Mechanism of Particle-Flux Separation and Generation of Dynamical Gauge Field in the Chern-Simons Theory with Fermions

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Abstract

SW 9543

We study nonperturbative aspects of the Chern-Simons (CS) gauge theory coupled with fermions by using spatial lattice regularization. This system is relevant to the composite-fermion approach to the fractional quantum Hall effect. It is expected that there are (at least) two phases in this system: One is the constrained phase in which the CS constraint is respected by quasiparticles, so certain amount of gauge fluxes are to be attached to each fermion. The other is the *dynamical* phase in which the CS constraint is irrelevant to quasiparticles, and a dynamical gauge field is generated as an *independent* field. We call this phenomenon of irrelevance of the CS constraint a particle-flux separation (PFS), since particles and fluxes move independently here. This PFS is characterized through dynamics of an auxiliary gauge field that glues particles and CS fluxes, and bears very close resemblance to the phenomenon of charge-spin separation in strongly-correlated electron system for high- T_c superconductivity. We show that there is a confinement-deconfinement (CD) phase transition in the gauge dynamics in question at certain critical temperature T_{CD} , which depends on the CS coefficient and the density of fermions. The constrained phase is identified with the confinement phase of gauge dynamics at $T > T_{CD}$, while the dynamical phase with PFS is identified with the deconfinement phase at $T < T_{CD}$. This result is applicable to the system of electrons in a uniform magnetic field at Landau filling factor $\nu = 1/2$. That is, there appears a dynamical gauge field that interacts weakly with the so-called composite fermions. This justifies several existing analyses based on perturbation theory, which conclude non-Fermi-liquid-like behavior of fermionic quasiparticles.

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1 Introduction

Recently, gauge field theories of fermions have attracted lots of interest in condensed matter physics. In the strongly-correlated electron system for high- T_c superconductivity, it is expected that the quasiparticles in the anomalous metallic phase are holons, spinons and dynamical gauge bosons; all these are to be generated as a result of the phenomenon called charge-spin separation (CSS)[1]. As we showed in the previous papers[2], a confinement-deconfinement (CD) phase transition of gauge theory at finite temperature naturally explains an experimentally observed phase transition (or crossover) between the anomalous and normal metallic states.

In the fractional quantum Hall effect (FQHE), the Chern-Simons (CS) gauge theory plays an essentially important role in the Ginzburg-Landau-type effective field theory[3]. The Hall liquid is characterized there by a condensation of bosonized electrons, which are nothing but electrons with odd number of flux quanta attaching to each of them. Being motivated by this success and the suggestion by Jain[4], Halperin, Lee and Read[5] studied a system of electrons in the half-filled Landau level by introducing a composite fermion field: a composite fermion is a fermionic composite of an electron and two flux quanta. A field theoretical model of this system is the CS gauge theory coupled with nonrelativistic fermions with a suitable CS coefficient. Analyses using the random-phase approximation (RPA)[5, 6] and the renormalization-group equations (RGE)[7] show that low-energy excitations of this model behave as a non-Fermi liquid. Recent experiments on the FQHE at the sequence of filling factors $\nu = p/(2p \pm 1)$ (p runs over positive integers)[8] provide an important clue to the possible non-Fermi-liquid-like nature of electrons at the half filling.

In the CS gauge theory, magnetic field strength of the CS gauge field is completely controlled by the fermion density, as dictated by the *local* CS constraint [See (2.4) below]. However, the most important point in the above RPA and RGE analyses of the CS gauge theory is the assumption that the CS constraint becomes irrelevant to quasiparticles at low energies, and consequently, a dynamical gauge field is generated.¹ Namely, there may take place a

¹We use the term quasiparticle as a particle picture associated with variables that diagonalize the Hamiltonian approximately. Such variables are called asymptotic fields and should be distinguished from the original Heisenberg fields. The point here is that the CS constraint should be always respected by the Heisenberg fields but may not by the asymptotic fields. See [2] for more details.

phenomenon that the binding implied by the CS constraint between an *original* fermion and magnetic fluxes of the CS field becomes physically irrelevant since the original fermions are *not* quasiparticles at all, and the *quasiparticles* themselves that are fermionic and magnetic fluxes are unbound and move separately. We call this phenomenon a particle-flux separation (PFS). When a PFS takes place, the CS gauge field is to describe an independent dynamical field as we shall see.

In RPA and RGE, the non-Fermi-liquid behavior of fermions is attributed to interactions with the transverse component of such a dynamical gauge field. The time component A_0 of the gauge field works as a Lagrange multiplier for the CS constraint. The RPA and RGE show that A_0 becomes massive due to fermion-loop effects, and so its fluctuations are suppressed at low energy. This is consistent with the relaxation of the CS constraint and the assumption of PFS.

In order to reach full understanding of what the CS constraint implies physically and to judge the validity of those calculations of RPA and RGE, we definitely need separate analyses that are capable to clarify appropriateness and limitation of the assumption of PFS. In the present paper, we shall address ourselves to this basic problem.

We stress that this phenomenon, PFS, is closely related to the CSS in the strongly-correlated electron system for high- T_c superconductivity. We shall employ the methods and techniques very similar to those used in Ref.[2] for studying the CSS. They make use of the notion of CD phase transition in lattice gauge theory and have proved themselves very efficient, leading to the correct results.

