GANIL

Langevin Equation with the Deterministic Algebraically Correlated Noise

A. Jasiński¹ and L. Srokowski^{1,2}

GANIL - BP 5027, F-14021 Caen Cedex, France

² Institute of Nuclear Physics, PL - 31-342 Kraków, Poland

SCAN-9510140

CERN LIBRARIES, GENEVA

203 43

GANIL P 95 25

Langevin Equation with the Deterministic Algebraically Correlated Noise

M. Płoszajczak¹ and T. Srokowski^{1,2}

¹ *GANIL, BP 5027, F-14021 Caen Cedex, France*

² *Institute of Nuclear Physics, PL – 31-342 Kraków, Poland*

Abstract

Stochastic differential equations with the deterministic, algebraically correlated noise are solved for a few model problems. The chaotic force with both exponential and algebraic temporal correlations is generated by the adjoined extended Sinai billiard with periodic boundary conditions. The correspondence between the autocorrelation function for the chaotic force and both the survival probability and the asymptotic energy distribution of escaping particles is found.

1 Introduction

For the description of dynamics of a classical many body system, there have been two main theoretical approaches proposed. The first one relies upon investigation of some kinetic rate equations and the second one uses the molecular dynamics approach. Both these approaches can be extended to include some elements of the full quantum problem such as the Pauli exclusion principle for fermions. In application to nuclear physics, the classical molecular dynamics (MD)[1] or its 'quantal' version the antisymmetrized molecular dynamics (AMD)[2] have been mainly studied in connection with the dynamics of the collision and the subsequent instabilities leading to the fragmentation[3] . Chaotic properties of nuclei have been discussed in the framework of the MD in connection with the heavy - ion scattering[4, 5]. It was shown that for sufficiently violent collisions, the colliding ions form a chaotic transient with the exponentially decaying survival probability. Moreover, one gets fast memory loss and the equilibration, corresponding to the excitation of many overlapping resonances (Ericson fluctuations), as well as the Lorentzian shape of the energy autocorrelation function of an S-matrix element $C_{ij}(\epsilon) = \langle S_{ij}^*(E) S_{ij}(E + \epsilon) \rangle_E$ [6]. For simple chaotic Hamiltonian systems containing essential elements of both direct reactions and isolated or overlapping resonances, the relevance of the random matrix theory of compound nucleus reactions has been demonstrated for quantities like the averaged fluctuating cross section and the correlations function $C(\epsilon)$ [7] . Recently, the velocity autocorrelation function (VACF) : $C(t) = \langle \mathbf{v}(t_0) \mathbf{v}(t_0 + t) \rangle$ has

been studied in the MD for the reaction $^{12}\text{C} + ^{12}\text{C}$ [8] . Two regimes have been demonstrated in this study. For central collisions , the VACF decays exponentially in time and the entire energy is dissipated at the very beginning of the reaction. The equilibration time is short, allowing the statistical properties of the compound nucleus to show up at the early stage of the reaction. On the contrary, in the peripheral collisions of ions, the algebraic $C(t) \sim t^{-\gamma}$ ($\gamma = 1$) , long time tail in the VACF has been demonstrated[9] . The force autocorrelation function (FACF) : $\tilde{C}(t) = \langle \mathbf{F}(t_0)\mathbf{F}(t) \rangle$ decays in the same way $\tilde{C}(t) \sim t^{-\gamma}$ ($\gamma = 1$) as can be seen in Fig. 1.

VACF is the principal quantity of interest from which many other useful quantities can be obtained by simple manipulations. The Fourier transform of $C(t)$ gives the power spectrum $S(\omega)$. For the peripheral collisions[8] , one finds $S(\omega) \sim |\ln \omega|$ and $S(\omega = 0) \equiv 2D = \infty$, where D is the diffusion constant[10] . The mean square displacement in configurational space can be obtained from the VACF using the Green-Kubo formula[12] , which yields in this case[8] :

$$\sigma^2(t) \equiv \langle (\mathbf{r}(t) - \mathbf{r}(t_0))^2 \rangle = 2 \int_{t_0}^t (t - \tau) C(\tau) d\tau \propto t \ln \frac{t}{t_0} - t + t_0 \quad , \quad (1)$$

i.e. the diffusion is anomalously enhanced (superdiffusion) and $D \equiv \lim_{t \rightarrow \infty} \sigma^2(t)/t$ diverges logarithmically. The same dependence as in the configurational space holds also for the mean-square displacement in the velocity space because diffusion in classical Hamiltonian systems proceeds in the phase space and no subspace is distinguished by the dynamics. One can also argue differently. The Green - Kubo

formula for the mean square displacement in the velocity space can be written substituting the VACF by the FACF in Eq. (1). Since $\tilde{C}(t)$ decays like $\sim 1/t$, therefore the momentum variance grows like $\tilde{\sigma} \sim t \ln t$. Direct numerical verification of this logarithmic deviation from the linear growth of $\tilde{\sigma}(t)$ is particularly difficult in small systems[8], where the small available phase space cuts short the diffusion before the logarithmic corrections can be seen with the naked eye. The same problem has been reported before for simpler systems such as billiards[11] or Lorentz gases[13, 14], which are easier to handle numerically than the MD and where the time-dependence is easier to decide. For heavy-ion collisions at grazing, independently of the MD results obtained in ref. [8], one expects particularly small number of open decay channels[15] and, therefore, both the existence of algebraic survival probability[16, 17] and the formation of long-lived orbiting complex[18].

In MD approach, the anomalous diffusion appears in peripheral collisions. In this case the projectile and target preserve their identity for a long time, i.e. the correlations fall off slowly and the interactions between nuclei are extremely weakly absorptive. The long-time rotation of weakly interacting reaction constituents is a necessary condition for the anomalous diffusion to take place. A decisive factor in that process are violent collisions of particles at the border between the projectile and target nuclei. For large impact parameters such collisions are rare and most of the particles are affected only by weak interactions from the side of their neighbours in the same nucleus, whereas the rest of the system continues its quasi-free

, rotational motion. Hence, in spite of the full chaoticity, interacting nuclei during a long period of time resemble an integrable, two-body system and their peripheral collision mimics the nonhyperbolic scattering[4] .

Studies of VACF in MD point to the existence of two different mechanisms[8] : the fusion dominated process at small angular momenta (impact parameters) with large number of open channels, and the diffusive process associated with orbiting with small number of decay channels. In the latter case, intensity of the diffusion process grows monotonically with time, i.e. the dissipation rate does not stabilize, as it would be the case for the normal Brownian diffusion, but instead it diverges logarithmically. Consequently, for peripheral collisions or for strongly elongated shapes from saddle to scission, it is more appropriate to use the time- or coordinate- dependent diffusion coefficient. The prediction of coexistence of the fusion dominated processes with the diffusion process in light heavy-ion collisions is in agreement with the coupled-channel analysis[19] and supported by experiments[20] .

The description of fission as a diffusion over a potential barrier in a collective variable was proposed long time ago by Kramers[21] . Kramers proposed also to use the Langevin equation to study this phenomenon. The slow collective motion with its high mass parameter is treated as a Brownian particle, whereas the fast nucleonic degrees of freedom form the heat bath. The diffusion process is driven by the coupling to the heat bath which models other equilibrated nuclear degrees of freedom, different from the shape degree related to fission. Effects of the rapid

exchange of energy with nucleonic motions are described in two ways: the average effect of the coupling between fission and the heat bath is described as the friction and the residual rapidly varying effect by the Langevin force. The friction then controls average behaviour whereas the fluctuating force produces a distribution around the average. The diffusion constant D is related to the friction constant β through the temperature T via the Einstein relation. The random force is phenomenologically assumed to be a Gaussian random variable with *no time correlations*, satisfying the fluctuation - dissipation theorem :

$$\langle F(t)F(t') \rangle = 2\gamma T \delta(t - t') \quad . \quad (2)$$

In the above equation, γ is the reduced friction ($\equiv \beta/m$), m is the mass associated with the collective fission mode, and T is the temperature of the heat bath.

These concepts have been fully appreciated with the discovery of deep-inelastic heavy-ion reactions[22] . A complete study of the fission process within this formalism was performed more recently [23, 24] and a long transient time was pointed out. Transients in fission, related to the time scale set by the friction constant, lead to higher pre-fission neutron multiplicities [23] than predicted by the statistical model[25] . The effect increases with the energy of the fissioning nucleus and seems to have been well documented experimentally[26, 27] .

The unusual algebraic VACF found in the MD are relevant for the formulation of the stochastic transport theory. To incorporate fluctuations into the semiclassical limit of the extended time-dependent Hartree-Fock approach with the Uehling -

Uhlenbeck collision term, it was proposed to consider the evolution of the single-particle density as a generalized Langevin process[28] . This approach is called usually Boltzmann - Langevin model. The analogy with the Langevin treatment of the fission process is close except that now the stochastic variable is the whole single-particle density and the correlated part of the two-body collisions acts on the single-particle density as a Langevin force characterized by its own correlation function. All parameters of such a generalized Langevin process are explicitly given in terms of the canonical microscopic values [28, 29] . The stochastic force is usually assumed to be δ - correlated in time[28, 30] , although the inclusion of a finite memory time in the stochastic force, which yields the Langevin approximation with the exponential correlated 'colored noise', has been discussed recently to describe coupling between high frequency collective vibrations and single particle degrees of freedom [31] . (For a recent review of stochastic theories with the colored noise see ref [32] .)

