Particle-Flux Separation in Chern-Simons Theory

and

the Landau-Level Gap of Composite Fermions

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Abstract

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We study a system of non-relativistic fermions in a uniform magnetic field near the half-filled Landau level ($\nu \simeq 1/2$) from a viewpoint of gauge theory. At exact half filling, quasi-excitations are so-called composite fermions and dynamically generated gauge bosons. A composite fermion (CF) is an electron with two flux quanta attaching it. Because of interaction between fermions and gauge bosons, there appears anomalous behavior of fermion propagator; vanishing of Fermi velocity. We start with this picture at half filling and follow the CF approach of Jain to the fractional quantum Hall effect near the half filling. We then make use of Chern-Simons bosonization to describe these CF's in a residual magnetic field. We calculate the effective Landau-level gap of CF, and find that it tends to vanish near the half filling due to the dynamical gauge field. Our calculation naturally explains the recent experiments reporting that the excitation gap of quantum Hall states behaves anomalously near the half filling, i.e., enhancement of the effective mass of quasi-particles. Throughout the calculation, we assume that there are two possible phases: in one phase, particles and fluxes are bound, while in the other phase, particles and fluxes are separated, i.e., the particle-flux separation occurs.

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Actually, it is easily seen that the case in which CF's fills up the first p (p is a positive integer) effective LL's, i.e., the effective filling factor of CF's, ν_{CF} , equals to p , corresponds to the electron filling factor $\nu = p/(2p + 1)$ [6]. Experimentally, FQHE is most clearly observed just at these values of the electron filling factor, $\nu = p/(2p + 1)$. This fact supports Jain's idea.

Furthermore, recent experiments [7] have observed anomalous behavior of gaps in the above FQH states; there is drastic enhancement of the CF mass near the half filling. To explain this fact theoretically, it is natural to expect that the dynamical gauge field, which appeared at the half filling, still survives even away but near the half filling and gives rise to some significant effect on FQH states.

In the present paper we shall study a 2-dimensional electron system in a uniform magnetic field near the half-filled LL ($\nu \simeq 1/2$). In particular, we shall investigate the effect of dynamical gauge field on CF's sitting in effective LL's. However, we stress that, at general filling factor, this problem is not easy at all, because gauge bosons have a rotationally invariant dispersion relation in momentum space, whereas fermions in LL's do not. The interesting case $\nu_{CF} = p$ is not an exception. In Ref. [8], the finite temperature compressibility of CF's was calculated by the RPA assuming applicability of perturbation theory for the case in which the chemical potential of CF's lies exactly at the middle of successive effective LL's. However, they faced some singular behavior of the renormalized energy gap in the limit $\nu \rightarrow 1/2$. This means that, as they discuss, a nonperturbative analysis is surely required for obtaining acceptable result.

In this paper, we carry out such a nonperturbative analysis, by considering another sequence of filling factors, $\nu_{CF} = (p - 1)/3$, or equivalently electron filling factor $\nu = (p - 1)/(2p + 1)$. Then, CF's fill up $[\nu_{CF}] = [(p - 1)/3]$ effective LL's, where $[x]$ is the maximum integer that does not exceed x . We shall focus on dynamics of CF's in the lowest-unfilled LL, ignoring mixing with lower filled LL's for the most of discussion. Therefore the effective filling factor of CF's in the lowest-unfilled LL becomes $\nu_{eff} = \nu_{CF} - [\nu_{CF}] = 0, 1/3$ or $2/3$. By employing the CS bosonization technique, these CF's in the lowest-unfilled LL are described by a boson field (bosonized CF's) coupled with a CS gauge field. So we end up with a

1 Introduction

At present, non-Fermi-liquid metals of 2-dimensional electron systems are certainly one of the most interesting topics in condensed matter physics. One example of non-Fermi liquid is the metallic phase of high- T_C cuprates exhibiting various anomalous behaviors. It is widely expected that a dynamical gauge field should exist there via the charge-spin separation and explain experiments consistently. Another interesting possibility of non-Fermi liquid is the electron system in a uniform magnetic field at Landau filling factor $\nu = 1/2$. It is suggested by Jain [1] that quasi-excitations of this system are so-called composite fermions. A composite fermion (CF) is nothing but an electron with two flux quanta attaching it.

The electron system with half-filled ($\nu = 1/2$) Landau level (LL) can be expressed by a Chern-Simons (CS) gauge theory coupled with non-relativistic CF's with a suitable CS coefficient. Halperin, Lee and Read [2] studied this CS gauge system by random-phase approximation (RPA), and argued that anomalous behavior of CF's appears because of their interaction with transverse gauge field: the effective mass of fermion diverges near Fermi surface. Essential point for this result is that the average of the CS magnetic field cancels out the external magnetic field, and residual gauge fluctuations behave as a dynamical (independent) gauge field.

One may naively expect that appearance of such a dynamical gauge field is not possible since the intrinsic CS local constraint equates the CS magnetic field and the density fluctuation of fermions. However, in the RPA or renormalization-group study of the CS theory, one can argue that the CS constraint becomes irrelevant for low-energy excitations, and, as a result, fermions become quasi-free, composing a Fermi surface with gapless excitations. This picture is consistent at least with experiments [3] and numerical studies [4]. This irrelevance of constraint implies the phenomenon that the CF's and the CS fluxes are detached and move separately. We call this phenomenon "particle-flux separation" in analogy with the charge-spin separation [5] in high- T_C superconductivity.

In the CF approach by Jain [1] to the fractional quantum Hall effect (FQHE), stability of FQHE is attributed to the energy gap of CF's due to the Landau-level quantization.

bosonized CF system interacting with the CS gauge field as well as the dynamical gauge field that comes from introduction of CF's themselves. As we show, this system is rather tractable, because introduction of bosonized CF's resolves the above mentioned difficulty.

The present paper is organized as follows. In Sect.2, we shall review CS gauge theories of fermions in a uniform magnetic field. Especially we emphasize that there are (at least) two different phases of CS gauge theory; one is the phase where the CS constraint is respected faithfully by quasi-excitations. The other is the phase where the particle-flux separation (PFS) takes place. In the former case, the CS gauge field is *not* an independent dynamical variable, but is expressed in terms of fluctuation of matter field. In the latter PFS case, the residual gauge field is an independent (dynamical) field.

In Sect.2.1, we explain a well-known example of the phase where the CS constraint is intact. It is a bosonized theory of FQHE with $\nu = 1/q$ (q is an odd integer) treated in Ref.[9, 10]. Here, electrons are represented by bosonized electrons coupled with a CS gauge field. FQH states or Hall liquids are then characterized by bose condensation of bosonized electrons. Density fluctuation of electrons controls flux fluctuation of the CS gauge field.

In Sect.2.2, we explain an example in which assumption of PFS works quite successful. It is the aforementioned CF theory of the state at $\nu = 1/2$ involving a dynamical gauge field. The fermions have a Fermi surface and interact with the dynamical gauge field. Non-Fermi liquid behaviors appear at low energy. Recent renormalization-group analyses of this system are given in Refs. [11, 12]

In Sect.3, we set up our model of electrons near the half filling [$\nu = (p - 1)/(2p + 1)$]. It is a hybrid of above two examples, being derived in two steps; Introduction of CF's and bosonization of the CF. Thus it contains one bosonized CF (BCF) field and two gauge fields; One is associated with the introduction of the CF and is treated as a dynamical gauge field, while the other is a CS gauge field associated with the bosonization, and its fluctuation is controlled by density fluctuation of CF's through the constraint.

In Sect.4, we study the model in loop expansion, assuming that the coupling constant of dynamical gauge field is small. We also neglect higher-order fluctuations of the BCF around its condensation. Especially, we calculate the effective LL gap of CF's. In the "mean-field

approximation" in which fluctuations of the gauge field are neglected, the gap is given by $\Delta\omega_c \sim \Delta B/m$, where ΔB is the effective external magnetic field for CF's and m is the effective CF mass. We study how the dynamical gauge field changes this effective LL gap. We find that m becomes divergent at $\nu = 1/2$. This investigation is very important, since recent experiments observed a phenomenon of drastic enhancement of the CF mass.

Section 5 is devoted for discussion. Here we present an argument that our result of LL gap is not restricted to the above fillings but is rather general and applicable to other fillings. Appendix deals with the Bogoliubov-type transformation in the path-integral formalism.

2 Chern-Simons gauge theories of fermions in a uniform magnetic field

In this section, we shall review electron systems in a uniform magnetic field from a viewpoint of gauge theory. As explained in the introduction, there are (at least) two phases in CS gauge theory, and transition between them is characterized by attaching (detaching) CS flux to (from) particles. As typical examples of these phases and also as a preparation for constructing a model of electron system near the half filling, we shall consider two cases; one is FQH state at $\nu = 1/q$ ($q = \text{odd integer}$) in Subsect.2.1, and the other is CF's at $\nu = 1/2$ in Subsect.2.2.