This paper is organized as follows. In Sect.2, we shall introduce a CS gauge model of fermions on a spatial lattice. With a specific choice of the CS coefficient and the fermion density, this model describes electrons in the half-filled Landau level, the fermions in the model being understood as composite fermions. Then we shall rewrite the model by introducing an auxiliary complex field living on links, the phase part of which will be identified with the gauge field that glues a fermion and CS magnetic fluxes. We explain that, if the dynamics of this gauge field is realized in a confinement phase, then the CS constraint is respected faithfully by the quasiparticles. On the other hand, if it is realized in a deconfinement phase, the PFS occurs, and, at the same time, this gauge field is not auxiliary any more but behaves as dynamical.

In Sect.3, we present a mean-field-theory type calculation that determines the absolute value of the above auxiliary complex field. It is shown that the auxiliary field has a nonvanishing amplitude at low temperatures (T). So a gauge field can be introduced as its phase degrees of freedom.

In Sect.4, a Ginzburg-Landau (GL) theory of the auxiliary gauge field is derived by using the hopping expansion of fermions. Its electric term and magnetic term are studied at general values of the CS coefficient and the fermion density. At low T , the electric term favors static configurations of the gauge field, i.e., configurations with *no* nontrivial winding numbers along the imaginary-time direction. This implies that it favors a deconfinement phase of gauge theory, where fluctuations of the gauge field are small and can be treated by perturbation theory. On the other hand, at high T , we show that it favors a confinement phase of the gauge theory, where fluctuations of the gauge field are large and violent. The magnetic term of the GL theory determines the relevant spatial configuration of gauge field. For some values of the CS coefficient and the fermion density, the uniform fluxless configuration has the lowest energy. In Sect.4.2, this result is applied to the electron system in the half-filled Landau level. In general cases, configurations with nontrivial fluxes are favored. Another interesting case is the 2-dimensional quantum XY spin model represented in terms of fermions of Jordan-Wigner type[9]. This case is discussed in some detail in Sect.4.3.

In Sect.5, discussion and conclusions are given. An interesting possibility of the third phase of the CS gauge theory is pointed out. It is a Higgs phase that supports superconductivity.

2 The model

We formulate the CS gauge theory of spinless fermions on a 2-dimensional spatial lattice. We start with the following Lagrangian density L in the imaginary-time formalism;

$$L = - \sum_x \bar{\psi}_x (\partial_0 - iA_0^{CS} - \mu_c) \psi_x - \frac{1}{2m} \sum_{x,j} (\bar{\psi}_{x+j} e^{i(A_{xj}^{CS} - eA_{xj}^{ex})} \psi_x + \text{H.c.}) - \frac{i}{2\pi q} \sum_{x,\mu\nu\lambda} \epsilon^{\mu\nu\lambda} A_{x\mu}^{CS} F_{\nu\lambda}^{CS}, \quad (2.1)$$

where x denotes the lattice sites and Greek indices $\mu, \nu, \dots$ take 0, 1, 2 and denote the directions (0; imaginary-time, 1, 2; spatial). $j = 1, 2$ (and i, k appearing later) denotes the spatial

directions and $\hat{j}$ are the unit vectors. ψ_x is a one-component Grassmann number to describe fermions, and the real field $A_{x\mu}^{CS}$ describes the CS gauge field. They are functions also of the imaginary time τ that runs from 0 to the inverse temperature $\beta = 1/T$. We suppress the argument τ for simplicity when no confusions would arise. The average fermion density $\rho = \langle \bar{\psi}_x \psi_x \rangle$ is set to a given number by adjusting the chemical potential μ_c . q and m are parameters of the model. We have introduced also an external vector potential A_{xi}^{ex} , and e is the charge of ψ . In most of the cases discussed below, A_{xi}^{ex} describes a uniform magnetic field B^{ex} perpendicular to the spatial 1-2 plane. The field strength $F_{\mu\nu}$ at x is given by

$$\begin{aligned} F_{ij} &= \Delta_i A_j^{CS} - \Delta_j A_i^{CS}, \\ F_{0i} &= \partial_0 A_i^{CS} - \Delta_i A_0^{CS}, \end{aligned} \quad (2.2)$$

where Δ_i is the difference operator on the lattice.

The Lagrangian (2.1) possesses a gauge symmetry under

$$\begin{aligned} \psi_x &\rightarrow e^{i\phi_x} \psi_x \\ A_{xi}^{CS} &\rightarrow A_{xi}^{CS} + \Delta_i \phi_x \\ A_{x0}^{CS} &\rightarrow A_{x0}^{CS} + \partial_0 \phi_x, \end{aligned} \quad (2.3)$$

up to surface terms.²

We shall study this model in the temporal gauge $A_{x0}^{CS} = 0$. The residual time-independent gauge symmetry is respected by requiring the physical states to satisfy the constraint $\delta L / \delta A_{x0}^{CS} = 0$, or equivalently, the CS constraint,

$$B_x^{CS} \equiv \epsilon_{ij} \Delta_i A_{xj}^{CS} = 2\pi q \hat{\rho}_x, \quad (2.4)$$

where $\hat{\rho}_x \equiv \bar{\psi}_x \psi_x$. From (2.4), it is obvious that the parameter q is the number of flux quanta attaching to each fermion. The general solution to (2.4) is obtained as follows,

$$A_{xi}^{CS} = 2\pi q \epsilon_{ij} \sum_{r'} \Delta_j G(r, r') \hat{\rho}_{r'} + \Delta_i \chi_x, \quad (2.5)$$

²Strictly speaking, the last CS term gives rise to extra non-gauge invariant terms, reflecting that the difference operators does not satisfy the Leibnitz's rule. This brings, however, no serious difficulties in later discussion: Our strating model , (2.8) or (2.11), is fully gauge invariant.