The hypothesis of the rapid decay of the force correlations holds when the motion of particles does not depend on the history of the system and , therefore, one may neglect possible correlations between the position of the Brownian particle and the fluctuating force. For central (low angular momentum) collisions, the MD yields fast decaying correlations in both velocity and acceleration (force)[8] . In this regime the conventional Langevin approach is applicable. However, for peripheral and/or strongly elongated shapes, the correlations decay algebraically[8] . Such slowly decaying correlations are known in various phenomena including the chemical reactions

in solutions[33] , ligands migration in biomolecules[34] , atomic diffusion through a periodic lattice[35] , Stark broadening[36] and many others. This regime is certainly beyond the standard Langevin approach and requires the consideration of the colored noise of arbitrarily long correlation time. The first important step in this direction was the theory of line shapes and relaxation in magnetic resonance systems through the study of the so called Kubo oscillator[37] . More recently, the Kubo-Anderson process[38] with the slowly decaying noise correlation function, the so called 'Kangaroo process' [39] , was used to explain the noise-induced Stark broadening[36] . In this paper we propose a new method which extends the Langevin approach for phenomena with either exponentially or algebraically decaying force correlations. In this exploratory studies, we shall investigate a simple two-dimensional Langevin-type equation :

$$\begin{aligned} \frac{d\mathbf{r}}{dt} &= \mathbf{v} \\ m \frac{d\mathbf{v}}{dt}(t) &= -\beta\mathbf{v}(t) - \frac{\partial V(r)}{\partial \mathbf{r}} + \mathbf{F}(t) \quad , \end{aligned} \quad (3)$$

where the potential V generates a conservative force. The friction term does not contain the memory kernel and, therefore, the noise generated by $\mathbf{F}(t)$ is called external.

The aim of the present paper is to study the properties of the Langevin equation for cases which are relevant in nuclear physics and in which the noise $\mathbf{F}(t)$ has algebraically decaying correlations: $\langle \mathbf{F}(0)\mathbf{F}(t) \rangle \sim 1/t$ and $\langle \mathbf{F}(t) \rangle = 0$. Since these conditions do not determine the noise uniquely, its choice is, from the math-

emational point of view, somewhat arbitrary. Physically, however, MD suggests a connection of the fluctuation term $\mathbf{F}(t)$ to the velocity of particle in the periodic Lorentz gas (the extended Sinai billiard). Properties of dynamical system decay appear to be determined by the long-time tails of the velocity autocorrelation function, connected with the existence of long free paths. Consequently, details of interactions involved and, therefore, the method of generation of those correlations, are of minor importance. In the Langevin problem with the exponentially correlated colored noise, one uses the Ornstein-Uhlenbeck process which is given by the second, adjoined, Langevin equation with δ -correlated noise. In the following, we simulate both exponential and algebraic correlated colored noise $\mathbf{F}(t)$ by the time series of particle velocity in the adjoined Sinai billiard. We will solve (3) both with and without conservative force. In the case when the conservative force is present, we consider radial motion for two cases with the circular symmetry : (i) the attractive potential and, (ii) the attractive potential with an external barrier. The main interest is to study the relation between the time evolution of $\mathbf{r}(t)$, $\mathbf{r}^2(t)$, $\mathbf{v}(t)$ and $\mathbf{v}^2(t)$, and the chaotic properties of $\mathbf{F}(t)$. In particular, we are interested in the relation between the FAF for different regimes of the billiard , on one side, and the thermalization property of the particles, their position and velocity distributions and the escape probability from the potential , on the other side. In chapter 2, we present our approach for the Langevin-type evolution of macroscopic variable with the deterministic, algebraically correlated noise. In sect. 2.1 , we briefly present

most important results known for the extended Sinai billiard, which are relevant for our problem. In sect. 2.2 , we discuss the main features of the macroscopic relation between friction force parameter, temperature and the diffusion constant as well as the thermodynamic properties for the correlated Langevin problem. In case of the Sinai billiard with an open horizon which yields the FAF : $\tilde{C}(t) \propto 1/t$, these macroscopic relations are modified with respect to the well known Einstein relations of the ordinary diffusion. We show also that the thermodynamic properties of particles in the correlated Langevin problem, such as the escape velocity and the velocity distribution of particles, are strongly dependent both on the regime of the billiard, and hence on the VAF (FAF), and on the strength of the stochastic and friction forces. The results obtained without the conservative force, with the circular attractive potential and with the external barrier are presented in sects. 3.1, 3.2 and 3.3, respectively. Finally, the conclusions and outlooks are given in chapter 4.

2 Langevin problem for the correlated stochastic force

In generalized Brownian motion theory[40] , the Hamilton equations can be rewritten in the form of a Langevin equation by making use of the projection operator technique. The total force acting on a Brownian particle or a relevant variable such

as the collective coordinate in fission, is divided into a systematic part and a random part. The slowly varying part describes the evolution of macroscopic variables. The fast varying part leads to the fluctuations around the most probable path and has a deterministic nature. In the conventional Langevin approach, it is usually assumed that the time-evolution of the fast varying random part is stochastic and the time rate of change is much faster than that of the systematic part. Consequently, it is assumed that the correlation function of the random part is δ - correlated or decays exponentially. It should be stressed that the random part can be regarded as an effect of some deterministic process. In fact, Markov processes were usually attributed to 'coarse-graining' procedures which, comparing with an underlying deterministic dynamics, lead to the information loss. It is possible, however, to prove the equivalence of both kinds of description, deterministic and stochastic one[41]. Roughly speaking, there exists a non-unitary and chaotic transformation without any loss of information, which is linking the deterministic and stochastic descriptions. On the other hand, the deterministic nature of the random force follows from the derivation in the generalized Brownian motion theory. In particular, both the magnitude of the impulses from the surrounding fluid on Brownian particles and the time interval between two successive collisions τ are deterministic. Moreover, τ is finite, even if it is very small compared with the observation time. The consequences for the Brownian motion of using such a 'deterministic', short-time correlated random driving force, generated by the chaotic sequence of iterates of some map has been

studied recently by Shimizu[42, 43] . In this paper the chaotic force is modeled by the time-sequence of the velocity of a particle in the 2D extended Sinai billiard. Such simulation of fast variables ('molecular environment') allows to study the motion of Brownian particles for broader class of random force correlations than studied by Shimizu[42, 43] . In particular, the algebraically decaying velocity correlations $C(t)$, which have been documented in the MD, can be generated and studied for the first time in the Langevin type of approach.

2.1 Extended Sinai billiard and the correlations for open and close horizons

The extended Sinai billiard (periodic Lorentz gas) consists of a single point particle moving in a two-dimensional array of fixed circular scatterers of radius R [44] . The point mass is scattered elastically from scatterers. We choose the square lattice with a lattice spacing equal to two and periodic boundary conditions. We choose also the particle velocity of unit length. The geometry of the billiard is shown in Fig. 2. The particle is reflected upon hitting arc of hard discs or meets the periodic boundary condition when it crosses the straight-line segment between hard discs. As seen in Fig. 2, the phase space is spanned by the arc length s with $0 < s < L = 2\pi R + 4l$ and by the tangential momentum $p = \cos \alpha$ with $p < 1$. The orbit consists of the succession of pairs $\{s_n(s_0), p_n(p_0)\}$ corresponding to the n th bounce when the initial condition was $\{s_0, p_0\}$. This dynamics is a mapping

M of the phase space $\{s, p\}$ into itself[45] :

$$\begin{pmatrix} s_{n+1} \\ p_{n+1} \end{pmatrix} = M \begin{pmatrix} s_n \\ p_n \end{pmatrix}, \quad (4)$$

with :

$$\frac{\partial(s_1, p_1)}{\partial(s_0, p_0)} = \det \begin{pmatrix} \partial s_1 / \partial s_0 & \partial s_1 / \partial p_0 \\ \partial p_1 / \partial s_0 & \partial p_1 / \partial p_0 \end{pmatrix} = 1. \quad (5)$$

The sequence of iterates (4) is uniquely determined as a function of the initial value.

The separation between disks completely determines the behaviour of the system.

If $R > 1$ ('the high-density regime' of the billiard) then the disks overlap and

the particle is trapped in a region bounded by four arcs of circles. This situation

corresponds to the closed horizon as the particle trajectory is bound. If $R < 1$ ('the

low-density regime' of the billiard) the particle sees an infinite horizon and may

move to an arbitrarily long distance between subsequent collisions, i.e. the length of

free paths is unbounded. At around $R = 1$, the Sinai billiard undergoes a transition

in its dynamics which has common features with the second-order phase transition.

This system is rich enough and its properties are sufficiently generic to provide a

model of the physical system ('the molecular environment'). On the other hand,

it is also simple enough to allow derivation of rigorous results[46]. We choose this

system to model erratic chaotic force acting on Brownian particles.