2.1 Example of constrained phase: FQH states and condensation of bosonized electrons

In this subsection, we shall consider a system of electrons at filling factor $\nu \equiv 2\pi\rho/(eB^{ex}) = 1/q$ ($q = \text{odd integer}$), where ρ is the density of electron and B^{ex} is the magnitude of the external magnetic field that points in z direction. The Hamiltonian of the system is given by

$$H_{1/q} = \frac{1}{2m} \int d^2x |(i\partial_i + eA_i^{ex})\psi_e|^2 + V(\psi_e^\dagger\psi_e), \quad (2.1)$$

where ψ_e is a spinless fermion field describing electrons, A_i^{ex} ($i = 1, 2$) is the vector potential for magnetic field B^{ex} and $V(\psi_e^\dagger \psi_e)$ represents a Coulombic-type potential between electrons whose explicit form is irrelevant for the following general discussion. As is well-known, Laughlin's wave function [13] is a good variational state of $H_{1/q}$ and it explains many of interesting phenomena of FQHE. After the appearance of Laughlin's wave function, some people expected that FQH states might support some type of long-range order, and a Ginzburg-Landau(GL) theory might be constructed in terms of its order parameter. A GL theory of FQHE was proposed in Ref. [9], which uses a bosonized electron field as an order parameter on the analogy of superfluidity.

Field theoretical study of the Hamiltonian (2.1) using bosonized electron was given in Ref. [10]. A bosonized electron field $\phi(x)$ is defined by

$$\phi(x) = \psi_e(x) \exp \left[-iq \int d^2y \theta(x-y) \hat{\rho}(y) \right], \quad (2.2)$$

where

$$\hat{\rho}(x) \equiv \psi_e^\dagger(x) \psi_e(x) = \phi^\dagger(x) \phi(x), \quad (2.3)$$

and $\theta(x)$ is the azimuthal angle function in 2-dimensional space,

$$\theta(x) = \arctan \frac{x^2}{x^1}.$$

From (2.2), it is obvious that $\phi(x)$ describes an electron with q flux quanta attaching to it. Since q is assumed to be an odd integer, $\phi(x)$ behaves as a bosonic operator; $\phi(x)$'s at different points *commute* each other. Substituting (2.2) into (2.1), we get

$$H_{1/q} = \frac{1}{2m} \int d^2x |(i\partial_i + eA_i^{ex} - A_i^{CS})\phi|^2 + V(\phi^\dagger \phi), \quad (2.4)$$

where the CS gauge field A_i^{CS} is represented in terms of $\hat{\rho}$ as

$$\begin{aligned} A_i^{CS}(x) &= q \int d^2y \partial_i \theta(x-y) \hat{\rho}(y) \\ &= 2\pi q \epsilon_{ij} \int d^2y \partial_j \mathcal{G}(x-y) \hat{\rho}(y), \end{aligned} \quad (2.5)$$

with the antisymmetric tensor ϵ_{ij} and the 2-dimensional Green's function,

$$\mathcal{G}(x) = \int \frac{d^2k}{(2\pi)^2} \frac{e^{ik \cdot x}}{k^2}. \quad (2.6)$$

This is nothing but the CS constraint. Here we note that the magnetic field $B^{CS}(x)$ associated with A_i^{CS} is given by

$$B^{CS}(x) \equiv \epsilon_{ij} \partial_i A_j^{CS}(x) = 2\pi q \hat{\rho}(x). \quad (2.7)$$

In addition to the Hamiltonian (2.4), we must impose hard-core properties of the boson field $\phi(x)$. As shown in Ref.[10], this can be incorporated by adding a δ -function-type repulsion term $(\phi^\dagger \phi)^2$ to the Hamiltonian. Then the total Hamiltonian takes the following form of Bogomol'nyi type,

$$H_{1/q}^{tot} = \int d^2x \left\{ \frac{1}{2m} |(i\partial_z + eA_z^{ex} - A_z^{CS})\phi|^2 + \frac{1}{2}\omega_c \phi^\dagger \phi \right\} + V(\phi^\dagger \phi), \quad (2.8)$$

where $z = x + iy$, $A_z^{ex} = A_1^{ex} - iA_2^{ex}$, etc. and $\omega_c = B^{ex}/m$.

In the Lagrangian formalism, we can reformulate the equivalent system in terms of CS gauge theory. In this case, the CS gauge field $A_\mu^{CS}(x)$ ($\mu = 0, 1, 2$) is an *independent* dynamical variable, and its dynamics is controlled by the CS term $q^{-1}\epsilon_{\mu\nu\lambda}A_\mu^{CS}\partial_\nu A_\lambda^{CS}$. The equation of motion of A_0^{CS} gives rise to the CS constraint (2.5, 2.7), so A_0^{CS} plays the role of Lagrange multiplier.

An interesting and important problem here is whether the CS constraint is respected by low-energy quasi-excitations or not. We recall that a similar problem arises in the t-J model, in which the local constraint excluding double-occupancy of electrons exists. There the problem is related with the phenomenon of so-called charge-spin separation (CSS). In the previous papers [14], two of the present authors discussed the CSS from a viewpoint of gauge dynamics and obtained interesting results. As explained there, there is a typical and well-understood example of the (ir)relevance of local constraint; the $O(N)$ nonlinear σ -model.

Let us briefly explain the point. The $O(N)$ model has a local constraint given by

$$\vec{n}(x)\vec{n}(x) = \sum_{i=1}^N n^i(x)n^i(x) = 1, \quad (2.9)$$

where $\vec{n}(x)$ is a N -component scalar field. The model is known to have two phases; a broken phase and a symmetric phase. In the broken phase, the global $O(N)$ symmetry is

spontaneously broken $\langle n^N(x) \rangle \neq 0$ to generate a long-range order. There exist $(N - 1)$ massless excitations (Nambu-Goldstone bosons), as expected from the constraint (2.9). This phase is correctly described by solving first the constraint (2.9) as

$$\begin{aligned} n^N(x) &= \left(1 - \sum_{i=1}^{N-1} n^i(x)n^i(x)\right)^{\frac{1}{2}} \\ &= 1 - \frac{1}{2} \sum_{i=1}^{N-1} n^i(x)n^i(x) + \dots \end{aligned}$$

On the other hand, in the symmetric phase, the $O(N)$ symmetry is intact and the field $n^i(x)$ does not have vacuum expectation value, and excitations (quasi-particles) are N scalar bosons with a common finite mass. This phase is correctly described by introducing a Lagrange multiplier for the constraint (2.9). The multiplier, instead of $n^i(x)$, acquires a nonvanishing expectation value, which is interpreted as the mass. These excitations *do not* obey the constraint.

In the t-J model [14], we found that there is a certain critical temperature across which the local constraint is respected or ignored by low-energy excitations. Similar aspect of the CS theory has not yet been analyzed in detail. However, we believe that there are (at least) two phases as in the $O(N)$ nonlinear σ -model and in the t-J model. In one phase, the CS constraint is respected by quasi-excitations, and the CS gauge field is expressed completely in terms of matter density by solving (2.5) [constrained phase]. In the other phase, the constraint is irrelevant and the CS gauge field becomes dynamical [dynamical phase]. If $A_0^{CS}(x)$ becomes massive at low energies, i.e., having less fluctuation, it is expected that this phase is realized, since $A_0^{CS}(x)$ is the Lagrange multiplier. At present, an order parameter to distinguish these two phases has not yet been identified and waits for investigation.

Let us assume that the system given by the Hamiltonian (2.8) is in the constrained phase. For a while, we shall ignore the potential term $V(\phi^\dagger\phi)$ for simplicity. At the classical level, the lowest energy state obviously satisfies the equation [15],

$$(i\partial_z + eA_z^{ex} - A_z^{CS})\phi = 0. \quad (2.10)$$

The vector potential of the magnetic field in the symmetric gauge is explicitly given as

$$A_i^{ex} = -\epsilon_{ij} \frac{B^{ex}}{2} x^j. \quad (2.11)$$

From (2.5), the translationary-invariant unique solution to (2.10) is given by

$$\phi = \sqrt{\rho}, \quad A_z^{CS} = eA_z^{ex}, \quad (2.12)$$

where ρ is the electron density $\rho = \langle \hat{\rho}(x) \rangle$. The second relation and Eq.(2.7) determine the filling factor ν as

$$\nu \equiv \frac{2\pi\rho}{eB^{ex}} = \frac{1}{q}. \quad (2.13)$$

There are also vortex-type solutions, but they are not homogeneous and so acquire extra energies if the potential $V(\phi^\dagger\phi)$ is turned on.