where $G(r, r')$ is the lattice Green function, r and r' in which are sites of the dual lattice and make dual pairs with x and x' , respectively [9]. The scalar field χ_x is a *time-independent* arbitrary function describing the longitudinal part of the gauge field. Under a time-independent residual gauge transformation, χ_x transforms as

$$\chi_x \rightarrow \chi_x + \phi_x. \quad (2.6)$$

Let us give an example of physical systems described by this model. It is the electron system in the half-filled Landau level. This system, being put on a lattice, is described by the model (2.1) and (2.5) with the choice $q = 2$. This can be seen as follows. We determine q by estimating the filling factor $\nu \equiv 2\pi\rho/eB^{ex}$ by imposing the condition,

$$\langle B^{CS} \rangle = eB^{ex}. \quad (2.7)$$

This condition and the average of (2.5), i.e., $\langle B^{CS} \rangle = 2\pi q\rho$, lead to the expression $\nu = 1/q$. Thus the half filling $\nu = 1/2$ corresponds to $q = 2$. In this case, ψ_x describes so-called composite fermions (See sect.4.2).

In the temporal gauge $A_0^{CS} = 0$, L is expressed in terms of ψ as³

$$L = -\sum \bar{\psi}_x(\partial_0 - \mu_c)\psi_x - \frac{1}{2m} \sum \bar{\psi}_{x+j}W_{x+j}\bar{W}_x\psi_x + \text{H.c.}, \quad (2.8)$$

$$W_x = \exp[iq \sum_{r'} \theta(x, r')(\bar{\psi}_{x'}\psi_{x'} - \rho) + i\chi_x], \quad (2.9)$$

where $\theta(x, r')$ is the multivalued azimuthal angle function in the 2-dimensional lattice, and we have used the identity

$$2\pi\epsilon_{ij}\Delta_j G(r, r') = \Delta_i\theta(x, r'). \quad (2.10)$$

We have also used $B^{ex} = 2\pi q\rho/e$ as explained above to solve A^{ex} in the form of (2.5) with $\hat{\rho} \rightarrow \rho, \chi \rightarrow 0$. With this choice of q , or B^{ex} , convergence of the sum in the exponent of (2.9) is guaranteed for asymptotically uniform distributions of fermions. W_x corresponds to the line integral $\exp(i \int^x (A_j^{CS} - eA_j^{ex})dx_j)$ in the continuum, where the contribution from the infinity is assumed negligible. Physically, W_x is nothing but the creation operator of q fluxes at x .

³In the path integral quantization, we note that the second hopping term in (2.8) should have been put in a normal-ordered form w.r.t. ψ_x 's. This can be worked out easily; e.g., by adopting the operator identity $\exp(\alpha\psi^\dagger\psi) = 1 + (\exp(\alpha) - 1)\psi^\dagger\psi$. Below, we assume this although not explicitly written down.

Let us introduce an auxiliary complex field V_{xj} sitting on the link $(x, x+j)$ to decouple the term $\bar{\psi}W\bar{W}\psi$ in (2.8). By using the Hubbard-Stratonovich transformation in path-integrals, we get the decoupled Lagrangian,

$$L = -\sum \bar{\psi}_x(\partial_0 - \mu_c)\psi_x + \sum \left[V_{xj}(\gamma \bar{W}_{x+j}W_x - \frac{1}{2m\gamma}\bar{\psi}_{x+j}\psi_x) + \text{H.c.} \right] - \sum |V_{xj}|^2 + \frac{1}{(2m\gamma)^2} \sum \bar{\psi}_{x+j}\bar{\psi}_x\psi_x\psi_{x+j}. \quad (2.11)$$

We introduced a parameter γ for dimensional reason. Its dimension is $[T]^{1/2}$. γ is important since it will determine the relative velocity of particles and fluxes in the dynamical phase. The equation of motion of V_{xj} is given by

$$V_{xj} = \gamma \bar{W}_x W_{x+j} - \frac{1}{2m\gamma} \bar{\psi}_x \psi_{x+j}. \quad (2.12)$$

Under the residual gauge transformation, V_{xj} transforms as

$$V_{xj} \rightarrow \exp(i\phi_x)V_{xj}\exp(-i\phi_{x+j}). \quad (2.13)$$

Therefore its phase degree of freedom behaves as a gauge field as announced in Introduction.

The physical origin of the four-Fermi term in the Lagrangian (2.11) is obvious. Fluctuations of the field V_{xj} generate an extra attraction (a repulsion) in the channel $\bar{\psi} - \psi$ ($\bar{\psi} - \bar{\psi}$ and $\psi - \psi$), besides an attraction in the $\psi - \bar{W}$ channel. The four-Fermi term "cancels" out this attraction or repulsion among fermions. In most of the following discussion, this term will be neglected, since its effect can be discussed separately as we shall do in Sect.5.

We shall study the dynamics of V_{xj} by integrating out the fermion field ψ_x . As we discussed in the previous papers[2, 10] on the CSS and the gauge theory of nonrelativistic fermions, the behavior of fermions ψ_x is to be distinguished according to whether the gauge dynamics of V_{xj} is realized in a confinement phase (I) in which V_{xj} *vanishes in average* or in a deconfinement phase (II) in which V_{xj} *develops a nonvanishing expectation value*.