The extended Sinai billiard, both for $R < 1$ and $R \geq 1$, belongs to the category

of so called K-systems[44], for which the nearby trajectories diverge exponentially

and the metric entropy is positive. The extended Sinai billiard is known to be

ergodic in two dimensions and numerical experiments in higher dimensions also indicate its ergodicity[14] . This means for example, that in two dimensions there exists an invariant asymptotic measure on the full three-dimensional manifold of constant energy. In spite of its very simple definition and its deterministic character, the extended Sinai billiard is an extremely chaotic system where any long time predictions concerning a single trajectory are impossible. So this system can be studied from the statistical point of view, similarly as complex multidimensional systems. Despite the ergodicity, the extended Sinai billiard exhibits long-time correlations which are typically associated with the existence of the KAM tori in the phase space. In the case of Sinai billiard with infinite horizon, there exist families of trajectories which do not collide with hard scatterers and therefore correspond to a regular motion as, for example, the particle motion in the rectangular box. Existence of these families is a reason of long-time tails in the correlation functions. The VACF changes from a stretched exponential decay[14] ($C(t) \sim \exp(-bt^{0.7})$) for $R > 1$ to an algebraic decay[47] ($C(t) \sim t^{-1}$) for $R < 1$. The self-generated diffusion process is Brownian in two dimensions and the diffusion constant exists in the limit $t \longrightarrow \infty$ if the horizon is finite, i.e. for $R > 1$. In case of the open horizon ($R < 1$), the $1/t$ tail of $C(t)$ implies that diffusion is anomalous and $D(t)$ behaves like $\sim \log t$, i.e. it shows a weak divergence. In this case, the distribution of free path lengths for large s is algebraic : $N(s) \sim s^{-3}$ [14].

The VACF with respect to the number of collisions : $C(n) = \langle v(k)v(k+n) \rangle$,

where $v(k)$ denotes the velocity after the k th collision, is : $|C(n)| \leq \exp(-n^\gamma)$ with $0 < \gamma \leq 1$ for a billiard with bounded horizon[46] . This means that in the extended Sinai billiard for $R > 1$ all correlations decay exponentially. In the case of an open horizon, it was conjectured [48] that correlations decay algebraically, as n^{-3} . For small n , the numerical experiments suggest the stretched exponential $C(n) \sim (-1)^n \exp(\kappa(R)n^\gamma)$ rather than the algebraic decay. At around $R = 1$, many quantities, including the probability density of free path length $N(s)$ and VACF's $C(n)$ and $C(t)$, exhibit a critical behaviour which resembles a second-order phase transition. In particular, the correlation length diverges and the length scale disappears. One can also define the order parameter :

$$\lim_{t \rightarrow \infty} \langle r^2(t) \rangle / (t \log t) \quad , \quad (6)$$

which is zero for $R > 1$ and changes to a finite value D_0 for $R < 1$.

2.2 The model

Let us consider a dissipative system which consists of many particles. Each particle obeys a simple damping which is considered to be an intrinsic damping in the system, i.e. independent of the fluctuation term. The 'deterministic' random force acting on a particle in the dissipative system is denoted by $\mathbf{F}(t)$. The force $\mathbf{F}(t)$, generated by the external dynamical system, is called in the following the deterministic chaotic force (DCF). In this work $\mathbf{F}(t) = \varepsilon \mathbf{u}$, where $\mathbf{u}$ is the velocity of particle in the billiard and ε is a force amplitude. Then, in the absence of external potential,

the velocity $\mathbf{v}(t)$ of particle in the dissipative system at time t is governed by the equation :

$$\frac{d\mathbf{v}}{dt}(t) = -\gamma\mathbf{v}(t) + \frac{1}{m}\mathbf{F}(t) \quad , \quad (7)$$

where γ is a reduced friction (damping) constant.

At each collision with the circular scatterer, the direction of the particle velocity changes what corresponds to the update of the chaotic force $\mathbf{F}(t)$. Otherwise the value of the force remains constant. The collisions are elastic so the length of the velocity vector remains constant. In the following discussion we take for the radius of the scattering disks always $R = 0.3$ and 1.2 , in the case of open and closed horizons, respectively. The distance between scattering centers is then $l = 2 - 2R$. The Eq. (7) is solved for the ensemble of initial conditions of the chaotic force. They are obtained by uniform sampling of positions of the particle inside the billiard, and of particle momentum components on the unit circle. Any final quantity $\langle A \rangle$ which is a function of $\mathbf{r}$ and $\mathbf{v}$, is obtained by averaging over this statistical ensemble. Operationally, we solve the Langevin Eq. (7) independently for any initial condition of the chaotic force (the Sinai billiard) and at any instant of time t we take the average for any observable $\langle A(\mathbf{r}(t), \mathbf{v}(t)) \rangle$.

3 Macroscopic relations and thermodynamic properties for the correlated Langevin problem

3.1 $V = 0$: The free particle case

Let us begin with a simple example of Brownian motion in the absence of any potential. If the friction force is absent then the velocity fluctuations grow extremely fast. In the case of the open horizon, when the free path of particle in the billiard may be very long, the Brownian particle is subjected to constant acceleration, in the same direction, for a long time. This leads to the behaviour which resembles the Levy flights. The diffusion of Brownian particles is rapid and the mean square displacement grows anomalously fast. The dependence of $\langle \mathbf{r}^2 \rangle$ on time (see Fig. 3) is $\langle \mathbf{r}^2 \rangle \sim t^\delta$ with $\delta \simeq 3.1$ and resembles the Richardson's law of a turbulent diffusion[49]. In this case, Eq. (7) is solved with the initial condition for the Brownian particle $\mathbf{u}(t=0) = 0$ and $x(t=0) = y(t=0) = 0$. The velocity variance $\langle \mathbf{v}^2 \rangle(t)$ for Brownian particles, can be calculated from the FACF $\tilde{C}(t)$, using the Green - Kubo formula. In the case of the open horizon, $\tilde{C}(t) \sim t^{-1}$ and, therefore, $\langle \mathbf{v}^2 \rangle(t) \sim t \log t$.

We will be mainly interested in the case when the intrinsic friction force is present. In the overdamped limit where the friction constant is large, the velocity correlations of the Brownian particle are evidently identical with those for the DCF. The mean square displacement $\langle \mathbf{r}^2(t) \rangle$ for $\gamma = 1$ and $\varepsilon = 1$ is shown in

Fig. 4. The chaotic force is generated by the billiard with the open horizon. In this case $\langle \mathbf{r}^2(t) \rangle$ is proportional to $t \log t$, i.e. the diffusion process is weakly (logarithmically) anomalous and the diffusion coefficient $D(t) \equiv (1/4) \langle \mathbf{r}^2(t) \rangle / t$ is $D(t) = D_0 \log t$. Existence of a finite intrinsic friction ($\gamma \neq 0$) leads to the equilibration, i.e. $\langle \mathbf{v}^2(t) \rangle$ reach the asymptotic value $\langle \mathbf{v}^2 \rangle_{eq}$ at later times. The equilibration rate depends obviously on the value of the intrinsic friction constant. $D(t)$, $\langle \mathbf{v}^2 \rangle_{eq}$, γ and ε are related through the equation :

$$\frac{m}{2} \frac{d^2}{dt^2} \langle \mathbf{r}^2 \rangle - m \langle \mathbf{v}^2 \rangle = -\frac{\gamma}{2} \frac{d}{dt} \langle \mathbf{r}^2 \rangle + \langle \mathbf{r} \mathbf{F} \rangle \quad . \quad (8)$$

For the conventional Langevin problem $\langle \mathbf{r} \mathbf{F} \rangle = 0$ and the above equation yields the relation linking the temperature $T = \langle \mathbf{v}^2 \rangle_{eq} / 2m$, γ and D : $D = T/\gamma$ (the Einstein relation). In our approach, $\langle \mathbf{r} \mathbf{F} \rangle(t) \neq 0$ and the relations among $D(t)$, $\langle \mathbf{v}^2 \rangle_{eq}$, γ and ε are obtained by comparing independently the time-dependent and the time-independent terms in (8). Keeping in mind the dependence of $\langle \mathbf{r}^2 \rangle$ on time in the open horizon case, we get terms which are either constant or proportional to $\log t$. This means that there are two equations relating $D(t)$, $\langle \mathbf{v}^2 \rangle_{eq}$, γ and ε . In a broad range of friction force parameters ($0.01 \leq \gamma \leq 10$) we find :

$$D_0 \sim \frac{\varepsilon^2}{\gamma^2} \quad . \quad (9)$$

The dependence of the velocity variance $\langle \mathbf{v}^2 \rangle$ on time is shown in Fig. 5 for $\gamma = 0.01$ and $\varepsilon = 1$. Fig. 6 shows the dependence of $\langle \mathbf{v}^2 \rangle_{eq}$ on the strength of

the friction force in a broad range of γ - values : $\gamma \geq 0.01$. This dependence can be parameterized by the function :

$$\langle \mathbf{v}^2 \rangle_{eq} = \varepsilon^2 \left(\frac{1}{\gamma^2} - \frac{c_1 \exp(-c_2 \gamma)}{\gamma^{c_3}} \right) , \quad (10)$$

with the optimal parameters $c_1 = 0.5$, $c_2 = 0.75$ and $c_3 = 2.187$. The fit is particularly impressive for large γ in the overdamped limit.