The uniform solution (2.12) obviously corresponds to a bose condensation and a Hall liquid. Quantum fluctuations around the ground state (2.12) are described by the field φ which is defined by

$$\phi = \sqrt{\rho} + \varphi. \quad (2.14)$$

By substituting (2.14) into (2.5), the constraint reads

$$\begin{aligned} A_i^{CS} - eA_i^{ex} &= 2\pi q\epsilon_{ij} \int d^2y \partial_j \mathcal{G}(x-y) (\phi^\dagger \phi(y) - \rho) \\ &= 2\pi q\epsilon_{ij} \int d^2y \partial_j \mathcal{G}(x-y) (\sqrt{\rho}(\varphi + \varphi^\dagger) + \varphi\varphi^\dagger). \end{aligned} \quad (2.15)$$

Then, by substituting (2.15) into (2.8), we obtain an effective Hamiltonian of φ . Up to quadratic terms in φ , it is given by

$$\begin{aligned} H_\varphi &= \int d^2x \left\{ \frac{1}{2m} \partial_i \varphi^\dagger \partial_i \varphi + \frac{\pi q \rho}{m} (\varphi + \varphi^\dagger)^2 \right\} \\ &\quad - \int d^2x d^2y \frac{\pi q^2 \rho^2}{m} [\varphi(x) + \varphi^\dagger(x)] \ln|x-y| [\varphi(y) + \varphi^\dagger(y)] \\ &\quad + \text{potential term}. \end{aligned} \quad (2.16)$$

Because the state (2.12) breaks the original global U(1) symmetry, $\phi \rightarrow e^{i\chi} \phi$, spontaneously, one may expect that a massless Nambu-Goldstone(NG) boson appears. However, this is not the case as we shall see [16].

From the CS constraint (2.15), we see that fluctuation of electron density $\varphi(x)$ induces fluctuation of gauge flux also. Therefore, the field $\varphi(x)$ is *not* describing simple particles but

particles with gauge fluxes attached to them. The energy spectrum of $\varphi(x)$ is easily obtained from (2.16). In terms of the Fourier transformed field $\varphi(\mathbf{k})$, we get

$$H_\varphi = \sum_{\mathbf{k}} \left(\epsilon(\mathbf{k}) \varphi(\mathbf{k}) \varphi^\dagger(\mathbf{k}) + U(\mathbf{k}) [2\varphi^\dagger(\mathbf{k}) \varphi(\mathbf{k}) + \varphi^\dagger(\mathbf{k}) \varphi^\dagger(-\mathbf{k}) + \varphi(\mathbf{k}) \varphi(-\mathbf{k})] + \frac{2(\pi q \rho)^2}{m|\mathbf{k}|^2} \right), \quad (2.17)$$

where $\epsilon(\mathbf{k})$ and $U(\mathbf{k})$ are explicitly given as follows for the case with no potential term [17],

$$\epsilon(\mathbf{k}) = \frac{\mathbf{k}^2}{2m}, \quad U(\mathbf{k}) = \frac{2(\pi q \rho)^2}{m\mathbf{k}^2} + \frac{\pi q \rho}{m}. \quad (2.18)$$

The terms with the coefficient $U(\mathbf{k})$ in (2.17) come from gauge-flux fluctuation. If there were no such terms, $\varphi(x)$ would describe a gapless excitation with the dispersion relation $E_0(\mathbf{k}) = \mathbf{k}^2/(2m)$, i.e., a NG boson. The Hamiltonian H_φ is diagonalized by the Bogoliubov transformation, and the energy spectrum $E(\mathbf{k})$ is obtained as

$$\begin{aligned} E(\mathbf{k}) &= [\epsilon^2(\mathbf{k}) + 4\epsilon(\mathbf{k})U(\mathbf{k})]^{1/2} \\ &= \frac{\mathbf{k}^2}{2m} + \omega_c. \end{aligned} \quad (2.19)$$

Therefore $\varphi(\mathbf{k})$ has an energy gap $\omega_c = eB^{ex}/m$, which is nothing but the well-known LL gap of electrons. Existence of the potential term does not modify the essential result. The field $\varphi(x)$ describes small fluctuation from the homogeneous ground state (2.12), and especially, $\varphi(\mathbf{k} = 0)$ describes uniform change of the electron density across the neighboring LL's separated by the gap ω_c . This fact is very important for later discussion on LL gap of CF's.

The above result that the energy gap equals to ω_c seems to justify our assumption that the system with the Hamiltonian (2.8) exists in the constrained phase of the CS theory. Actually, as the picture of the LL quantization is describable solely in terms of electrons, $\varphi(x)$ must be accompanied by gauge fluxes and there should be no dynamical gauge field. Of course, to be conclusive, study of quasi-excitations with fractional statistics and charge is also required. These excitations are non-uniform states and to be described by vortex-like solutions in the present formalism. Actually, in Ref.[10], it is shown that these vortex-like solutions correspond to Laughlin's quasi-particles in FQH state. (For this point see also discussion given in Ref.[18] which employs a collective-field method.)

2.2 Example of particle-flux separation: CF's at $\nu = 1/2$

In this subsection, we shall review recent development [11, 12] in the study of electron system in a uniform magnetic field at filling factor $\nu = 1/2$. It follows the work of Halperin, Lee and Read [2] who employed Jain's CF approach [1].

The Hamiltonian is given by Eq.(2.1) with the choice $q = 2$. As in (2.2), let us introduce a CF operator $\psi_{cf}(x)$ by

$$\psi_{cf}(x) = \psi_e(x) \exp \left[-2i \int d^2y \theta(x-y) \hat{\rho}(y) \right]. \quad (2.20)$$

As $q = 2$ in the present case, $\psi_{cf}(x)$ describes electrons with two flux quanta attaching to each of them, so it obeys Fermi statistics. In terms of $\psi_{cf}(x)$, the Hamiltonian $H_{1/2}$ is written as

$$H_{1/2} = \frac{1}{2m} \int d^2x |(i\partial_t + eA_t^{ex} - A_t^{CS})\psi_{cf}|^2 + V(\psi_{cf}^\dagger \psi_{cf}), \quad (2.21)$$

with

$$A_t^{CS} - eA_t^{ex} = 4\pi\epsilon_{ij} \int d^2y \partial_j \mathcal{G}(x-y) (\psi_{cf}^\dagger \psi_{cf}(y) - \rho), \quad (2.22)$$

where we have used the relation (2.13), $\nu = 2\pi\rho/(eB^{ex}) = 1/2$.

In contrast to FQH state of Subsect.2.1, let us assume that the CS gauge theory is in the dynamical phase. From the existing works, this assumption is known to lead us to interesting and reasonable results. As discussed in the previous subsection, introducing the CS gauge term in the Lagrangian formalism is useful for describing that phase. Practical calculation summing bubble diagrams of fermions [2, 12] shows that A_0^{CS} , the Lagrange multiplier of the CS constraint, becomes massive. This indicates that the CS constraint is irrelevant at low energies [19].

Let us derive the Lagrangian of composite-fermion-gauge system according to Refs.[2, 12]. First we consider a generalized potential term, which take the form,

$$V(\psi_e^\dagger \psi_e) = \int d\mathbf{k} (\hat{\rho}(\mathbf{k}) - \rho) \frac{1}{k^b} (\hat{\rho}(\mathbf{k}) - \rho), \quad (2.23)$$

where

$$\hat{\rho}(\mathbf{k}) = \int \frac{d^2k}{(2\pi)^2} e^{i\mathbf{k} \cdot \mathbf{x}} \psi_e^\dagger \psi_e(x), \quad (2.24)$$

where b is a parameter. For the Coulomb potential, $b = 1$. Next, we introduce an *independent* (*dynamical*) gauge field A_i ,

$$A_i \equiv A_i^{CS} - e A_i^{ex}. \quad (2.25)$$

To obtain its kinetic term, we substitute (2.22) and (2.25) into the potential $V(\psi_{cf}^\dagger \psi_{cf})$ of (2.23). Then the Lagrangian of the CF ψ_{cf} and the gauge field A_i is given as follows;

$$\begin{aligned} L &= \int d^2x \bar{\psi}_{cf}(x) \left(\partial_\tau - \mu - \frac{1}{2m} D_i D_i \right) \psi_{cf}(x) \\ &\quad + \frac{1}{2} \int d^2x d^2y B(x) V_B(x-y) B(y), \\ D_i &= \partial_i + ig A_i, \\ B(x) &= \epsilon_{ij} \partial_i A_j(x), \\ V_B(x) &= \int \frac{d^2k}{(2\pi)^2} \frac{e^{i\mathbf{k} \cdot \mathbf{x}}}{k^b}, \end{aligned} \quad (2.26)$$

where we have introduced chemical potential μ to control the fermion density and a gauge coupling constant g for later convenience.

If the gauge interaction is weak enough, the Lagrangian (2.26) shows that CF's are quasi-free and have a Fermi surface, obeying Landau's Fermi liquid theory. Renormalization group studies [12] and RPA [2, 11] show that there appears some important modification in low-energy behavior of the CF propagator due to dynamical fluctuations of A_i .

The above derivation of the Lagrangian (2.26) is not rigorous. In particular, there is ambiguity to identify the gauge-field kinetic term using the CS constraint. So far, its usefulness is supported only by its interesting and reasonable outcomes. In fact, numerical calculations [4] and experiments [3] support Fermi-liquid-like behavior of quasi-excitations in the half-filled state. Therefore, we believe that the model (2.26) is a good effective theory for describing low-energy excitations at $\nu = 1/2$.