(I) If the gauge dynamics is in the confinement phase, neither ψ_x nor W_x has its own hopping amplitude; only charge neutral compounds like $\bar{W}_x\psi_x$ may hop and appear as physical objects. ψ_x "particles" couple strongly with the gauge potential (2.5), which is generated by ψ_x 's themselves. In this case, no mean-field theory treating ψ and W as quasiparticles is applicable for solving the model (2.1). Instead, the so-called strong-coupling expansion, an expansion

w.r.t. powers of V_{xj} , is applicable. For the electron system at $\nu = 1/2$, the product $\bar{W}_x \psi_x$ describes nothing but the original electron operator up to an irrelevant phase.⁴ Therefore, the quasiparticles are not the composite fermions ψ but the original electrons themselves.

(II) If the gauge dynamics is in the deconfinement phase, there is a finite hopping amplitude for ψ_x which is proportional to $\langle V_{xj} \rangle$, so ψ_x may hop and become the quasiparticles. Also the gauge field, the phase of V_{xj} , should behave as quasiparticles, since kinetic terms of V_{xj} are to be generated through hoppings of ψ_x 's. From the form of coupling to ψ_x 's, we see that this dynamical gauge field corresponds to $A_{xj}^{CS} - eA_{xj}^{ex}$, that is the local fluctuations of the original CS gauge field around the external electromagnetic field. ψ_x and V_{xj} are interacting perturbatively with each other.⁵ This phase is still quite nontrivial, due to perturbative but non-negligible gauge interactions. Many studies have been done on the low-energy properties of this fermion-gauge-field system [5, 6, 7]. The RGE study exhibits that (i) there is a nontrivial infrared fixed point and (ii) marginal-Fermi-liquid-like behavior shows up at low energies.

As we shall see in the subsequent sections, there is a critical temperature T_{CD} , at which a confinement-deconfinement (CD) phase transition takes place for physically interesting cases of our model. Therefore, analyses like RPA and RGE, being based on perturbation theory with a small gauge coupling, are validated in the region of $T < T_{CD}$.

In Sect.3, we shall first determine the *amplitude* of V_{xj} by a mean-field-type analysis; it is nonvanishing at low T . Then, in Sect.4, we shall study the dynamics of phase degrees of freedom of V_{xj} by using a Ginzburg-Landau-type theory of V_{xj} 's. In Sect.3 and Sect.4, we adopt a general strategy: (i) to study the region around T_{CD} , we use a hopping expansion, an expansion in powers of V_{xj} , because the average of V_{xj} is small there; (ii) to study the region near $T = 0$, we use a generalized mean-field theory assuming a uniform distribution of fermions, $\langle \bar{\psi}_{x+j} \psi_x \rangle = \text{constant}$, because such a distribution is expected to give a lower energy.

⁴More precisely, the electron operator ψ_e is expressed as $\psi_e = \exp(-2i \sum \theta(x, r') \bar{\psi}_{x'} \psi_{x'} - i\chi_x) \psi_x$.

⁵It is interesting to consider a *dual* description of the present approach in which a composite fermion is viewed as a compound of an electron and CS fluxes. There, a gauge field $\tilde{V}_{xj}$ dual to V_{xj} controls the interaction between the *original electron* and the CS fluxes. In the *confinement* phase of $\tilde{V}_{xj}$, composite fermions are quasiparticles.

3 Calculation of the amplitude of V_{xj}

To study the dynamics of phase degrees of freedom of V_{xj} , it is necessary to know the behavior of its amplitude, $|V_{xj}|$. For example, its phase can be defined only when the amplitude is nonvanishing.

As in the case of the CSS, this can be done by a mean-field-type calculation. For the present model (2.11), we put $V_{xj} = V_0$, where a "mean field" V_0 is a real link-independent constant. It is general enough as long as the system does not prefer "dimer" configurations where the amplitude may vary link by link. Then, the mean-field Lagrangian is given as

$$L_{MF} = -\sum \bar{\psi}_x(\partial_0 - \mu_c)\psi_x - \sum (|V_0|^2 + \gamma^2) + \sum \left[V_0(\gamma \bar{W}_{x+j} W_x - \frac{1}{2m\gamma} \bar{\psi}_{x+j} \psi_x) + \text{H.c.} \right], \quad (3.1)$$

where we have omitted the four-Fermi term as announced.

The effective potential of V_0 is obtained by integrating out the field ψ_x . If there were no contributions from the term $V_0 W_{x+j} \bar{W}_x$, the effective potential could be calculated exactly, because L_{MF} reduces to a system of free fermions. This type of calculations (or for more complicated cases) was performed, for example, in the mean-field theory for the t-J model [11], and it is shown that there is a critical temperature T_{MF} , below which V_0 develops a nonvanishing expectation value.

We shall see how the term $V_0 W \bar{W}$ affects the above result.⁶ We consider two cases separately; the case of high T in Sect.3.1 and the case of low T in Sect.3.2. The result is illustrated in Fig.1.

3.1 The single-link potential in the hopping expansion

At high T , the hopping expansion is a reliable method because the expectation value of V_{xj} is small at T near T_{CD} , $T < T_{CD}$, or zero at $T > T_{CD}$, when the CD transition is of second or higher order. In the hopping expansion, one assumes that amplitudes of hopping processes of fermions and fluxes are small and expands $\exp\left(\int d\tau L_{MF}\right)$ in powers of V_0 , and integrates out

⁶From (2.12), it is obvious that the term $V_{xj} \bar{W}_{x+j} W_x$ reflects the fact that the system is not of a simple bilinear form of fermions; the fermions have nonlocal and nonpolynomial interactions.

the fermion field. As a result, we obtain a GL theory of V_{xj} . The full hopping expansion will be performed in Sect.4. Below in this subsection we calculate the single-link potential $P(V_0)$ of V_0 to get a rough estimation of V_0 .