In the case of closed horizon $\langle \mathbf{r}^2 \rangle$ depends linearly on time and $\langle \mathbf{r} \mathbf{F} \rangle$ is a negative constant. Thus, (8) gives the ordinary Einstein relation but the temperature T is slightly larger then the average kinetic energy $\langle \mathbf{v}^2 \rangle_{eq} / 2m$. This difference diminishes with γ and in the limit of large γ one finds $\langle \mathbf{r} \mathbf{F} \rangle = 0$, as for the conventional Langevin problem.

3.2 Particles in the circular attractive potential well

In this section we shall consider the motion of the Brownian particle in the circular attractive potential. This example may be considered as an idealization of the evaporation process from, for example, an excited atomic nucleus or a polyatomic molecule. The potential well is defined as :

$$\begin{aligned} V(r) &= V_0 \left(1 - \left(\frac{|\mathbf{r}|}{r_B} \right)^2 \right) & \text{for } |\mathbf{r}| \leq r_B \\ V(r) &= 0 & \text{for } |\mathbf{r}| > r_B \end{aligned} , \quad (11)$$

where V_0 and r_B are the depth and the radius of the potential, respectively. Inside the potential, the motion of particle is given by the Langevin equation (3) with the

correlated DCF (4) . Otherwise the particle is free. Initially ($t = 0$), the particle rests at the bottom of the well ($|\mathbf{r}| = 0$). At later times, the chaotic force $\mathbf{F}(t)$ accelerates the particle which may eventually escape from the well. $\mathbf{F}(t)$ fluctuates according to the deterministic rule of the billiard what leads to the stochastization of the Brownian particle motion.

The finiteness of the time interval between two subsequent variations of the chaotic force, which is a characteristic difference of the present approach with respect to the conventional Langevin problem, makes possible the escape of particle from the potential well before their motion is randomized due to multiple kicks (fluctuations) of the DCF. We may call these particles 'pre-randomized'. Their energy distribution depends on the relative magnitude of the chaotic force as compared to the size of the potential (depth and radius) and the strength of the intrinsic friction. Fig. 7a shows the energy distribution $P(E)$ for particles escaping from the container in the case when the DCF is generated by billiard with the open horizon. Since the particle motion outside the container is free, this distribution is equivalent to the asymptotic distribution at $|\mathbf{r}| = \infty$. Results shown in Fig. 7a have been obtained for $\varepsilon = 1, \gamma = 0.02$ and the parameters of the potential are $V_0 = -40, r_B = 50$. The mass of the particle is taken to be 1 . The distribution $P(E)$ has a form of a sharp peak superimposed on a broad structure. This peak corresponds to trajectories subjected to the constant acceleration due to the chaotic force (free path in the Sinai billiard), until the border of the potential is reached. The amplitude ε of the

accelerating force, the friction force slowing down the particle, and the parameters of the external container determine the position of this peak for pre-randomized particles. It moves towards smaller energies with increasing both the depth of the potential V_0 and the friction force constant γ . In our case, the peak for pre-randomized particles disappears for $V_0 \leq -50$. The broad structure in $P(E)$ is given by 'randomized' particles. The shape of this bump (see the dashed curve in Fig. 7a) is Gaussian : $P(E) \sim \exp(-E^2/2\sigma^2)$ with $\sigma = 60.5$. It should be stressed that the energy tail for randomized particles is not exponential, as one would expect for the thermally equilibrated system.

For a fixed potential $V(r)$, the decrease of an amplitude of the DCF leads to the increase of an average number of the DCF fluctuations before the trajectory of the Brownian particle leaves the potential. The energy distribution $P(E)$ for a small amplitude DCF ($\varepsilon = 0.25$) is plotted in Fig. 7b. Similarly as in Fig. 7a, the DCF is generated by the billiard with the open horizon. In this case, $P(E)$ decreases exponentially for large energies as expected for an 'equilibrated' system. Fig. 8 shows the distribution $P(E)$ in the case of the closed horizon. The friction parameter is $\gamma = 0.001$. Parameters of both the DCF and the spherical potential well are the same as chosen before for the open horizon (Fig. 7a). The time-correlations decay fast and the length of the free path between the two subsequent DCF fluctuations is always bounded. As a result, one never finds the peak for pre-randomized particles and the energy distribution $P(E)$ is always exponential. It should be noticed that

the effective viscosity is significantly smaller when the horizon is open than when it is bounded. In the former case ($\tilde{C}(t) \sim 1/t$), dissipation mechanism is associated with the presence of the long free paths between successive collisions and the size of the attractive potential yields a natural cutoff scale for the length of the particle orbit, in a somewhat similar way as for the one-body dissipation mechanism[50] based on the concept of a Knudsen gas[51]. In the latter case ($\tilde{C}(t) \sim \exp(-\alpha t^{\alpha'})$), the dissipation is similar to the two-body hydrodynamic dissipation[52] and results from a short mean free path of the Brownian particles.

The form of the energy distribution of particles escaping from the container depends on the correlation properties of the DCF. One finds a sharp peak superimposed on a broad Gaussian distribution if FDCF decays algebraically ($\sim 1/t$) and the amplitude of the DCF is sufficiently large, so that the average number of fluctuations of the DCF preceding emission of the particle is small. The decrease of an amplitude of the DCF, leads to an increase of the average number of the DCF fluctuations which are necessary for the particle emission to happen. The potential cavity enslaves particles for a long time and the long-time correlations, resulting from the openness of the horizon, cannot be seen easily. For an external observer, this phenomenon is hardly distinguishable from the effect of the short-time correlated chaotic force. The energy spectrum of emitted particles becomes in this case exponential, as should be for the emission from thermally equilibrated system. As said before, a similar particle enslavement can be achieved when the FDCF decays fast

(exponentially), i.e. when the horizon is closed. This enslavement is independent of both the size of the potential and the amplitude of the DCF, and the exponential tail of $P(E)$ is generic in this case.

The escape from the potential requires acceleration by the chaotic force to climb the well. Frequent collisions with the scatterers (frequent changes of the applied force) reduce the mean acceleration. In the absence of an external potential, all trajectories in the billiard, both long and short ones, provide a sufficient change of the particle momentum to allow its escape. All those particles which by chance appear sufficiently close to the distance $r = r_B$ from the center (the absorbing barrier), may escape. The survival probability of the system is thus proportional to the number of particles, $N(t)$, still inside the potential well at a time t . The time-variation of this number depends only on the phase-space density of particles which is just proportional to the number of particles : $\dot{N}(t) \sim N(t)$, i.e. $N(t) = N(t=0) \exp(-at)$ and a is a constant. $N(t)$ is shown in Figs. 9 and 10. In the case of FACF decaying algebraically ($\sim 1/t$) and for sufficiently large amplitude of the DCF, $N(t)$ approaches its asymptotic behaviour $\sim 1/t$ (see Fig. 9a) in a short time. The potential $V(r)$, ε and γ are the same as in Fig. 7a. On the contrary, the survival probability for small ε decays first exponentially (see Fig. 9b), following an analogous tendency in the energy distribution $P(E)$ of escaping particles (see Fig. 7b) and the algebraic $\sim 1/t$ tail in the survival time distribution begins to be visible only after a very long time. Finally, for the exponentially decaying FACF,

$N(t)$ decreases also exponentially : $N(t) \sim \exp(-\Gamma t)$ (see Fig. 10) and the decay width is $\Gamma = 2.5 \cdot 10^{-4}$. Hence, there is a close interrelation between the behaviour of the survival probability and the energy distribution of emitted particles and, moreover, both these distributions depend on the nature of the chaotic force.

To understand this behaviour more in details, let us investigate features of the ensemble of trajectories in the billiard corresponding to those Brownian particles which survive inside the circular container up to time t . As said before, the initial conditions of the chaotic force acting on each Brownian particle, are obtained by uniform sampling of the positions of the particle inside the billiard, and of the momentum components of the particle on the unit circle. At later times, we keep account of only those trajectories in the billiard for which the corresponding Brownian trajectory is found inside the potential, i.e. we count only the surviving particles. Without this constraint, as described in sect. 2.1, the distribution of path lengths would depend only on whether the horizon is closed or open and the average path length would be constant in time. However, since the escape probability may not be the same for trajectories driven by long and short free paths in the billiard, that distribution can be different if we consider only particles inside the container. The mean free path in the extended Sinai billiard for those surviving particles, $\langle s \rangle_B$, is plotted as a function of time in Fig. 11. The horizon is open and the calculations are done for the two sets of parameters : $\varepsilon = 1$, $\gamma = 0.02$ (the solid line) and $\varepsilon = 0.25$, $\gamma = 0.001$ (the dashed line). In both cases the parameters of the poten-

tial are $V_0 = -50$ and $r_B = 50$. $\langle s_B \rangle(t)$ changes qualitatively with the strength ε of the DCF. For the weak DCF ($\varepsilon = 0.25$), $\langle s \rangle_B$ grows extremely slowly in time and in the considered range of times this growth is hardly visible. On the contrary, in the case of the strong DCF ($\varepsilon = 1$), $\langle s_B \rangle(t)$ grows fast. The first case corresponds to the exponential distributions of both the energy of emitted particles $P(E)$ (Fig. 7b) and the survival probability $N(t)$ (Fig. 9b) though, in the latter case, the algebraic $1/t$ -tail emerges at late times. The case of the strong stochastic force ($\varepsilon = 1$) is connected with the Gaussian energy distribution $P(E)$ (Fig. 7a) and the algebraic tail $\sim 1/t$ in $N(t)$ (Fig. 9a).