Result of the renormalization-group study [12] is summarized as follows. The most important point is that, after integrating out high-momentum modes in renormalization-group transformation, there appears a Landau screening term $|\omega|/k$ (ω is energy and k is magnitude of momentum) in the gauge field propagator. The low-energy behavior of fermion propagator depends on value of b . For $b < 1$, the dissipative term of gauge field propaga-

tor gives nontrivial anomalous dimension to the fermion field. Also the Fermi velocity is renormalized to vanish at low-energy limit. As a result, fermion propagator has a branch cut rather than a pole, just like the Luttinger liquid in one dimension [20]. For $b = 1$, the weight of quasiparticles decreases at low energy as $1/\ln \omega$, where ω is energy of the fermion. Obviously, this behavior is that of marginal Fermi liquid [21]. For $b > 1$, the model has a trivial low-energy fixed point of Landau's Fermi liquid.

3 Model of CF system near the half-filled state

In this section we establish our model of electrons near the half filling. There are two steps in its derivation. The first step is the introduction of CF's, which we discuss in Sect.3.1. This is motivated by the example of Sect.2.2, and a dynamical gauge field is introduced. The second step is the bosonization of CF's, which we discuss in Sect.3.2. This is motivated by the example of Sect.2.1, and a constrained CS gauge field is introduced.

3.1 Introduction of composite fermions

The Hamiltonian of electrons in magnetic field B^{ex} is given by

$$\begin{aligned} H_{nhf} &= \frac{1}{2m} \int d^2x |(i\partial_i + eA_i^{ex})\psi_e|^2 + V(\psi_e^\dagger \psi_e), \\ \epsilon_{ij} \partial_i A_j^{ex} &= B^{ex}. \end{aligned} \quad (3.1)$$

We introduce the CF operator by using transformation (2.20) to obtain

$$H_{nhf} = \frac{1}{2m} \int d^2x |(i\partial_i + eA_i^{ex} - A_i^{CS})\psi_{cf}|^2 + V(\psi_{cf}^\dagger \psi_{cf}), \quad (3.2)$$

with the constraint,

$$A_i^{CS} = 4\pi\epsilon_{ij} \int d^2y \partial_j \mathcal{G}(x-y) \psi_{cf}^\dagger \psi_{cf}(y). \quad (3.3)$$

Then we choose the $\nu = 1/2$ state as the reference state and measure everything from it.

For this purpose we define the following quantities,

$$A_i^{ex} = A_i^{1/2} + \Delta A_i^{ex}.$$

$$\begin{aligned}
\epsilon_{ij}\partial_i A_j^{1/2} &= B_{1/2} \equiv \frac{4\pi\rho}{e}, \\
\epsilon_{ij}\partial_i \Delta A_j^{ex} &= \Delta B \equiv B^{ex} - B_{1/2}, \\
A_i &\equiv A_i^{CS} - eA_i^{1/2}, \\
B &\equiv \epsilon_{ij}\partial_i A_j = e\left(\frac{4\pi}{e}\psi_{cf}^\dagger\psi_{cf} - B_{1/2}\right).
\end{aligned} \tag{3.4}$$

Using these quantities and substituting the last relation in (3.4) into the potential term of the Hamiltonian (3.2) as in Subsect.2.2, we obtain

$$\begin{aligned}
H_{nhf} &= \frac{1}{2m} \int d^2x |(\partial_i - ie\Delta A_i^{ex} + igA_i)\psi_{cf}|^2 \\
&+ \frac{1}{2} \int d^2x d^2y B(x)V_B(x-y)B(y),
\end{aligned} \tag{3.5}$$

where we have introduced the gauge coupling constant g as in Sect.2.2. Through the above substitution, the potential term produces extra terms; terms proportional to $\psi_{cf}^\dagger(x)\psi_{cf}(x)$ and a constant term. These terms have been omitted in H_{nhf} since they are to be absorbed into the chemical potential term and the irrelevant constant term. Now we assume particle-flux separation and discard the CS constraint in (3.4). So we treat the field A_i as a dynamical variable.

In this CF representation, CF's are effectively put in the residual uniform magnetic field ΔB . The corresponding effective filling factor of the CF, ν_{CF} , is given as

$$\nu_{CF} = \frac{2\pi\rho}{e\Delta B}. \tag{3.6}$$

As explained in Introduction, in Jain's approach to FQHE, the most stable FQH states are given by integer fillings of CF's, $\nu_{CF} = p$, which corresponds to the electron filling factor $\nu = p/(2p+1)$ [6]. If the dynamical gauge field A_i were absent, the LL gap of CF's or energy gap of FQH state at $\nu = p/(2p+1)$ should scale as $\Delta\omega_c = e\Delta B/m$, where m is the effective mass of CF's. Actually, experiments [3] show that energy gaps of FQH states for relatively small p can be fitted by this formula with a *constant* m . However, recent experiments [7] observed drastic enhancement of the mass m for sufficiently large p , i.e., the gap tends to vanish very rapidly as $\nu \rightarrow 1/2$. Theoretical explanation for this phenomenon is required.

We study how the dynamical gauge field A_i changes the above picture of the CF, especially its effect on the effective LL gap. From (3.5), it is obvious that the effective magnetic field which CF's feel is not ΔA_i^{ex} but $\Delta A_i^{ex} - (g/e)A_i$. As A_i is dynamical and fluctuates (independently of the CF density), it may change the LL structure of CF's drastically.

As explained in Introduction, it is not easy to study the model (3.5) at general filling factor. Therefore, we shall focus on the specific filling factor of $\nu = (p-1)/(2p+1)$, which corresponds to $\nu_{CF} = (p-1)/3$ [6]. As we let $p \rightarrow \infty$, ν approaches $1/2$ as desired. In this case, CF's fill up $[\nu_{CF}]$ LL's, and there remains a partially-filled LL with effective filling factor $\nu_{eff} \equiv \nu_{CF} - [\nu_{CF}] = 1/3$ or $2/3$, for $p = 3N - 1$ or $p = 3N$ (N is an integer), respectively. In most of the discussion, we shall ignore mixing effects of LL's, and concentrate on the behavior of CF's in the lowest unfilled LL.

The reason why we consider above values of ν_{eff} is that one can make use of the bosonization technique for these values. Actually, for $\nu_{eff} = 1/(2n+1)$ ($n =$ positive integer), the dynamics of CF's in the lowest unfilled LL can be well-studied by employing a bosonized CF (BCF) operator, just as in the case of FQH state discussed in Subsect.2.1. [Remember $\nu = 1/q = 1/(2n+1)$ there.] This is the topics of the next subsection. We have selected $n = 1$ above, since this seems to give the most stable QH states but our treatment applies equally well also for other values of n , for which we have $\nu_{cf} = (p-1)/(2n+1)$ and $\nu = (p-1)/(2pn+1)$, thus corresponding to the states near $\nu = 1/(2n)$.

3.2 Bosonization of CF's

We focus on the dynamics of CF's in the lowest unfilled LL and write the corresponding field as ψ_{eff} . We shall ignore effects of LL mixing in the later discussion. After replacing ψ_{cf} by ψ_{eff} in H_{nhf} (3.5), we introduce the BCF field ϕ into H_{nhf} via the bosonization formula (2.2) [22],

$$\phi(x) = \psi_{eff}(x) \exp \left[-2i\alpha_{eff} \int d^2y \theta(x-y) \psi_{eff}^\dagger(y) \psi_{eff}(y) \right], \quad (3.7)$$

where

$$2\alpha_{eff} = (2n+1), \quad (3.8)$$

$$\rho_{eff} \equiv \langle \psi_{eff}^\dagger(x) \psi_{eff}(x) \rangle = \langle \phi^\dagger(x) \phi(x) \rangle. \quad (3.9)$$

where $n = 1, 2, 3, \dots$. As explained, this choice of α_{eff} gives rise to

$$\nu_{eff} \equiv \frac{2\pi\rho_{eff}}{e\Delta B} = \frac{1}{2\alpha_{eff}} = \frac{1}{2n+1}, \quad (3.10)$$

for the effective Landau filling factor ν_{eff} of CF's in the lowest-unfilled LL.