To be explicit, one needs the fermion propagator of the following form;

$$\begin{aligned}\langle \psi_x(\tau_1) \bar{\psi}_y(\tau_2) \rangle &= \delta_{x,y} \cdot G_\psi(\tau_1 - \tau_2) \\ G_\psi(\tau) &\equiv \frac{e^{-\mu_c \tau}}{1 + e^{-\beta \mu_c}} [\theta(\tau) - e^{-\beta \mu_c} \theta(-\tau)] \\ &= e^{-\mu_c \tau} [(1 - \rho) \theta(\tau) - \rho \theta(-\tau)],\end{aligned}\tag{3.2}$$

where we used the following expression of the fermion density in the leading order;

$$\rho = \lim_{\tau_2 \rightarrow \tau_1 + 0} \langle \psi(\tau_2) \psi(\tau_1) \rangle = \frac{e^{-\beta \mu_c}}{1 + e^{-\beta \mu_c}}.\tag{3.3}$$

From (3.1), the single-link potential at the tree level, $P_{\text{tree}}(V_0)$, is given by

$$P_{\text{tree}}(V_0) = V_0^2.\tag{3.4}$$

In the second order of the hopping expansion, the fermion hopping term $\bar{\psi}_{x+j} V_0 \psi_x$ gives the contribution,

$$\begin{aligned}\Delta P_\psi^{(2)} &= -\left(\frac{V_0}{2m\gamma}\right)^2 \frac{1}{\beta} \\ &\quad \times \int_0^\beta d\tau_1 d\tau_2 \langle \bar{\psi}_x(\tau_1) \psi_{x+j}(\tau_1) \bar{\psi}_{x+j}(\tau_2) \psi_x(\tau_2) \rangle \\ &= -\left(\frac{V_0}{2m\gamma}\right)^2 \beta \rho (1 - \rho),\end{aligned}\tag{3.5}$$

where we have used (3.2) and (3.3) together with the Wick's theorem.

The contribution from the flux hopping term $V_0 W_{x+j} \bar{W}_x$ is also calculated in a similar way. To this end, it is useful to use the following expression of $W_{x+j} \bar{W}_x$;

$$W_{x+j} \bar{W}_x = \exp[2\pi q i \sum_{r'} \epsilon_{jl} \Delta_l G(r, r') (\hat{\rho}(x') - \rho) + i \Delta_j \chi_x].\tag{3.6}$$

From (3.6), the contribution to the potential $P(V_0)$ is written as

$$\begin{aligned}\Delta P_W^{(2)} &= -V_0^2 \frac{\gamma^2}{\beta} \int_0^\beta d\tau_1 d\tau_2 \langle W_{x+j} \bar{W}_x(\tau_1) W_x \bar{W}_{x+j}(\tau_2) \rangle \\ &= -V_0^2 \frac{\gamma^2}{\beta} \int_0^\beta d\tau_1 d\tau_2 \langle \exp[2\pi q i \sum_{r'} \epsilon_{jl} \Delta_l G(r, r') (\bar{\psi}_{x'} \psi_{x'}(\tau_1) \\ &\quad - \bar{\psi}_{x'} \psi_{x'}(\tau_2))] \rangle.\end{aligned}\tag{3.7}$$

The expectation value $\langle W\bar{W}W\bar{W} \rangle$ in (3.7) is easily evaluated by the hopping expansion. On each site x and for an arbitrary α , the leading order of the hopping expansion involves the following averages,

$$\begin{aligned}
& \langle e^{\alpha(\bar{\psi}_x\psi_x(\tau_1)-\bar{\psi}_x\psi_x(\tau_2))} \rangle \\
&= \langle [1 + (e^\alpha - 1)\bar{\psi}_x\psi_x(\tau_1)][1 + (e^{-\alpha} - 1)\bar{\psi}_x\psi_x(\tau_2)] \rangle \\
&= 1 + (e^\alpha - 1)\rho + (e^{-\alpha} - 1)\rho + (e^\alpha - 1)(e^{-\alpha} - 1)\rho \\
&= 1.
\end{aligned} \tag{3.8}$$

In passing from the first line to the second, we take into account that the W_x variables, hence the first line, had been normal-ordered w.r.t. ψ 's in path integral formalism. As a result, in the second order of the hopping expansion, we get

$$\Delta P_W^{(2)} = -V_0^2\gamma^2\beta. \tag{3.9}$$

By collecting (3.4), (3.5) and (3.9), we get

$$P(V_0) = [1 - \beta(\frac{\rho(1-\rho)}{\gamma^2 + 4m^2\gamma^2})]V_0^2, \tag{3.10}$$

which indicates that, at low T , the origin $V_0 = 0$ is not stable and so V_0 should have a nonvanishing expectation value. The critical temperature T_{MF} is estimated as

$$T_{MF} \sim \gamma^2 + \frac{\rho(1-\rho)}{4m^2\gamma^2}. \tag{3.11}$$

Let us calculate the fourth-order terms of the hopping expansion. The fermion hopping term gives

$$\begin{aligned}
\Delta P_\psi^{(4)} &= \frac{1}{2}\left(\frac{V_0}{2m\gamma}\right)^4 \frac{1}{\beta} \prod_{i=1}^4 \int d\tau_i G_\psi(\tau_i - \tau_{i+1}) \\
&= \frac{1}{2}C_\psi\left(\frac{V_0}{2m\gamma}\right)^4\beta^3,
\end{aligned} \tag{3.12}$$

$$C_\psi \equiv \frac{1}{4!}\left\{-4\rho(1-\rho)^3 + 12\rho^2(1-\rho)^2 - 4\rho^3(1-\rho)\right\}, \tag{3.13}$$

where $\tau_5 = \tau_1$. We note that the case of half filling (See Sect.4.2) requires $\rho = 1/2$, which leads to $C_\psi = 1/96 > 0$.