In the absence of the potential or in the case of a very weak DCF, both short and long free paths in the billiard have equal chance to bring the particle outside the spherical cavity. Therefore, the ratio between short and long trajectories in the billiard for surviving Brownian particles remains constant in time. The time-variation of $N(t)$ is then proportional to the phase-space density of particles and, hence, it decreases exponentially in time. For strong DCF situation is quite different. Here, the relative number of long free paths rapidly increases in time. In order to understand the reason of this growth, we plot a typical trajectory, corresponding to long free path in the billiard (Fig. 12). It appears to be very regular. The constant DCF and the conservative force from the potential act like the gravitation and elastic forces in the simple problem of the oscillatory motion of a string. The friction force slows down the Brownian particle which finally stops at the point where the con-

stant DCF exactly compensates the conservative force[53] . Since such trajectories do not have much chance to escape, therefore their weight in the ensemble increases and $\langle s_B \rangle$ grows in time. The stoppage of Brownian particle, favouring long free paths, is decisive for its properties. The Brownian particle is at standstill as long as the particle in the billiard stays on a free path. At some time however, the particle in the billiard hits the scatterer and the long path finishes. This changes the balance of forces and the Brownian particle begins to move irregularly. The decay process starts again from $v_x = v_y = 0$ and, usually, one observes a rapid stochastization of the particle motion and the particle escapes. The long-time decay can therefore be considered as starting from some quasi-stationary distribution of motionless particles, corresponding to long paths in the billiard. As long as the particle in the billiard continues its free motion, the balance of forces ensures that the Brownian particle remains inside the absorbing barrier. The larger becomes deviation from the free path direction, the better is a chance that the Brownian particle escapes. Therefore, the survival probability up to time t is just proportional to the average projection of the velocity of particle in the billiard on its initial velocity, connected with the free path direction. i.e. to the velocity autocorrelation function $C(t) \sim 1/t$. For $\varepsilon = 0.25$, $\langle s_B \rangle$ grows slowly because the stochastization coming after the termination of the long free path in the billiard is weak and has a small chance to kick the particle out of the potential well. The particle is driven by many short free paths and either escapes or the free path becomes long again. In the latter case,

the whole process starts from the beginning. Eventually, however, $\langle s_B \rangle$ becomes large, long free paths prevail and, asymptotically, the survival probability turns into the algebraic shape.

For $\varepsilon = 0.25$, if time is not too large, and in the case of the short-time correlations (closed horizon), short free paths in the billiard dominate. The system is in the equilibrium. The velocity of a particle coming to the absorbing barrier is a sum of a large number of small random variables (the Sinai billiard loses memory fast) and, therefore, its distribution is Maxwellian. On the other hand, for $\varepsilon = 1$, the decay process starting at later times from the configuration of particles at rest is fast because DCF is sufficiently strong to bring the trajectory to the absorbing barrier. Usually, a small number of its fluctuations is enough for that. Hence, the final velocity of a particle at the absorbing barrier is the sum of small number of components, which yields approximately the Gaussian kinetic energy distribution $\sim \exp(-const v^4)$. This form of the energy distribution results from the apparent pre-equilibrium character of the emission process. Summarizing, for the strong DCF generated by the billiard with open horizon, the decay times and the energy spectra of escaping particles result from the dominance of long free paths. In spite of long life-times, the deexcitation process has features of the 'fast' process due to the liberation of 'frozen' trajectories.

3.3 Passage of the particles over the potential barrier

In this section we shall investigate the motion of the Brownian particle over the potential barrier as a simple model of fission. We shall consider the dissipative dynamics towards 'scission' for the DCF (4) generated by the extended Sinai billiard with both closed and open horizons. The potential is defined as :

$$\begin{aligned} V(r) &= V_0(1 - (\frac{|\mathbf{r}|}{r_B})^2) + V_1 & \text{for } |\mathbf{r}| \leq r_B \\ V(r) &= V_1(1 - (\frac{|\mathbf{r}|}{r_{SC}})^2) & \text{for } r_B < |\mathbf{r}| < r_{SC} \\ V(r) &= 0 & \text{for } |\mathbf{r}| > r_{SC} \end{aligned} \quad (12)$$

Let the particle be initially at the bottom of the potential well ($|\mathbf{r}| = 0$) at $t = 0$. At later times, the DCF $\mathbf{F}(t)$ moves the particle away from its ground state. The particle is accelerated and may have eventually enough energy to reach the top of the barrier $V(r) = V_1$ at $|\mathbf{r}| = r_B$. For $|\mathbf{r}| > r_B$, the potential is down sloping accelerating particles towards the 'scission' at $|\mathbf{r}| = r_{SC}$. The motion continues to be erratic and the DCF may decelerate or even turn back the particle into the attractive region for $|\mathbf{r}| < r_B$, though the probability of such an event is small. In its attractive part, the shape of the potential (12) is identical to the one considered before (11) in sect. 3.2. For $|\mathbf{r}| \geq r_{SC}$, particle moves freely. Hence, taking different measures of the particle distribution at $|\mathbf{r}| = r_{SC}$ is equivalent to determining its asymptotic distribution at $|\mathbf{r}| = \infty$. In the following, we assume $r_B = 50$ and $r_{SC} = 150$.

Fig. 13 shows the energy distribution of particles $P(E)$ at scission in the case when the DCF is generated by billiard with the open horizon. Results have been obtained for $\varepsilon = 1$, $\gamma = 0.02$. The depth of the attractive part is either $V_0 = -50$ (the solid line) or $V_0 = -80$ (the dashed line) and the barrier height is $V_1 = 5.6$. The asymptotic distributions $P(E)$ at $|\mathbf{r}| = r_{SC}$, both for $V_0 = -50$ and $V_0 = -80$, exhibit a peak at $E \simeq 80$ for pre-randomized particles which is superimposed on a broad structure. This peak corresponds to trajectories subjected to the constant acceleration towards scission due to the chaotic force (long path in the Sinai billiard) in region of an external barrier. Obviously, the position of the peak depends not only on the amplitude ε of the accelerating force and the magnitude of friction force slowing down the particle but also on the size of the barrier. The broad structure in $P(E)$ for large E has a Gaussian shape : $P(E) \sim \exp -(2\alpha E + E^2)/2\sigma^2$ with $\alpha = 82.5$ and $\sigma = 65$. The distribution $P(E)$ is given approximately by the convolution of two Gaussian distributions : $P_1(\tilde{E}_0)$ corresponding to the energy distribution of particles at the barrier ($|\mathbf{r}| = r_B$), and $P_2(E, \tilde{E}_0)$ corresponding to the conditional probability of finding the particle with energy E at $|\mathbf{r}| = r_{SC}$ if its initial energy at $|\mathbf{r}| = r_B$ was $\tilde{E}_0$. Width of this distribution is close to the one shown in Fig. 7a without the external barrier. Similarly as seen before in Fig. 7a, the energy spectrum $P(E)$ differs from the spectrum of the thermally equilibrated system. It is interesting to notice that $P(E)$ is almost independent of the depth of the attractive part of the potential.

The decrease of magnitude of the DCF leads to the increase of average number of the DCF fluctuations which are necessary for the particle emission to happen. The energy distribution of particles $P(E)$ at scission for the weak DCF ($\varepsilon = 0.25$) is shown in Fig. 14. Results have been obtained for $\gamma = 0.001$ and the depth of the attractive part of the potential is $V_0 = -50$. The asymptotic distribution at $|\mathbf{r}| = r_{SC}$ has again a Gaussian shape : $P(E) \sim \exp -(2\alpha E + E^2)/2\sigma^2$ with $\alpha = 11.3$ and $\sigma = 20.2$. This shape is a convolution of the exponential energy distribution of particles $P_1(\tilde{E}_0)$ at $|\mathbf{r}| = r_B$ (see Fig. 7b) and the Gaussian distribution $P_2(E, \tilde{E}_0)$ at $|\mathbf{r}| = r_{SC}$ with the supplementary condition that the initial energy at $|\mathbf{r}| = r_B$ was $\tilde{E}_0$. In the barrier region, the conservative potential is repulsive and the long free paths may show up independently of the amplitude of the DCF. Therefore, the form of $P(E)$ as shown in Fig. 14 is generic for any weak DCF generated by the billiard with open horizon.