The action in the imaginary-time path-integral formalism is given by

$$\begin{aligned} S = & \int d\tau d^2x \left\{ \bar{\phi}(x, \tau) \partial_\tau \phi(x, \tau) + \frac{1}{2m} |(\mathcal{D}_1 + i\mathcal{D}_2)\phi(x, \tau)|^2 \right\} \\ & + \frac{1}{2} \int d\tau d^2x d^2y B(x, \tau) V_B(x-y) B(y, \tau), \end{aligned} \quad (3.11)$$

where

$$\begin{aligned} B(x, \tau) &= \epsilon_{ij} \partial_i A_j(x, \tau) \\ \mathcal{D}_i &\equiv \partial_i + i\mathcal{A}_i(x, \tau), \quad \mathcal{A}_i(x, \tau) \equiv a_i(x, \tau) + gA_i(x, \tau), \end{aligned} \quad (3.12)$$

$$a_i(x, \tau) \equiv 2\alpha_{eff}\epsilon_{ij} \int d^2y \partial_i \mathcal{G}(x-y) \Delta\rho(y, \tau), \quad (3.13)$$

$$\Delta\rho(x, \tau) \equiv \bar{\phi}\phi(x, \tau) - \rho_{eff}. \quad (3.14)$$

$$V_B(x-y) \equiv v_B \int \underline{dq} q^{-b} e^{iq \cdot (x-y)}, \quad (3.15)$$

$$\underline{dq} \equiv \frac{d^2q}{(2\pi)^2}.$$

and we have introduced the parameter v_B for simple dimensional reasoning. We shall work in the Coulomb gauge,

$$\partial_i A_i(x, \tau) = 0. \quad (3.16)$$

Then $A_i(x, \tau)$ can be represented as

$$A_i(x, \tau) = \epsilon_{ij} \int d^2y \partial_j \mathcal{G}(x-y) B(y, \tau). \quad (3.17)$$

4 Landau-level gap of CF's

In this section, we calculate the LL gap of CF's. In Sect.4.1 we consider the Bose condensation of BCF and derive an effective Hamiltonian for the fluctuating field around the

condensation. In Sect.4.2.1 we find that the gap vanishes on the tree level, and discuss physical interpretation of the result. In Sect.4.2.2 we check that the one-loop correction does not change the result.

4.1 Bose condensation

To study the model (3.11) let us first consider the bose condensation. As in the FQH state discussed in Subsect.2.1, the classical solution for the ground state is given by

$$\phi(x, \tau) = \sqrt{\rho_{eff}}. \quad (4.1)$$

We shall quantize the deviation $\eta(x, \tau)$ from this solution,

$$\phi(x, \tau) = \sqrt{\rho_{eff}} + \eta(x, \tau). \quad (4.2)$$

By substituting this expression, the action is represented in terms of $\eta(x, \tau)$ as

$$\begin{aligned} S = & \int d\tau d^2x \left[\bar{\eta}(x, \tau) \partial_\tau \eta(x, \tau) + \frac{1}{2m} |\partial_i \eta(x, \tau)|^2 + \frac{2\alpha_{eff}}{2m} \{ \Delta \rho(x, \tau) \}^2 \right] \\ & + \int d\tau d^2x d^2y \frac{(2\alpha_{eff})^2 \rho_{eff}}{2m} \Delta \rho(x, \tau) \mathcal{G}(x-y) \Delta \rho(y, \tau) \\ & + \frac{1}{2} \int d\tau d^2x d^2y B(x, \tau) \left\{ V_B(x-y) + \frac{g^2 \rho_{eff}}{m} \mathcal{G}(x-y) \right\} B(y, \tau) \\ & + \int d\tau d^2x \frac{g}{2m} \Delta \rho(x, \tau) B(x, \tau) + \int d\tau d^2x d^2y g \frac{2\alpha_{eff} \rho_{eff}}{m} \Delta \rho(x, \tau) \mathcal{G}(x-y) B(y, \tau) \\ & + \int d\tau d^2x d^2y \frac{(-i)}{m} \epsilon_{ij} \partial_i \bar{\eta}(x, \tau) \partial_j \eta(x, \tau) \mathcal{G}(x-y) \{ 2\alpha_{eff} \Delta \rho(y, \tau) + g B(y, \tau) \} \\ & + \int d\tau d^2x d^2y d^2z \frac{1}{2m} \partial_i \mathcal{G}(x-y) \partial_i \mathcal{G}(x-z) \\ & \times \Delta \rho(x, \tau) \{ 2\alpha_{eff} \Delta \rho(y, \tau) + g B(y, \tau) \} \{ 2\alpha_{eff} \Delta \rho(z, \tau) + g B(z, \tau) \}. \end{aligned} \quad (4.3)$$

In this form, there are mixing terms between the fluctuation η and the dynamical-flux B . They come from the couplings $(g/m)\bar{\phi}A_i\partial_i\phi$ and $(1/m)\bar{\phi}a_iA_i\phi$. In order to have a clear picture of physical degrees of freedom, let us make a change of variables so as to diagonalize the quadratic terms in the action. To this end, we first introduce the following function.

$$\mathcal{W}(x-y) = V_B(x-y) + \frac{g^2 \rho_{eff}}{m} \mathcal{G}(x-y). \quad (4.4)$$

$$\mathcal{W}^{-1}(x-y) = \int \frac{dq}{(2\pi)^2} \frac{q^2}{F(q)} e^{iq \cdot (x-y)}. \quad (4.5)$$

where

$$F(q) \equiv v_B q^{2-b} + \Delta M, \quad \Delta M \equiv \frac{g^2 \rho_{eff}}{m} = \frac{g^2 e \Delta B}{2m\alpha_{eff}}. \quad (4.6)$$

Using this function, we introduce a new variable $\tilde{B}(x, \tau)$,

$$\tilde{B}(x, \tau) \equiv B(x, \tau) + \int d^2 y \mathcal{W}^{-1}(x-y) \delta \rho(y, \tau), \quad (4.7)$$

$$\delta \rho(x, \tau) \equiv \frac{g}{2m} \Delta \rho(x, \tau) + g \Delta \omega_c \int d^2 y \mathcal{G}(x-y) \Delta \rho(y, \tau), \quad (4.8)$$

$$\Delta \omega_c \equiv \frac{2\alpha_{eff} \rho_{eff}}{m} = \frac{e \Delta B}{m}. \quad (4.9)$$

In terms of $\tilde{B}(x, \tau)$, the action takes a form of

$$\begin{aligned} S = & \int d\tau d^2 x \left[\bar{\eta}(x, \tau) \partial_\tau \eta(x, \tau) + \frac{1}{2m} |\partial_i \eta(x, \tau)|^2 + \frac{\Delta \omega_c}{2\rho_{eff}} \{\Delta \rho(x, \tau)\}^2 \right] \\ & + \int d\tau d^2 x d^2 y \frac{m \Delta \omega_c^2}{2\rho_{eff}} \Delta \rho(x, \tau) \mathcal{G}(x-y) \Delta \rho(y, \tau) \\ & - \frac{1}{2} \int d\tau d^2 x d^2 y \delta \rho(x, \tau) \mathcal{W}^{-1}(x-y) \delta \rho(y, \tau) \\ & + \frac{1}{2} \int d\tau d^2 x d^2 y \tilde{B}(x, \tau) \mathcal{W}(x-y) \tilde{B}(y, \tau) \\ & + \int d\tau d^2 x d^2 y \frac{(-i)}{m} \epsilon_{ij} \partial_i \bar{\eta}(x, \tau) \partial_j \eta(x, \tau) \mathcal{G}(x-y) \\ & \times \left\{ 2\alpha_{eff} \Delta \rho(y, \tau) - g \int d^2 z \mathcal{W}^{-1} \delta \rho(z, \tau) + g \tilde{B}(y, \tau) \right\} \\ & + \int d\tau d^2 x d^2 y d^2 z \frac{1}{2m} \partial_i \mathcal{G}(x-y) \partial_i \mathcal{G}(x-z) \Delta \rho(x, \tau) \\ & \times \left\{ 2\alpha_{eff} \Delta \rho(y, \tau) - g \int d^2 u \mathcal{W}^{-1} \delta \rho(u, \tau) + g \tilde{B}(y, \tau) \right\} \\ & \times \left\{ 2\alpha_{eff} \Delta \rho(z, \tau) - g \int d^2 u' \mathcal{W}^{-1} \delta \rho(u', \tau) + g \tilde{B}(z, \tau) \right\}. \end{aligned} \quad (4.10)$$

There are no mixing terms anymore in the quadratic terms of (4.10). Of course, interactions create mixing terms, but practical calculation shows that they appear in higher-order terms than one loop. In the following discussion, we shall consider only one-loop effects.

From the action (4.10), we see that $\tilde{B}$ fluctuates around zero, which means that the "ground state" of the dynamical gauge flux B has some correlation with the matter field. This correlation effect generates nonvanishing magnetic field B spontaneously due to matter fluctuation $\Delta \rho$. Another point to notice is that the dynamical gauge field acquires a mass ΔM by the Anderson-Higgs mechanism with the BCF condensation (4.1). This suppresses

the fluctuation of the gauge field and makes the perturbative calculation reliable. The above two facts become very important in the later discussion on the LL gap of CF's.