There is another important fourth-order term in the potential of V_0 which comes from the combination of $V_0 W_{x+j} \bar{W}_x$ and $V_0 \bar{\psi}_{y+\hat{k}} \psi_y$. This contribution is nothing but the connected part defined as

$$\begin{aligned} & \langle W \bar{W} W \bar{W} \bar{\psi} \psi \bar{\psi} \psi \rangle_c \\ &= \langle W \bar{W} W \bar{W} \bar{\psi} \psi \bar{\psi} \psi \rangle - \langle W \bar{W} W \bar{W} \rangle \cdot \langle \bar{\psi} \psi \bar{\psi} \psi \rangle, \end{aligned} \quad (3.14)$$

where the subscript c implies the connected part. Explicitly, it reads for each link $(x, x + \hat{j})$ as

$$\begin{aligned} \Delta P_W^{(4)} &= -\frac{V_0^4}{(2m)^2 \beta} \prod_{i=1}^4 \int d\tau_i \langle W_{x+j} \bar{W}_x(\tau_1) W_x \bar{W}_{x+j}(\tau_2) \\ &\quad \times \sum_{y,k} \bar{\psi}_{y+\hat{k}}(\tau_3) \psi_y(\tau_3) \bar{\psi}_y(\tau_4) \psi_{y+\hat{k}}(\tau_4) \rangle_c \\ &= -\frac{V_0^4}{(2m)^2 \beta} \sum_{y,k} \prod_{i=1}^4 \int d\tau_i \langle e^{2\pi q i \epsilon_{jl} \Delta_l G(r, y+\hat{k})} \bar{\psi}_{y+\hat{k}} \psi_{y+\hat{k}}(\tau_1) \\ &\quad \times e^{2\pi q i \epsilon_{jl} \Delta_l G(r, y)} \bar{\psi}_y \psi_y(\tau_1) \cdot e^{-2\pi q i \epsilon_{jl} \Delta_l G(r, y+\hat{k})} \bar{\psi}_{y+\hat{k}} \psi_{y+\hat{k}}(\tau_2) \\ &\quad \times e^{-2\pi q i \epsilon_{jl} \Delta_l G(r, y)} \bar{\psi}_y \psi_y(\tau_2) \cdot \bar{\psi}_{y+\hat{k}}(\tau_3) \psi_y(\tau_3) \bar{\psi}_y(\tau_4) \psi_{y+\hat{k}}(\tau_4) \rangle_c, \end{aligned} \quad (3.15)$$

where we have used (3.8) for averages at sites other than y and $y + \hat{k}$. It is not difficult to calculate the matrix element in (3.15). One way to do this is as follows; (i) first fix the ordering of τ 's, say, $\tau_1 > \tau_3 > \tau_2 > \tau_4$, (ii) second rewrite (3.15) as an average in the operator formalism w.r.t. the Hamiltonian $H_0 = -\mu_c \sum_x \bar{\psi}_x \psi_x$ to proceed as

$$\begin{aligned} & \langle e^{2\pi q i \epsilon_{jl} \Delta_l G(r, y+\hat{k})} \bar{\psi}_{y+\hat{k}} \psi_{y+\hat{k}} \cdot e^{2\pi q i \epsilon_{jl} \Delta_l G(r, y)} \bar{\psi}_y \psi_y \cdot \bar{\psi}_{y+\hat{k}} \psi_y \\ & \times e^{-2\pi q i \epsilon_{jl} \Delta_l G(r, y+\hat{k})} \bar{\psi}_{y+\hat{k}} \psi_{y+\hat{k}} \cdot e^{-2\pi q i \epsilon_{jl} \Delta_l G(r, y)} \bar{\psi}_y \psi_y \bar{\psi}_y \psi_{y+\hat{k}} \rangle_c \\ &= \langle e^{2\pi q i \epsilon_{jl} \Delta_l G(r, y+\hat{k})} \bar{\psi}_{y+\hat{k}} \psi_{y+\hat{k}} \cdot \bar{\psi}_{y+\hat{k}} \cdot e^{-2\pi q i \epsilon_{jl} \Delta_l G(r, y+\hat{k})} \bar{\psi}_{y+\hat{k}} \psi_{y+\hat{k}} \cdot \psi_{y+\hat{k}} \rangle \\ & \times \langle e^{2\pi q i \epsilon_{jl} \Delta_l G(r, y)} \bar{\psi}_y \psi_y \cdot \psi_y \cdot e^{-2\pi q i \epsilon_{jl} \Delta_l G(r, y)} \bar{\psi}_y \psi_y \cdot \bar{\psi}_y \rangle - \rho(1 - \rho) \\ &= \rho(1 - \rho) \left(e^{2\pi q i \epsilon_{jl} \Delta_k^y \Delta_l^r G(r, y)} - 1 \right). \end{aligned} \quad (3.16)$$

To obtain the last line, we notice that only the relevant state is the empty state at y with the probability $(1 - \rho)$ and the occupied state at $y + \hat{k}$ with the probability ρ . The second term $\rho(1 - \rho)$ is the disconnected term. Similarly, for $\tau_2 > \tau_3 > \tau_1 > \tau_4$, the matrix element is given by

$$\rho(1 - \rho) \left(e^{-2\pi q i \epsilon_{jl} \Delta_k^y \Delta_l^r G(r, y)} - 1 \right). \quad (3.17)$$

