \lambda} \text{Tr} \left( \rho e^{\lambda a^\dagger - \lambda a} \right), \tag{3} \]
in the complex plane of the field amplitude for a squeezed vacuum and a squeezed states along with their shifted counterparts.

How the evolution (1) of squeezed vacuum under phase shift can be utilized in improving interferometry?

Homodyne detector is depicted in Fig. 3. The input signal is mixed by a beam splitter with an intense laser beam \( |\tilde{z}\rangle \) (local oscillator) and the resulting two output fields are detected by a pair of photo counters. Homodyne output is the difference photocurrent rescaled by the local oscillator intensity
\[ \hat{x} = \frac{1}{|e|} (a^\dagger b + a b^\dagger). \tag{4} \]

For sufficient intense local oscillator the probability distribution of the homodyne photocurrent approaches [7] that of field quadrature \( a_\varphi = 1/2( ae^{-i\varphi} + a^\dagger e^{i\varphi}) \) of the input field, \( \varphi \) being the phase difference between the signal and the local oscillator. I consider a detection scheme in which each experimental event consists in a couple of independent homodyne measurements with \( \varphi \) shifted by \( \pi/2 \) each other. This can be easily achieved by changing the optical path of the local oscillator. The outcomes of such an experiment can be represented as points in the complex plane \( \alpha = x + iy = \rho e^{i\phi} \) of field
amplitude. It can be shown [6] that they are distributed according with the Wigner function of the considered state

\[ p(x, y) = p_\phi(x)p_{\phi+\pi/2}(y) \equiv W(x + iy, x - iy). \]  

(5)

The phase value inferred from each event is the polar angle of the point itself. The experimental histogram of the phase distribution is obtained by dividing the plane into angular bins and then counting the number of points which fall into each bin. In Fig. 4 a Monte Carlo simulation of the above experimental procedure is illustrated for a squeezed vacuum with \( \bar{n} = 3 \).

The phase probability distribution is the marginal distribution, integrated over the radius, of the Wigner function (5)

\[ p(\phi) = \int_0^\infty \rho d\rho \, W(\rho e^{i\phi}, \rho e^{-i\phi}) \]

\[ = \frac{1}{2\pi} \int_0^\infty \frac{1}{e^{-2\rho} \cos^2(\phi - \phi_0) + e^{2\rho} \sin^2(\phi - \phi_0)} \]

\[ \approx \frac{1}{2\pi} \frac{\bar{n}}{1 + 16\bar{n}^2 \sin^2(\phi - \phi_0)}. \]  

(6)

It exhibits sharp peaks of equal height at \( \phi = \phi_0, \phi_0 \pm \pi \) and deep minima (also of equal height) at \( \phi = \phi_0 \pm \pi/2 \) (see Fig. 5). When fluctuations in the monitored parameter change the stationary value \( \phi_0 \), the phase distribution rigidly translates as it happens for classical interference fringes. The presence of multiple peaks (phase bifurcation [8]) would seem to indicate a loss of phase information, but the fact that probability distribution rigidly translates under phase shift reverses this assertion as the bifurcation itself can be utilized as a multiple check for improving sensitivity.

A good parameter to evaluate the phase sensitivity of the present detection scheme is given by the full width half maximum (FWHM) of the distribution peaks. From Eq. (6) we obtain

\[ h_{1/2} \approx \frac{\bar{n}}{4\pi}, \]  

(7)

for the half maximum height and correspondingly a phase sensitivity given by

\[ \delta\phi \approx \frac{1}{4\bar{n}}. \]  

(8)
In Eq. (8) is already taken into account the squeezing enhancement described by Eq. (1).

In conclusion, a high-sensitive interferometric scheme based on phase properties of squeezed vacuum has been presented. It involves couples of homodyne measurements whose outcomes are points in the complex plane of field amplitude. The phase distribution is then obtained as marginal probability integrated over the radius. Multiple peaks of squeezed vacuum phase probability becomes useful for detection as the whole distribution rigidly translates under phase shifts. The resulting phase sensitivity is largely improved in comparison with shot noise limit and it scales as $\delta \phi \simeq \frac{1}{4} n^{-1}$ with respect to the input photons number.

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References

Caption to figures

Figure 1: Outline of a generic interferometric scheme. Stable configuration corresponds to a fixed phase shift between the signal and the local oscillator, which, in turn, leads to some distribution outcomes. Fluctuations in the parameter under examination change the optical path of the input signal leading to a different distribution for the detector outcomes.

Figure 2: Wigner distribution function $W(\alpha, \bar{n})$ for shifted squeezed vacuum and squeezed states. In (a) a squeezed vacuum with $\bar{n} = 4$ initially with zero squeezing phase is subjected to a $\pi/8$ shift and the resulting squeezed vacuum show a squeezing phase equal to $\psi = \pi/4$. In (b) a squeezed states with total number of photons $\bar{n} = 4$ and $|\alpha| = 1$ is initially prepared with equal squeezing and signal phase in order to minimize phase fluctuations. After the same shift occurred in (a) squeezing phase is twice than the signal one and phase distribution is broadened.

Figure 3: Outline of homodyne detector. The input signal is represented by the mode $a$ whereas $b$ support a highly excited coherent state, which provides the local oscillator. Optical path of $LO$ can be tuned in order to switch the phase difference between the two modes from 0 to $\pi/2$. 
Figure 4: Monte Carlo simulation of the suggested experimental procedure on a squeezed vacuum with $\bar{n} = 3$. The outcomes distribution in the complex plane and the resulting marginal phase distribution are reported.

Figure 5: Phase probability distribution for the input squeezed vacuum and for the squeezed vacuum after than a $\pi/8$ shift is occurred. The plots are for a state with $\bar{n} = 3$. 