The action (4.10) is highly nonlocal, and therefore we shall work with the Fourier transformed fields,

$$\begin{aligned}
S = & \int \underline{d\omega dk} \left[\{i\omega + \epsilon(k)\} \bar{\eta}(k, \omega) \eta(k, \omega) \right. \\
& + \frac{1}{2\rho_{eff}} \left\{ \Delta\omega_c + \frac{\Delta\omega_c^2}{2\epsilon(k)} - \frac{\Delta M \{\epsilon(k) + \Delta\omega_c\}^2}{\epsilon(k)F(k)} \right\} \Delta\rho(-k, -\omega) \Delta\rho(k, \omega) \Big] \\
& + \frac{1}{2} \int \underline{d\epsilon dq} F(q) b(-q, -\epsilon) b(q, \epsilon) \\
& + \int \underline{d\omega d\epsilon dk} \underline{dq} \frac{i(k \times q)}{mq^2} \bar{\eta}(k+q, \omega+\epsilon) \eta(k, \omega) \\
& \times \left[gqb(q, \epsilon) + \frac{m}{\rho_{eff}} \left\{ \left(1 - \frac{\Delta M}{F(q)}\right) \Delta\omega_c - \frac{\Delta M}{F(q)} \epsilon(q) \right\} \Delta\rho(q, \epsilon) \right] \\
& + \int \underline{d\epsilon d\epsilon'} \underline{dq dq'} \frac{(-q \cdot q')}{2mq^2 q'^2} \Delta\rho(-q-q', -\epsilon-\epsilon') \\
& \times \left[gqb(q, \epsilon) + \frac{m}{\rho_{eff}} \left\{ \left(1 - \frac{\Delta M}{F(q)}\right) \Delta\omega_c - \frac{\Delta M}{F(q)} \epsilon(q) \right\} \Delta\rho(q, \epsilon) \right] \\
& \times \left[gqb(q', \epsilon') + \frac{m}{\rho_{eff}} \left\{ \left(1 - \frac{\Delta M}{F(q')} \right) \Delta\omega_c - \frac{\Delta M}{F(q')} \epsilon(q') \right\} \Delta\rho(q', \epsilon') \right]. \quad (4.11)
\end{aligned}$$

where

$$\eta(x, \tau) = \int \underline{d\omega dk} e^{i\omega\tau + i\mathbf{k} \cdot \mathbf{x}} \eta(k, \omega), \quad (4.12)$$

$$\tilde{B}(x, \tau) = \int \underline{d\epsilon dq} e^{i\epsilon\tau + i\mathbf{q} \cdot \mathbf{x}} qb(q, \epsilon), \quad (4.13)$$

$$\Delta\rho(x, \tau) = \int \underline{d\omega dk} e^{i\omega\tau + i\mathbf{k} \cdot \mathbf{x}} \Delta\rho(k, \omega), \quad (4.14)$$

$$\begin{aligned}
\Delta\rho(k, \omega) = & \sqrt{\rho_{eff}} \{ \eta(k, \omega) + \bar{\eta}(-k, -\omega) \} \\
& + \int \underline{d\epsilon dq} \bar{\eta}(k, \omega) \eta(k+q, \omega+\epsilon), \quad (4.15)
\end{aligned}$$

$$\epsilon(k) \equiv \frac{\mathbf{k}^2}{2m}, \quad (4.16)$$

$$\mathbf{k} \times \mathbf{q} \equiv \epsilon_{ij} k_i q_j. \quad (4.17)$$

We are interested in the effect of the dynamical gauge field, and therefore we neglect the self interactions of $\eta(k, \omega)$. Also, because the total number of electrons in the system

conserves, there holds a condition $\int d^2x \Delta\rho(x, \tau) = \Delta\rho(k=0, \omega) = 0$. Taking these simplifications into account, the final form of the action (for the one-loop calculation) is given as follows;

$$\begin{aligned}
S = & \int \underline{d\omega} \underline{dk} [\{i\omega + X(k)\} \bar{\eta}(k, \omega) \eta(k, \omega) \\
& + Y(k) \{ \bar{\eta}(k, \omega) \bar{\eta}(-k, -\omega) + \eta(k, \omega) \eta(-k, -\omega) \}] \\
& + \frac{1}{2} \int \underline{d\epsilon} \underline{dq} F(q) b(-q, -\epsilon) b(q, \epsilon) \\
& + \int \underline{dk} \underline{dq} \frac{ig}{m} \Gamma_O(k, q) b(q, \epsilon) \bar{\eta}(k+q, \omega+\epsilon) \eta(k, \omega) \\
& + \int \underline{dk} \underline{dq} \frac{g}{m} \Gamma_I(k, q) b(q, \epsilon) \{ 2\bar{\eta}(k+q, \omega+\epsilon) \eta(k, \omega) \\
& + \bar{\eta}(k+q, \omega+\epsilon) \bar{\eta}(-k, -\omega) + \eta(-k-q, -\omega-\epsilon) \eta(k, \omega) \}, \tag{4.18}
\end{aligned}$$

where

$$X(k) \equiv \epsilon(k) + \Delta\omega_c + \frac{\Delta\omega_c^2}{2\epsilon(k)} - \frac{\{\epsilon(k) + \Delta\omega_c\}^2}{2\epsilon(k)} \frac{\Delta M}{F(k)}, \tag{4.19}$$

$$Y(k) \equiv \frac{1}{2} \left[\Delta\omega_c + \frac{\Delta\omega_c^2}{2\epsilon(k)} - \frac{\{\epsilon(k) + \Delta\omega_c\}^2}{2\epsilon(k)} \frac{\Delta M}{F(k)} \right], \tag{4.20}$$

$$\Gamma_O(k, q) \equiv \frac{k \times q}{q}, \tag{4.21}$$

$$\Gamma_I(k, q) \equiv \frac{1}{4} \left\{ \frac{(k+q) \cdot q}{q} I(k+q) - \frac{k \cdot q}{q} I(k) \right\}, \tag{4.22}$$

$$I(k) \equiv \left(1 - \frac{\Delta M}{F(k)} \right) \frac{\Delta\omega_c}{\epsilon(k)} - \frac{\Delta M}{F(k)}. \tag{4.23}$$

In the following subsection, we calculate the energy gap of $\eta(k, \omega)$, which is identified with the LL gap of CF's.

4.2 Energy gap of quasi-excitations

4.2.1 Tree level

We shall quantize the model constructed in the previous subsection. The partition function is given as

$$\begin{aligned}
Z &= \int [d\bar{\eta}][d\eta][dB] \exp(-S) \\
&= \int [d\bar{\eta}][d\eta][db] \exp(-S). \tag{4.24}
\end{aligned}$$

From (4.18) and (4.24), the tree-level propagators of $\eta(k, \omega)$ and $b(q, \epsilon)$ are easily obtained as

$$\begin{aligned}\langle \eta(k, \omega) \bar{\eta}(k', \omega') \rangle_0 &\equiv \bar{\delta}(k - k') \bar{\delta}(\omega - \omega') \Delta_0(k, \omega), \\ \Delta_0(k, \omega) &\equiv \frac{-i\omega + X(k)}{\{\omega^2 + E^2(k)\}},\end{aligned}\tag{4.25}$$

$$\begin{aligned}\langle \eta(k, \omega) \eta(k', \omega') \rangle_0 &= \langle \bar{\eta}(k, \omega) \bar{\eta}(k', \omega') \rangle_0 \equiv \bar{\delta}(k + k') \bar{\delta}(\omega + \omega') \tilde{\Delta}_0(k, \omega), \\ \tilde{\Delta}_0(k, \omega) &\equiv \frac{-2Y(k)}{\{\omega^2 + E^2(k)\}},\end{aligned}\tag{4.26}$$

$$\begin{aligned}\langle b(q, \epsilon) b(q', \epsilon') \rangle_0 &\equiv \bar{\delta}(q + q') \bar{\delta}(\epsilon + \epsilon') D_0(q, \epsilon), \\ D_0(q, \epsilon) &\equiv \frac{1}{F(q)},\end{aligned}\tag{4.27}$$

where

$$\begin{aligned}E^2(k) &\equiv X^2(k) - 4Y^2(k) = \left\{ 1 - \frac{\Delta M}{F(k)} \right\} \{\epsilon(k) + \Delta\omega_c\}^2, \\ \bar{\delta}(k) &\equiv (2\pi)^2 \delta(k_1) \delta(k_2), \quad \bar{\delta}(\omega) \equiv (2\pi) \delta(\omega).\end{aligned}\tag{4.28}$$

By integrating out $b(q, \epsilon)$ perturbatively, we can obtain the effective action of $\eta(k, \omega)$ and calculate its energy spectrum. In this subsection, let us consider tree-level result, ignoring the interaction terms in (4.18).

From the first and the second terms on the RHS of (4.18), it is obvious that the energy spectrum of $\eta(k, \omega)$ is obtained by the Bogoliubov transformation. In Appendix, we explicitly show how the Bogoliubov transformation is performed in the path-integral formalism. At the tree level, the final result is summarized as follows: From the propagators (4.25) and (4.26), the quadratic terms of $\eta(k, \omega)$ in the action (4.18) is diagonalized as

$$S_{tree} = \int \underline{d\omega dk} \{ i\omega + \mathcal{E}_0(k) \} \bar{\varphi}(k, \omega) \varphi(k, \omega),\tag{4.29}$$

where

$$\eta(k, \omega) = g(k) \varphi(k, \omega) - h(k) \bar{\varphi}(-k, -\omega),\tag{4.30}$$

with

$$\mathcal{E}_0(k) = E(k),\tag{4.31}$$

$$g^2(k) = \frac{1}{2} \left\{ 1 + \frac{X(k)}{E(k)} \right\}, \quad h^2(k) = \frac{1}{2} \left\{ -1 + \frac{X(k)}{E(k)} \right\}.\tag{4.32}$$

From (4.28), (4.6) and (4.16), the energy spectrum $\mathcal{E}_0(k)$ on the tree level is given by

$$\mathcal{E}_0(k) = \left(\frac{k^2}{2m} + \Delta\omega_c \right) \left(\frac{v_B k^{2-b}}{v_B k^{2-b} + \Delta M} \right)^{1/2}. \quad (4.33)$$

The second factor $[v_B k^{2-b}/(v_B k^{2-b} + \Delta M)]^{1/2}$ comes from the mixing of $\eta(x, \tau)$ and $B(x, \tau)$, and it makes the excitation energy $\mathcal{E}_0(k)$ at $k = 0$ to vanish as long as $\Delta M \neq 0$. Thus the effective LL gap of CF's tends to vanish.