The other time orderings give either (3.16), (3.17) or vanishing results. From the long-distance behavior of the Green function $G(r, y) \sim \ln|r - y|$, the sum of (3.16) and (3.17) is estimated as

$$\begin{aligned} C_W(q) &\equiv \sum_{y,k} \left(e^{2\pi q i \epsilon_{jl} \Delta_k^y \Delta_l^r G(r,y)} - 1 \right) + \text{c.c.} \\ &\sim -(2\pi q)^2 \int dr \frac{1}{r^3} + \dots \end{aligned} \quad (3.18)$$

In this way, we finally obtain

$$\Delta P_W^{(4)} = -\frac{\rho(1-\rho)}{2} C_W(q) \frac{V_0^4 \beta^3}{(2m)^2}. \quad (3.19)$$

By including these fourth-order terms $\Delta P_\psi^{(4)}$ and $\Delta P_W^{(4)}$, the effective potential $P(V_0)$ now takes a conventional GL form;

$$\begin{aligned} P(V_0) &= (T - T_{MF}) V_0^2 + C_4 V_0^4, \\ C_4 &= \beta^3 \left[\frac{C_\psi}{2(2m\gamma)^4} - \frac{C_W}{(2m)^2} \right], \end{aligned} \quad (3.20)$$

with $C_4 > 0$ for $\rho = 1/2$. Therefore, at $T > T_{MF}$, $V_0 = 0$, and below and near T_{MF} , V_0 has the typical behavior of an order parameter of a second-order phase transition,

$$V_0 \sim \frac{1}{\sqrt{2C_4}} \sqrt{T_{MF} - T}. \quad (3.21)$$

3.2 The mean-field theory at low temperatures

Let us consider the behavior of V_0 at very low T . From the result in Sect.3.1, one expects that the amplitude V_0 develops well in this region. To obtain an explicit result we assume that the expectation value of V_{x_j} itself may develop a stable nonvanishing value there. We shall find below that this assumption gives a self-consistent result. We note that finiteness of $\langle V_{x_j} \rangle$ generally favors homogeneous fermion distributions, because it generates a hopping term of ψ_x in (3.1) that affords fermions to hop randomly over the lattice. In this case, we can set $\bar{\psi}_x \psi_x \rightarrow \rho$ in (2.9) with good accuracy. Then the term $\bar{V}_{x_j} W_{x+j} \bar{W}_x$ reduces to $V_{x_j} \exp(i\Delta_j \chi_x)$, which is nothing but the conventional gauge coupling to the Higgs field $\exp(i\chi_x)$. Mean-field theories of the gauge-Higgs model have been studied in various ways. We can set $\chi_x = 0$ as a gauge-fixing condition to the unitary gauge. Then, we conclude that the term $\bar{V}_{x_j} W_{x+j} \bar{W}_x$

supplies the term $-\sum V_0$ for the effective potential of V_0 . This is linear in V_0 and prefers, by itself, a positive value of V_0 .

Taking the above linear term into account, we start with the mean-field Hamiltonian,

$$\begin{aligned} H_{MF} &= \frac{V_0}{2m\gamma} \sum (\psi_{x+i}^\dagger \psi_x + \text{H.c.}) - \mu_c \sum \psi_x^\dagger \psi_x \\ &+ 2NV_0^2 - 4\gamma NV_0, \end{aligned} \quad (3.22)$$

where N is the total number of lattice sites. Since this describes a system of free fermions, its partition function Z_{MF} is written down as

$$\begin{aligned} Z_{MF} &= \text{Tr} \exp(-\beta H_{MF}) \\ &= \exp(-\beta F_{MF}) \\ &= e^{-\beta[N(2V_0^2 - 4\gamma V_0)]} \prod_k \left[1 + \exp(-\beta[\frac{V_0}{m\gamma} \sum_i \cos k_i - \mu_c]) \right], \end{aligned} \quad (3.23)$$

where k_i is the lattice momentum. The free energy per site is thus given by

$$\frac{F_{MF}}{N} = 2V_0^2 - 4\gamma V_0 - \frac{1}{\beta N} \sum_k \ln[1 + \exp(-\beta[\frac{V_0}{m\gamma} \sum_i \cos k_i - \mu_c])]. \quad (3.24)$$

The value of V_0 is determined by minimizing F_{MF}/N ,

$$\begin{aligned} \frac{\partial}{\partial V_0} \frac{F_{MF}}{N} &= 4V_0 - 4\gamma + \frac{1}{m\gamma N} \sum_k (\sum_i \cos k_i) f(k) \\ &= 0, \end{aligned} \quad (3.25)$$

where $f(k)$ is the Fermi distribution function,

$$f(k) = \frac{\exp\{-\beta[\frac{V_0}{m\gamma} \sum_i \cos k_i - \mu_c]\}}{1 + \exp\{-\beta[\frac{V_0}{m\gamma} \sum_i \cos k_i - \mu_c]\}}. \quad (3.26)$$

μ_c is determined by the condition for ρ ,

$$\begin{aligned} \rho &= \frac{1}{\beta N Z_{MF}} \frac{\partial}{\partial \mu_c} Z_{MF} \\ &= \frac{1}{N} \sum_k f(k). \end{aligned} \quad (3.27)$$

The set of mean-field equations (3.25) and (3.27) can be solved numerically. In Fig.1, the results are given for the interesting case of $\rho = 1/2$, as will be explained in Sect.4.2. For this case,

one sets $\mu_c = 0$, because each pair of a point (k_1, k_2) and its associated partner $(k_1 + \pi, k_2 + \pi)$ contributes $1/2$ for ρ , since $(1/2)[f(k_1, k_2) + f(k_1 + \pi, k_2 + \pi)] = 1/2$ for $\mu_c = 0$.