Fig. 15 shows the distribution $P(E)$ in the case of the closed horizon. The friction parameter is $\gamma = 0.001$. Amplitude of the DCF is $\varepsilon = 1$ and the attractive circular potential has the same depth $V_0 = -40$ as in the analogous case without an external barrier (see Fig. 8). Barrier height is $V_1 = 5.6$. The time-correlations decay fast. The length of the free path between two subsequent DCF fluctuations is bounded and the energy distribution $P(E)$ is always exponential. Notice also that the velocities of particles are significantly smaller than in the case of open horizon because strong accelerations due to the long free paths in the billiard are

now lacking.

Particles which reached the top of the barrier at $|\mathbf{r}| = r_B$ may further reach the scission point even in the absence of the DCF, i.e. the down-sloping potential for $r > r_B$ introduces a net flow towards scission. The DCF provides an erratic component superimposed on this coherent motion and may either further accelerate particles down the potential hill or decelerate or even return them back to the attractive region. Frequent collisions with the scatterers (frequent changes of the applied force) reduce the mean acceleration and system has more chance to equilibrate. The survival probability of the system is proportional to the number of particles, $N(t)$, still inside $r = r_{SC}$ at a time t . The time-variation of this number is shown in Figs. 16, 17 and 18. In the case of FDCF decaying algebraically ($\sim 1/t$) and for sufficiently large amplitude of the DCF, $N(t)$ approaches fast its asymptotic dependence $\sim 1/t$ (see Fig. 16), independently of the depth of the attractive part of the potential. The potential $V(r)$, ε and γ are the same as in Fig. 13. The survival probability for small ε decays first exponentially (see Fig. 17), following an analogous tendency in the energy distribution $P(E)$ of the escaping particles (see Fig. 14), and an algebraic tail $\sim 1/t$ in this distribution is visible only asymptotically. Finally, for the exponentially decaying FDCF, $N(t)$ decreases exponentially (see Fig. 18) with the decay width $\Gamma = 2.3 \cdot 10^{-4}$, which is similar to the case without an external barrier. The features found in the case of the potential with the external barrier are qualitatively similar to those corresponding to the emission from the spherical

container and discussed in sect. 3.2. The behaviour of both the survival probability and the energy distribution are closely interrelated and depend directly on the nature of the chaotic force. The features of the escape probability find their explanation in the properties of the trajectories in the billiard, in particular the long ones, which were used to generate the DCF.

4 Summary and Outlook

Close collisions of heavy ions, in which nuclei experience a considerable overlap of their density distributions, may lead either to the formation of a compound nucleus or to the deeply inelastic dissipative processes in which the system breaks apart before the compound nucleus is formed, i.e. before a complete statistical equilibrium is achieved. In this latter case, there exist 'collective' degrees of freedom which relax very slowly as compared to the relaxation of single particle degrees of freedom or to the contact time of colliding heavy ions. This process is associated with the mass transfer, kinetic energy loss and angular momentum dissipation. The phenomenological description of slowly relaxing, collective degrees of freedom employs the transport equations or the Langevin equations[54]. With respect to the collective variables, the dissipative collisions are similar to the induced fission in the highly excited nucleus and, indeed, the Langevin approach has been applied for the description of this process for a long time[21]. Irreversibility of dissipative processes is

related to the memory loss, quantitatively described by various time-dependent correlation functions. Analyzing those functions is important because they determine essentially the transport properties, in particular the diffusion coefficients.

Recently, the velocity autocorrelation function has been determined in the MD approach with the realistic nuclear potential[8] and the slowly decaying algebraic velocity and force correlations have been demonstrated for peripheral collisions of nuclei. The decay of both VACF and FACF seems to be universal ($\sim t^{-\gamma}$ with $\gamma = 1$) and originates from the long free paths between collisions, similarly as in the strongly chaotic (ergodic) extended Sinai billiard with open horizon. This means also that the decay of the force (velocity) correlations is independent of both the details of the potential, in particular its short range features, and the fermionic/bosonic nature of the particles involved[55]. In this work we have shown how such slowly decaying correlations can be incorporated in the Langevin approach which is a usual framework for the description of the dissipative reactions and/or the induced fission of hot nucleus. The main problem is the generation of the properly correlated noise which drives a Brownian particle in each microscopic realization of its trajectory. It was solved by introducing the adjoined extended Sinai billiard, for which the VACF of the particle in the billiard is known to decay either slowly (algebraically : $\sim 1/t$) or fast (exponentially), depending on whether the horizon is open or bounded. In each event of the Langevin evolution, the erratic Langevin force is proportional to the instantaneous velocity of the particle in the adjoined billiard. This procedure

should be relevant for a broad class of problems, involving correlations, in nuclear physics, condensed matter physics and chemistry.

The way of generating the chaotic force in the Langevin equation , as described in the present work, is attractive also from the conceptual point of view as it provides a unified procedure of simulating the molecular environment in a theory of the Brownian motion. In one and the same theoretical framework, one may be able to describe dissipative systems in which viscosity is associated either with fast decaying temporal correlations and short mean free paths of particles, like for the hydrodynamical two-body viscosity mechanism[52] , or with slowly (algebraically) decaying temporal correlations and the dominance of long mean free paths. The latter case resembles somewhat the one-body dissipation mechanism for rarefied (Knudsen) gas[50] with a mean free path comparable or larger than the dimension of the container, although it differs from the Knudsen gas by the salient features of the VACF (FACF). The correspondence between this one-body dissipation mechanism, in particular the window dissipation, and the dissipation mechanism discussed in this work which is associated with long-time auto-correlation of the DCF, will be published separately[56] .

The adjoined extended Sinai billiard determines the overall time scale of the force fluctuations. The Langevin force is defined by means of the particle velocity in the billiard and, hence, it is deterministic. This property of the force can be elucidated from the derivation of the Langevin-type equation from the Hamilton

equation[40] . In our exploratory study, we assumed for simplicity that the amplitude of the Langevin force is constant. The generalization of the above scheme to the force of a variable amplitude is straightforward. The friction force was assumed to be an intrinsic property of the system, unrelated to the properties of the driving noise. Hence, there is no fluctuation - dissipation theorem. Nevertheless, for any initial condition in the Langevin equations, system drives towards an asymptotic stable state and the relations between the diffusion coefficient D_0 , the intrinsic friction force constant γ , the amplitude of the Langevin force ε and the average kinetic energy in this asymptotic stable state $\langle \mathbf{v}^2 \rangle_{eq}$, can be determined. The studies of the memory effects in the friction term which are generated by the long-time correlated Langevin force and can be consistently described within a framework of the generalized Brownian motion theory[40] , will be published elsewhere[56] .

For particles escaping from the attractive potential, either with or without external barrier, we have found an entangled relation between the FCF of the DCF, on one side, and both the survival probability and the asymptotic energy distribution of particles, on the other side. For fast (exponentially) decaying FCF, the survival probability decays exponentially and the asymptotic energy distribution of particles is always Maxwellian. For the FCF decaying as $\tilde{C}(t) \sim 1/t$, the situation is more involved. For a shallow potential , the asymptotic energy distribution of escaping particles exhibits a pronounced peak corresponding to pre-randomized particles which are associated with long trajectories in the adjoined billiard with the open

horizon and leave the potential without any collision with particles of the molecular environment. With increasing depth (size) of the potential, this peak is shifted gradually to lower energies and finally it disappears. The peak for pre-randomized particles is superimposed on top of the Gaussian distribution. The Gaussian shape of the energy distribution is connected with the randomized particles. They can stay inside the potential well for a long time, never reaching the equilibration state. Decreasing magnitude of the fluctuating force leads, first of all, to the disappearance of the 'pre-randomized peak' and, moreover, the smooth part changes its shape gradually from the Gaussian distribution to the Maxwellian one. In the survival probability distribution one sees a similar tendency accompanying the decrease of the magnitude of the Langevin force. In the case of a strong force, the survival probability approaches the asymptotic $\sim 1/t$ dependence in a short time. With decreasing magnitude of the Langevin force, the exponential modifications of this dependence show up for short and intermediate times and the algebraic $1/t$ -tail is seen only asymptotically. The passage time from exponential to algebraic regime of the decay law moves gradually towards higher values with decreasing magnitude of the DCF.

The simple laws relating the FACF to the properties of both the decay probability of the system $p(t)$ and the asymptotic energy distribution $P(E)$ are challenging. For peripheral collisions of particle aggregates such as heavy nuclei, the diffusion is anomalously enhanced and the diffusion coefficient grows from $D(t_0) = 0$ logarith-

mically in time. Hence, depending on the duration time of the process, the time-averaged diffusion constant may exhibit considerably different values. From this stand point, the long-lived orbiting dinuclear complexes, found in medium-heavy ion reactions, are particularly attractive for theoretical studies. On the one hand, classical MD and Langevin studies of this work predict characteristic dependencies for $N(t)$ and $P(E)$, associated with the anomalous diffusion. On the other hand, small number of open decay channels, which one expects in these configurations, yields by independent quantal arguments the algebraically decaying survival probability $N(t)$. The correspondence between these two formulations, if any, remains an intriguing open question.