To obtain the above result, the following two points are essential;

- We made an assumption that there is a dynamical gauge field $A_i(x, \tau)$ which behaves independently of $\eta(x, \tau)$
- By a coherent condensation of BCF $\phi(x, \tau)$, the dynamical gauge field becomes massive and short-ranged.

This gaplessness can be understood in the following picture. For comparison, let us first recall the example of Subsect.2.1, where there is no dynamical gauge field and the field $\eta(x, \tau)$ has a mass. The reason why the would-be NG boson $\eta(x, \tau)$ becomes massive in that case is that there are CS flux tubes attached to it and fluctuation of $\eta(x, \tau)$ necessarily induces fluctuation of CS magnetic fluxes, reflecting our assumption of the constrained phase of the CS gauge theory. The compound of $\eta(x, \tau)$ particle and flux, which is nothing but an electron density fluctuation, acquires a mass gap. Mathematically, this mechanism is realized by a cancellation of the zero mode of $\eta(x, \tau)$ by a long-range mode reflecting the CS constraint [16]. On the contrary, in the present case, there is an additional dynamical gauge field A_i . The flux fluctuation induced by $\eta(x, \tau)$ fluctuation is partly canceled (shielded) by the dynamical gauge field, so the energy of $\eta(x, \tau)$ is drastically decreased. This phenomenon is easily seen from the fact that in the action (4.10) magnetic flux of the dynamical gauge field appears in the combination $\tilde{B}(x, \tau)$. The dynamical fluxes move to shield the constrained fluxes orienting in the opposite direction to reduce the energy of the system [23]. Especially, as the dynamical gauge field $A_i(x, \tau)$ is massive, the correlation of $A_i(x, \tau)$ itself is short-ranged. Therefore, the distorted "ground state" configuration of the dynamical gauge field, which is generated by the "external field" $\Delta\rho(x, \tau)$, has less energy compared with the case

$$= \frac{g^2}{2m} \epsilon(k) \int \underline{dq} \sin^2 \phi \left\{ \frac{\epsilon(k+q)}{E(k+q)} + \frac{E(k+q)}{\epsilon(k+q)} \right\} \frac{1}{F(q)}, \quad (4.36)$$

$$\begin{aligned} \mathcal{I}_{O-}(k, \omega) &\equiv \left(\frac{ig}{m} \right)^2 \int \underline{d\epsilon} \underline{dq} \Gamma_O(k, q) \Gamma_O(-k, -q) \tilde{\Delta}_0(k+q, \omega + \epsilon) D_0(q, \epsilon) \\ &= -\frac{g^2}{2m} \epsilon(k) \int \underline{dq} \sin^2 \phi \left\{ \frac{\epsilon(k+q)}{E(k+q)} - \frac{E(k+q)}{\epsilon(k+q)} \right\} \frac{1}{F(q)}, \end{aligned} \quad (4.37)$$

$$\begin{aligned} \mathcal{I}_{I+}(k, \omega) &\equiv \left(\frac{g}{m} \right)^2 \int \underline{d\epsilon} \underline{dq} \Gamma_I(k, q) \Gamma_I(k+q, -q) \Delta_0(k+q, \omega + \epsilon) D_0(q, \epsilon) \\ &= \frac{g^2}{32m} \epsilon(k) \int \underline{dq} \left\{ \frac{(k+q) \cdot q}{kq} I(k+q) - \frac{k \cdot q}{kq} I(k) \right\}^2 \\ &\quad \times \left\{ \frac{\epsilon(k+q)}{E(k+q)} + \frac{E(k+q)}{\epsilon(k+q)} \right\} \frac{1}{F(q)}, \end{aligned} \quad (4.38)$$

$$\begin{aligned} \mathcal{I}_{I-}(k, \omega) &\equiv \left(\frac{g}{m} \right)^2 \int \underline{d\epsilon} \underline{dq} \Gamma_I(k, q) \Gamma_I(-k, -q) \tilde{\Delta}_0(k+q, \omega + \epsilon) D_0(q, \epsilon) \\ &= \frac{g^2}{32m} \epsilon(k) \int \underline{dq} \left\{ \frac{(k+q) \cdot q}{kq} I(k+q) - \frac{k \cdot q}{kq} I(k) \right\}^2 \\ &\quad \times \left\{ \frac{\epsilon(k+q)}{E(k+q)} - \frac{E(k+q)}{\epsilon(k+q)} \right\} \frac{1}{F(q)}. \end{aligned} \quad (4.39)$$

In terms of (4.34) and (4.35), the corrected propagators of $\eta(k, \omega)$ is expressed as

$$\begin{aligned} \Delta(k, \omega) &= \Delta_0(k, \omega) + \Delta_0(k, \omega) \Sigma(k, \omega) \Delta_0(k, \omega) + \dots \\ &= \left\{ \Delta_0^{-1}(k, \omega) - \Sigma(k, \omega) \right\}^{-1} \end{aligned} \quad (4.40)$$

where

$$\Delta(k, \omega) \equiv \begin{pmatrix} \Delta(k, \omega) & \tilde{\Delta}(k, \omega) \\ \tilde{\Delta}(-k, -\omega) & \Delta(-k, -\omega) \end{pmatrix}, \quad (4.41)$$

$$\Delta_0(k, \omega) \equiv \begin{pmatrix} 4\Delta_0(k, \omega) & \tilde{\Delta}_0(k, \omega) \\ \tilde{\Delta}_0(-k, -\omega) & \Delta_0(-k, -\omega) \end{pmatrix}, \quad (4.42)$$

$$\Sigma(k, \omega) \equiv \begin{pmatrix} \Sigma(k, \omega) & \tilde{\Sigma}(k, \omega) \\ \tilde{\Sigma}(-k, -\omega) & \Sigma(-k, -\omega) \end{pmatrix}. \quad (4.43)$$

In the present model, the self energies are independent of the frequency ω ,

$$\Sigma(k, \omega) = \Sigma(k), \quad (4.44)$$

$$\tilde{\Sigma}(k, \omega) = \tilde{\Sigma}(k). \quad (4.45)$$

of massless gauge field. This shielding mechanism of two kinds of fluxes naturally explains the gaplessness.

Recent experiments [7] observed the phenomenon of drastic enhancement of the CF mass near the half-filled state, which means that the energy gap of FQH states tends to vanish very rapidly as $\nu \rightarrow 1/2$. Our results explain this phenomenon. In other words, these experiments supports the existence of dynamical gauge field near the half filling.

In the present paper, we have started from the half-filled state and treated the dynamical gauge field as a totally independent dynamical variable. However, at $\nu \neq 1/2$, there may be some correlation between $A_i(x, \tau)$ and the CS gauge field $a_i(x, \tau)$ which is composed of $\eta(x, \tau)$. Therefore the above shielding mechanism may not be complete away from $\nu = 1/2$. In order to have a quantitative prediction for the gap of FQH states $\Delta\omega_c^*$ as a function of ΔB , this correlation must be taken into account. This problem is under study and result will be reproted in a forthcoming paper.

In the following subsection, we shall study the one-loop correction to the energy spectrum of $\eta(x, \tau)$ in order to check the stability of the result obtained on the tree level.