The induced fission is also important for our discussion because in this process the shape changes from the compact configurations, where a normal diffusion is expected, to the strongly elongated configurations, close to the scission point, where the geometrical conditions are similar to those of the peripheral collisions and where the diffusion could be anomalously enhanced. One would then expect different time-averaged diffusion coefficients and, hence, different dissipations for low and high fissility systems which are characterized by substantially different path length from saddle to scission. Such a tendency has been suggested in the simultaneous analysis of the fission probabilities and the prescission neutron multiplicities in the hybrid Langevin model of Fröbrich et al.[57]. The existing data on prescission neutron multiplicities and fission fragment kinetic energy are clearly incompatible

with hydrodynamical two-body viscosity and hence with the fast decay of the temporal correlations. A much better description is obtained using the one-body/long-path type dissipation[58] . This is an indication that the dissipation mechanism for strongly elongated shapes, proposed in the present work, could be the correct one. Much systematic work has still to be done to address appropriately this challenging open problem.

Acknowledgements

The authors wish to thank Y. Abe, S. Ayik and P. Danielewicz for their interest in the present work and useful comments. The work was partly supported by KBN Grant No. 2 P302 157 04.

References

- [1] K. Möhring, T. Srokowski and D. H. E. Gross, *Nucl. Phys.* **A533** (1991), 333.
- [2] S. Drożdż, J. Okołowicz and M. Płoszajczak, *Phys. Lett.* **109B** (1982), 145;
E. Caurier, B. Grammaticos and T. Sami, *Phys. Lett.* **109B** (1982), 150;
H. Feldmeier, *Nucl. Phys.* **A515** (1990), 147;
A. Ono, H. Horiuchi, T. Maruyama and A. Ohnishi, *Phys. Rev. Lett.* **68** (1992), 2898.
- [3] W. Bauhoff, E. Caurier, B. Grammaticos and M. Płoszajczak, *Phys. Rev.* **C32** (1985), 1915;
T. Maruyama, A. Ono, A. Onishi and H. Horiuchi, *Prog. Theor. Phys.* **87** (1992), 1367;
H. Horiuchi, *Kyoto Univ. Preprint* KUNS 1283;
M. Belkacem, V. Latora and A. Bonasera, *Phys. Lett.* **B326** (1994), 21;
V. Latora, M. Belkacem and A. Bonasera, *Phys. Rev. Lett.* **73** (1994), 1765;
S. Pratt, C. Montoya and F. Ronning, *Phys. Lett.* **B349** (1995), 261.
- [4] M. Baldo, E. G. Lanza and A. Rapisarda, *Chaos* **3** (1993), 691;
C. H. Dasso, M. Gallardo and M. Saraceno, *Nucl. Phys.* **A549** (1992), 265.
- [5] T. Srokowski, J. Okołowicz, S. Drożdż and A. Budzanowski, *Phys. Rev. Lett.* **71** (1993), 2867;
S. Drożdż, J. Okołowicz, T. Srokowski and A. Budzanowski, *Phys. Rev.* **C51** (1995), 2630.
- [6] Y.-C. Lai, R. Blümel, E. Ott and C. Grebogi, *Phys. Rev. Lett.* **68** (1992), 3491.

- [7] C. H. Lewenkopf and H. A. Weidenüller, *Phys. Rev. Lett.* **68** (1992), 3511.
- [8] T. Srokowski and M. Płoszajczak, *Phys. Rev. Lett.* **75** (1995), 209.
- [9] This finding is supported by the observation of the powerlaw decay $p(t) \sim t^{-z}$ for the quasi-elastic reaction $^{12}\text{C} + ^{12}\text{C}$ at grazing collisions[5] .
- [10] The same logarithmic power spectrum has been found for the Lorentz gas of hard discs (the extended Sinai billiard) [11]. The similarity of the diffusive behaviour for systems as different as the MD and the Lorentz gas of hard discs, follows from the fact that the powerlaw tail of the velocity autocorrelation function is due to the existence of long free paths. This behaviour is insensitive to the details of the potential, in particular to its short distance features.
- [11] A. Zacherl, T. Geisel, J. Nierwetberg and G. Radons, *Phys. Lett.* **114A** (1986), 317.
- [12] D. A. Mc Quarrie. "Statistical Mechanics" , Harper & Row, New York, 1976.
- [13] J. Machta and R. Zwanzig, *Phys. Rev. Lett.* **25** (1983), 1959.
- [14] J. P. Bouchaud and P. L. Le Doussal, *J. Stat. Phys.* **41** (1985), 225.
- [15] R. M. Anjos, N. Added, N. Carlin, L. Fante, Jr., M. C. S. Figueira, R. Matheus, E. M. Szanto, C. Tereiro and Szanto de Toledo. *Phys. Rev.* **C49** (1994), 2018.
- [16] H. L. Harney, F.-M. Dittes and A. Müller, *Ann. Phys. (N.Y.)* **220** (1992), 159.
- [17] C. H. Lewenkopf and H. A. Weidenmüller, *Ann. Phys. (N.Y.)* **212** (1991), 53;
F. M. Izrailev, D. Saher and V. V. Sokolov, *Phys. Rev.* **E48** (1994), 130.
- [18] D. Shapira, R. Novotny, Y. D. Chan, K. A. Erb, J. L. C. Ford Jr., J. C. Peng and J. D. Moses, *Phys. Lett.* **B114** (1982), 111.

- [19] C. Beck, Y. Abe, N. Aissaoui, B. Djerroud and F. Haas, *Phys. Rev.* **C49** (1994), 2618.
- [20] A. Ray, D. Shapira, J. Gomez del Campo, H. J. Kim, C. Beck, B. Djerroud, B. Heusch, D. Blumenthal and B. Shivakumar, *Phys. Rev.* **C44** (1991), 514.
- [21] H.A. Kramers, *Physica* VII 4 (1940), 284.
- [22] J. Wilczyński, *Phys. Lett.* **B47** (1973), 484;
D. H. E. Gross and H. Kalinowski, *Phys. Lett.* **B48** (1974), 302.
- [23] P. Grangé, J.Q. Lie and H. A. Weidenmüller, *Phys. Rev.* **C27** (1983), 2063.
- [24] Y. Abe, C. Grégoire and H. Delagrangé, *J. Phys. (Paris)* **C4** (1986), 47.
- [25] N. Bohr and J. A. Wheeler, *Phys. Rev.* **56** (1939), 426.
- [26] D. Hilscher and H. Rossner, *Ann. Phys. Fr.* **17** (1992), 471.
- [27] D. J. Hinde, D. Hilscher and H. Rossner, *Nucl. Phys.* **A538** (1992), 243c.
- [28] S. Ayik and C. Grégoire, *Phys. Lett.* **B212** (1988), 269; *Nucl. Phys.* **A513** (1990), 187.
- [29] J. Randrup and S. Ayik, *Nucl. Phys.* **A572** (1994), 489.
- [30] J. Randrup and B. Remaud, *Nucl. Phys.* **A514** (1990), 339;
P.-G. Reinhard, E. Suraud, S. Ayik, *Ann. Phys. (N.Y.)* **213** (1992), 204;
P.-G. Reinhard, E. Suraud, *Ann. Phys. (N.Y.)* **216** (1992), 987.
- [31] S. Ayik, *Z. Phys.* **A350** (1994), 45;
S. Ayik and J. Randrup, "Effect of Memory Time on the Agitation of Unstable Modes in Nuclear Matter", *LBL Preprint* LBL-35745;

Y.B. Ivanov and S. Ayik, "Medium - Modified Interaction Induced by Fluctuations",
GSI Preprint GSI-95-17.

- [32] P. Hänggi and P. Jung, *Adv. Chem. Phys.* **89** (1995), 229.
- [33] R. F. Grote and J. T. Hynes, *J. Chem. Phys.* **73** (1980), 2715.
- [34] P. Hänggi, *J. Stat. Phys.* **30** (1983), 401.
- [35] A. Igarashi and T. Munakata, *J. Phys. Soc. Jpn.* **57** (1988), 2439.
- [36] A. Brissaud and U. Frisch, *J. Quant. Spectrosc. Radiat. Transfer* **11** (1971), 1767.
- [37] R. Kubo, *J. Math. Phys.* **4** (1962), 174.
- [38] P. W. Anderson, *J. Phys. Soc. J.* **9** (1954), 316;
R. Kubo, *J. Phys. Soc. J.* **9** (1954), 935.
- [39] A. Brissaud and U. Frisch, *J. Math. Phys.* **15** (1974), 524.
- [40] H. Mori, *Prog. Theor. Phys.* **33** (1965), 423;
R. Kubo, *Rep. Prog. Phys.* **29** (1966), 255.
- [41] B. Misra, I. Prigogine and M. Courbage, *Physica* **A98** (1979), 1;
I. E. Antoniou and K. E. Gustafson, *Physica* **A197** (1993), 153.
- [42] T. Shimizu, *Physica* **A212** (1994), 61.
- [43] T. Shimizu, *Physica* **A164** (1990), 123; *ibid.* **A196** (1993), 42; *ibid.* **A195** (1993),
113.
- [44] Ya. G. Sinai, *Russ. Math. Surv.* **25** (1970), 137.
- [45] M. V. Berry, *Eur. J. Phys.* **2** (1981), 91.

- [46] Ya. G. Sinai and L. A. Bunimovich, *Comm. Math. Phys.* **78** (1980), 247; *ibid.* **78** (1980), 479.
- [47] B. Friedman and R. F. Martin, *Phys. Lett.* **A105** (1984), 2.
- [48] L. A. Bunimovich, in "Italian - Soviet Symposium on Mathematical Problems of Statistical Physics" , p. 47, 1983.
- [49] M. F. Schlesinger, B. W. West and J. Klafter, *Phys. Rev. Lett.* **58** (1987), 1100.
- [50] J. Błocki, Y. Boneh, J. R. Nix, J. Randrup, M. Robel, A. J. Sierk and W. J. Swiatecki, *Ann. Phys. (N.Y.)* **113** (1978), 330.
- [51] M. H. C. Knudsen, "The Kinetic Theory of Gases", Wiley, New York, 1950.
- [52] G. Wegmann, *Phys. Lett.* **B50** (1974), 327.
- [53] In the absence of friction, the Brownian particle evolves on a quasi-periodic orbit.
- [54] W. Nörenberg and H. A. Weidenmüller, "Introduction to the Theory of Heavy-Ion Collisions", *Lecture Notes in Physics*, Vol. 51, Springer Verlag, Berlin, Heidelberg, New York, 1980.
- [55] S. Drożdż, J. Okołowicz, M. Płoszajczak and T. Srokowski, to be published.
- [56] M. Płoszajczak and T. Srokowski, to be published.
- [57] P. Fröbrich, I. I. Gontchar and N. D. Mavlitov, *Nucl. Phys.* **A556** (1993), 281.
- [58] T. Wada, Y. Abe and N. Carjan, *Phys. Rev. Lett.* **70** (1993), 3538.

Figure captions

Fig. 1

The force autocorrelation function $\hat{C}(t) = \langle \mathbf{F}(0)\mathbf{F}(t) \rangle$ for the peripheral reaction $^{12}\text{C} + ^{12}\text{C}$ at $l = 16\hbar$ and $E_{cm} = 20\text{MeV}$. The collision is described using the molecular dynamics with α - particles as the elementary constituents. $\mathbf{F}(t)$ is the 18 - dimensional velocity vector and $\langle \rangle$ means the average over an ensemble of events. For more details of the model see the discussion in ref. [8].

Fig. 2

Geometry of the Sinai billiard, equivalent to the periodic Lorentz gas. The dashed line denotes periodic boundary conditions. The particle is reflected elastically at the solid lines. The lattice spacing is 2 and the separation distance between hard disks is $2 - 2R$.

Fig. 3

The mean square displacement $\langle \mathbf{r}^2(t) \rangle$ of the Brownian particle in the absence of the intrinsic friction ($\gamma = 0$). The amplitude of the chaotic force is $\varepsilon = 1$ and the FACF is $\sim 1/t$. The diffusion process closely approximates the Richardson's law of turbulent diffusion[49].

Fig. 4

The mean square displacement $\langle \mathbf{r}^2(t) \rangle$ divided by $\log t$ in the overdamped case

($\gamma = 1$) for $\varepsilon = 1$ with the FACF of the chaotic force $\sim 1/t$. The diffusion process is weakly anomalous and the diffusion coefficient, $D(t) \equiv (1/4) \langle \mathbf{r}^2(t) \rangle / t \sim \log t$, diverges logarithmically.

Fig. 5

The dependence of the velocity variance on time for $\gamma = 0.01$. The FACF of the chaotic force is $\sim 1/t$ and $\varepsilon = 1$.

Fig. 6

The dependence of the velocity variance in the equilibrium $\langle \mathbf{v}^2 \rangle_{eq}$ on the value of the intrinsic friction constant γ . The FACF of the chaotic force is $\sim 1/t$ and $\varepsilon = 1$. This dependence can be fitted for $\gamma \geq 0.01$ with a phenomenological function (10).

Fig. 7

The asymptotic energy distribution of the particles escaping from the spherical potential (solid line) of radius $r_B = 50$.

(a) The deterministic chaotic force with the FACF proportional to $1/t$ (open horizon in the billiard) and amplitude $\varepsilon = 1$. The constant of the intrinsic friction is $\gamma = 0.02$. The potential has the depth $V_0 = -40$. The dashed line shows the Gaussian shape fit.

(b) The same as in (a) but for the amplitude $\varepsilon = 0.25$ and the friction constant $\gamma = 0.001$. The potential has the depth $V_0 = -50$.

For more details see the description in text.

Fig. 8

The asymptotic energy distribution of the particles escaping from the circular attractive potential (solid line) of depth $V_0 = -40$ and radius $r_B = 50$ for the deterministic chaotic force with the exponentially decaying correlations and amplitude $\varepsilon = 1$. The constant of the intrinsic friction is $\gamma = 0.001$. The dashed line shows the exponential fit.

Fig. 9

Time variation of the particle number inside the circular attractive potential of depth $V_0 = -50$ and radius $r_B = 50$. The absorbing barrier is at $r = r_B$.

(a) The deterministic chaotic force with the FACF proportional to $1/t$ and amplitude $\varepsilon = 1$. The constant of the intrinsic friction is $\gamma = 0.02$. The dashed line shows the $1/t$ -dependence.

(b) The same as in (a) but for the amplitude $\varepsilon = 0.25$ and the intrinsic friction constant $\gamma = 0.001$.

For more details see the description in text.

Fig. 10

Time variation of the particle number inside the circular attractive potential of depth $V_0 = -40$ and radius $r_B = 50$ for the deterministic chaotic force with the exponentially decaying correlations and amplitude $\varepsilon = 1$. The constant of the intrinsic friction is $\gamma = 0.001$. The absorbing barrier is at $r = r_B$. The dashed line shows the exponential fit.

Fig. 11

Time-variation of the mean free paths of particles in the extended Sinai billiard corresponding to the non-escaping trajectories of the Langevin particles in the circular container. Parameters of the potential, the deterministic chaotic force and the friction are the same as given in caption of Fig. 9. The solid and dashed lines correspond to $\varepsilon = 1$ and $\varepsilon = 0.25$ respectively.

Fig. 12

Spatial coordinates of an exemplaric trajectory of the Brownian particle, corresponding to the very long trajectory in the extended Sinai billiard. At $t \sim 560$, trajectory in the billiard bounces off the scatterer for the first time. At this time, the Brownian particle is at rest in the point at which the applied deterministic chaotic force is exactly compensated by the conservative force.

Fig. 13

The asymptotic energy distributions of particles escaping over a potential barrier of height $V_1 = 5.6$ from the potentials of different depth : $V_0 = -50$ (the solid line), $V_0 = -80$ (the dashed line). Radius of the potential is $r_B = 50$ and the 'scission' radius at which the particle energies are registred is $r_{SC} = 150$. The constant of the intrinsic friction is $\gamma = 0.02$. The deterministic chaotic force with the FACF proportional to $1/t$ has an amplitude $\varepsilon = 1$. For more details see the description in text.

Fig. 14

The asymptotic energy distributions of particles escaping over a potential barrier of height $V_1 = 5.6$ from the potential of depth : $V_0 = -50$. The deterministic chaotic force with the FACF proportional to $1/t$ and amplitude $\varepsilon = 0.25$ is used. For more details see the caption of Fig. 13 and the description in the text.

Fig. 15

The asymptotic energy distributions of particles escaping over a potential barrier ($V_1 = 5.6$) from the potential of depth : $V_0 = -40$, for the deterministic chaotic force with exponentially decaying correlations and the amplitude $\varepsilon = 1$. The constant of the intrinsic friction is $\gamma = 0.001$. The dashed line shows the exponential fit. For more details see the caption of Fig. 13 and the description in the text.

Fig. 16

Time variation of the particle number for $|\mathbf{r}| < r_{SC}$, where $r_{SC} = 150$ is the 'scission' radius. Particles escape over a potential barrier of height $V_1 = 5.6$ from a circular attractive potential of depth either $V_0 = -50$ (the solid line) or $V_0 = -80$ (the dashed line) and radius $r_B = 50$. The constant of the intrinsic friction is $\gamma = 0.02$. The deterministic chaotic force with the FACF proportional to $1/t$ has an amplitude $\varepsilon = 1$.

Fig. 17

Time variation of the particle number for $|\mathbf{r}| < r_{SC}$. The depth of the attractive region is $V_0 = -50$. The deterministic chaotic force with the FACF proportional to $1/t$ has an amplitude $\varepsilon = 0.25$. For more details see the caption of Fig. 16 and the discussion in the text.

Fig. 18

Time variation of the particle number for $|\mathbf{r}| < r_{SC}$, for the deterministic chaotic force with the exponentially decaying correlations and amplitude $\varepsilon = 1$. Particles escape over the potential barrier of height $V_1 = 5.6$ from the potential of depth $V_0 = -40$ and radius $r_B = 50$. The constant of the intrinsic friction is $\gamma = 0.001$.

Fig.1

Fig. 2

Fig.3

Fig. 4

Fig. 5

Fig.6

Fig. 7

Fig. 8

Fig. 9

Fig.10

Fig. 11

Fig. 12

Fig. 13

Fig. 14

Fig. 15

Fig.16

Fig.17

Fig.18