4.2.2 One-loop corrections

In this subsection, we study one-loop correction to the energy spectrum of quasi-excitations by integrating out the gauge field $b(q, \epsilon)$ in (4.24). Correction to the self energy of $\eta(k, \omega)$ is obtained by evaluating the Feynman diagrams given in Figures 1-3. From (4.25), (4.26) and the interaction terms in the action (4.18), it is obvious that the self energies and the propagators have a (2×2) -matrix structure. The $(\eta - \bar{\eta})$ self energy $\Sigma(k, \omega)$ and the $(\eta - \eta)$ self energy $\tilde{\Sigma}(k, \omega)$, which equals to the $(\bar{\eta} - \bar{\eta})$ self energy, are obtained as follows.

$$\Sigma(k, \omega) = I_{O+}(k, \omega) + 8I_{I+}(k, \omega) + 8I_{I-}(k, \omega), \quad (4.34)$$

$$\tilde{\Sigma}(k, \omega) = I_{O-}(k, \omega) + 8I_{I+}(k, \omega) + 8I_{I-}(k, \omega), \quad (4.35)$$

where the integrals $I_{O+}(k, \omega)$ etc. are given by

$$I_{O+}(k, \omega) \equiv \left(\frac{ig}{m}\right)^2 \int \frac{d\epsilon}{2\pi} \frac{dq}{2\pi} \Gamma_O(k, q) \Gamma_O(k+q, -q) \Delta_0(k+q, \omega+\epsilon) D_0(q, \epsilon)$$

In this case, we can use the formulae obtained in Appendix. As shown there, the energy spectrum of $\eta(k, \omega)$ is given in terms of the propagators (4.40) as,

$$\mathcal{E}^2(k) = [\det\{\Delta(k, 0)\}]^{-1}. \quad (4.46)$$

From (4.40), (4.34) and (4.35), the explicit expression of $\mathcal{E}(k)$ is given by

$$\begin{aligned} \mathcal{E}^2(k) &= [\det\{\Delta(k, 0)\}]^{-1} \\ &= \det\{\Delta_0^{-1}(k, 0) - \Sigma(k)\} \\ &= \{X(k) - \Sigma(k)\}^2 - \{2Y(k) - \tilde{\Sigma}(k)\}^2 \\ &= \left[1 - \frac{1}{\epsilon(k)}\{\Sigma(k) - \tilde{\Sigma}(k)\}\right] \left[E^2(k) - \epsilon(k)\{\Sigma(k) + \tilde{\Sigma}(k)\}\right] \\ &= \left[1 - \frac{g^2}{m}\mathcal{I}_1(k)\right] \left[E^2(k) - \frac{g^2}{m}\epsilon^2(k)\mathcal{I}_2(k)\right], \end{aligned} \quad (4.47)$$

where

$$\mathcal{I}_1(k) \equiv \int \underline{dq} \sin^2 \phi \frac{\epsilon(k+q)}{E(k+q)} \frac{1}{F(q)}, \quad (4.48)$$

$$\begin{aligned} \mathcal{I}_2(k) &\equiv \int \underline{dq} \sin^2 \phi \frac{E(k+q)}{\epsilon(k+q)} \frac{1}{F(q)} \\ &\quad + \int \underline{dq} \left\{ \frac{(k+q) \cdot q}{kq} I(k+q) - \frac{k \cdot q}{kq} I(k) \right\}^2 \frac{\epsilon(k+q)}{E(k+q)} \frac{1}{F(q)}. \end{aligned} \quad (4.49)$$

It is easily verified that the integrals (4.48) and (4.49) are finite constants for $k \rightarrow 0$, and so the quasi-excitation remains gapless, $\mathcal{E}(0) = 0$.

This one-loop result supports the stability of the tree-level result. According to the interpretation of gaplessness by the shielding mechanism argued in Subsec.4.1, the gaplessness reflects the NG nature of η field, so the higher-order corrections should not generate a gap.

5 Conclusion

In this paper, we studied an electron system in a uniform magnetic field near the half-filled Landau level. We started from Jain's picture of CF's at $\nu = 1/2$, and investigated the influence of the gauge-field fluctuation on the effective LL gap of CF's. It is very difficult to

study this problem for the system at general filling factor. Therefore, we considered the case of the specific filling factor like $\nu = (p - 1)/(2p + 1)$ (p is a positive integer). In this case, the system of CF's can be studied rather easily by using a BCF field, which is motivated by the investigation on the usual FQH state at $\nu = 1/q$ [13, 9, 10].

We studied the ground state and the quasi-excitations of the CF system interacting with the dynamical gauge field. We found that the effective LL gap of CF's tends to vanish because of the interaction with the transverse gauge field. This result holds for arbitrary value of the parameter b , $0 \leq b < 2$. We presented an intuitive picture to explain the result, which is based on the shielding mechanism. The dynamical fluxes shield, at least partly, the constrained fluxes which are attached to BCF's tightly. The origin of the smallness of observed energy gap is traced back to the NG nature of BCF field, η . These effects, especially the strong correlation between the constrained fluxes and BCF's, are highly nonperturbative, and it is very difficult to describe them in the framework of conventional perturbation theory.

Our study focused on specific filling factors for technical reasons. Unfortunately, we do not know a convincing method to analyze how the energy gap behaves in the system at general filling factor. However, at filling factor slightly away from $\nu = (p - 1)/(2p + 1)$, important quasi-excitations, besides those across the effective LL gap, are anyons with fractional statistics and charge, and they are vortex-like solitons in the present formalism of the bosonized fermion. These anyons are also expressed in terms of a boson field (bosonized anyon field) and a CS gauge field with a suitable CS coefficient. Then, the system is described by the action similar to (3.11) with two boson fields $\phi(x, \tau)$ and $\phi_{\text{anyon}}(x, \tau)$. At some filling factor, not only the BCF field but also the bosonized anyon field condense, just like the hierarchical state in the conventional FQHE. In this state, behavior of the dynamical gauge field and the effective LL gap of CF's can be studied by a similar method presented in this paper. Detailed investigations will be reported in a future publication.

In the present approach, the finiteness of $\Delta\omega_c^*$ at $\nu \neq 1/2$ implies that the two gauge fields, the dynamical gauge field and the CS constrained gauge field are not completely independent, but there is a small but finite correlation between them. According to our interpretation given in Sect.4.1, this correlation breaks a complete shielding between two

kinds of fluxes and generates a small but finite energy gap to BCF. This correlation effect is under study and the result will be reported in a forthcoming paper.

Appendix

A Bogoliubov transformation and the energy gap in path integrals

In this Appendix, we consider the Bogoliubov transformation in the path-integral formalism. Let us start with the following general model for complex field $\eta(k, \omega)$;

$$\begin{aligned} S_\eta &= \int [\{i\omega + X(k)\} \bar{\eta}(k, \omega) \eta(k, \omega) \\ &\quad + Y(k) \{\bar{\eta}(k, \omega) \bar{\eta}(-k, -\omega) + \eta(k, \omega) \eta(-k, -\omega)\}] \\ &= \frac{1}{2} \int \bar{\boldsymbol{\eta}}(k, \omega) \boldsymbol{\Delta}_0^{-1}(k, \omega) \boldsymbol{\eta}(k, \omega), \end{aligned} \quad (\text{A.1})$$

where

$$\boldsymbol{\eta}(k, \omega) \equiv \begin{pmatrix} \eta(k, \omega) \\ \bar{\eta}(-k, -\omega) \end{pmatrix}, \quad (\text{A.2})$$

$$\boldsymbol{\Delta}_0^{-1}(k, \omega) \equiv \begin{pmatrix} i\omega + X(k) & 2Y(k) \\ 2Y(k) & -i\omega + X(k) \end{pmatrix}. \quad (\text{A.3})$$

$$\boldsymbol{\Delta}_0(k, \omega) \equiv \frac{1}{\omega^2 + E^2(k)} \begin{pmatrix} -i\omega + X(k) & -2Y(k) \\ -2Y(k) & i\omega + X(k) \end{pmatrix}, \quad (\text{A.4})$$

$$E^2(k) \equiv X^2(k) - 4Y^2(k). \quad (\text{A.5})$$

Below we assume that $X(k), Y(k)$ are real and symmetric functions of k .

Let us introduce a new field φ as follows,

$$\eta(k, \omega) = g(k) \varphi(k, \omega) - h(k) \bar{\varphi}(-k, -\omega), \quad (\text{A.6})$$

where $g(k)$ and $h(k)$ are real and symmetric functions of k and satisfy the condition,

$$g^2(k) - h^2(k) = 1, \quad (\text{A.7})$$

to keep the integration measure unchanged (Jacobian = 1). We choose $g(k)$ and $h(k)$ to diagonalize the action in the following form:

$$S_\eta = \int \frac{d\omega dk}{(2\pi)^2} \{i\omega + \mathcal{E}_0(k)\} \bar{\varphi}(k, \omega) \varphi(k, \omega). \quad (\text{A.8})$$

This requires the relations,

$$\begin{aligned} g^2(k) + h^2(k) &= \frac{\mathcal{E}_0(k)X(k)}{E^2(k)}, \\ g(k)h(k) &= \frac{\mathcal{E}_0(k)Y(k)}{E^2(k)}. \end{aligned} \tag{A.9}$$

Eqs.(A.7,A.9) are solved as

$$\mathcal{E}_0(k) = E(k), \tag{A.10}$$

$$g^2(k) = \frac{1}{2} \left\{ 1 + \frac{X(k)}{E(k)} \right\}, \quad h^2(k) = \frac{1}{2} \left\{ -1 + \frac{X(k)}{E(k)} \right\}. \tag{A.11}$$

We note that the energy $\mathcal{E}_0(k)$ of excitations can be obtained from the expression, $\mathcal{E}_0(k)^2 = [\det \mathbf{\Delta}_0(k, 0)]^{-1}$.

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Fig. 1. Our action is not diagonal , so that there are three kinds of propagator for η (and $\bar{\eta}$). A white circle means η and a black circle means $\bar{\eta}$. A wavy line is the propagator of b field.

Fig. 2. There are four types of vertices in our model. The index "0" means $k \times q$ type, and "1" means $k \cdot q$ type.

Fig. 3. Graphical representation of the elements of η field self-energy matrix.