To understand the results well, we illustrate the case $T = 0, \rho = 1/2$ that is tractable without numerical study. First, we note that, at $T = 0$, the Fermi distribution $f(k)$ becomes a step function,

$$\lim_{\beta \rightarrow \infty} f(k) = \theta\left(-\frac{V_0}{m\gamma} \sum_i \cos k_i\right). \quad (3.28)$$

Thus, the mean-field equation (3.25) reduces as

$$\begin{aligned} 4V_0 - 4\gamma &= -\frac{1}{m\gamma N} \sum_k \left(\sum_i \cos k_i\right) \theta\left(-\frac{V_0}{m\gamma} \sum_i \cos k_i\right) \\ &= \text{sign}\left(\frac{V_0}{m}\right) \frac{2}{(2\pi)^2 m\gamma} \int_0^\pi dk_1 \int_{-\pi+k_1}^{\pi-k_1} dk_2 (\cos k_1 + \cos k_2) \\ &= \text{sign}\left(\frac{V_0}{m\gamma}\right) \frac{4}{\pi^2 m\gamma}, \end{aligned} \quad (3.29)$$

where we replaced the momentum sum by a momentum integral by taking the thermodynamic limit $N \rightarrow \infty$. For $V_0 > 0$ and $m\gamma > 0$ this gives

$$V_0 = \gamma + \frac{1}{\pi^2 m\gamma}. \quad (3.30)$$

We note that the first term in V_0 , γ , comes from the linear term in the potential discussed before and even if this term were missing, V_0 would remain nonvanishing.

4 The GL theory and the confinement-deconfinement phase transition

In this section, we shall study the dynamics of phase degrees of freedom of V_{xj} 's, i.e., the dynamics of a U(1) gauge field. By integrating over fermions using the hopping expansion, we obtain an effective theory of V_{xj} , the GL theory. It takes a form of lattice gauge theory and its electric and magnetic terms are calculated explicitly. By applying the knowledge of Polyakov-Susskind theory of CD phase transition to this theory, we can discuss the possibility of CD phase transition and PFS.

Explicitly, after calculating the GL theory in Sect.4.1, we study two interesting cases; (1) composite fermions in the half-filled Landau level in Sect.4.2, and (2) the quantum XY spin model with spin $S = 1/2$ in Sect.4.3. We show for these two cases that there is a CD phase transition, and at $T < T_{CD}$ the gauge dynamics is in the deconfinement phase and the PFS takes place. We also argue that the system selects the spatially uniform configuration of V_{xj} 's near $T = 0$.

It is known for both of these cases that the mean-field-type calculations with perturbative (weak) gauge interactions [5, 7, 10, 13] supply qualitatively correct pictures near $T = 0$. The above results of this section support the assumption used in these existing mean-field-type calculations.

4.1 The effective gauge theory

In this subsection, we derive the effective gauge theory of $U(1)$ link variables U_{xj} defined through

$$V_{xj} = V_0 U_{xj}, \quad U_{xj} = \exp(iA_{xj}), \quad (4.1)$$

where $V_0 > 0$ and A_{xj} is the gauge potential. In the conventional tight-binding model of fermions,

$$H_{tb} = t \sum_{xj} \bar{\psi}_{x+j} \psi_x + \text{H.c.}, \quad (4.2)$$

where t is the hopping amplitude, the hopping expansion is convergent only at high temperature $T \gtrsim t$. However, in the present case, the effective hopping parameter is $V_0 U_{xj}/(2m)$, and fluctuations of U_{xj} afford the hopping expansion to converge even at lower T .

We start with the following Lagrangian,

$$\begin{aligned} L = & - \sum \bar{\psi}_x (\partial_0 - \mu_c) \psi_x - \frac{V_0}{2m} \sum \bar{\psi}_{x+j} U_{xj} \psi_x + \text{H.c.} \\ & + \sum V_0 \bar{U}_{xj} W_{x+j} \bar{W}_x + \text{H.c.} - \sum |V_0|^2. \end{aligned} \quad (4.3)$$

The effective action $A[U]$ of the gauge theory of U_{xj} is defined as

$$\exp(A[U]) = \int [\prod_x d\bar{\psi}_x d\psi_x] \exp\left(\int_0^\beta d\tau L\right). \quad (4.4)$$

In the hopping expansion performed below, we shall consider the following canonical form of $A[U]$;

$$\begin{aligned}
A[U] &= A_e + A_m, \\
A_e &= C_e \int d\tau \sum_{xj} \partial_0 \bar{U}_{xj} \partial_0 U_{xj}, \\
A_m &= C_m \sum_x U_{x2,0} U_{x+\hat{2},1,0} \bar{U}_{x+\hat{1},2,0} \bar{U}_{x1,0} + \text{H.c.},
\end{aligned} \tag{4.5}$$

where the subscripts e and m indicate the "electric term" and the "magnetic term", respectively. The variable $U_{xj,0}$ in A_m is the zero mode in Fourier expansion as introduced later in (4.8).⁷ Each of them consists of the contributions from the fermion hopping term $\bar{\psi}U\psi$ and the flux hopping term $\bar{W}UW$ in (4.3);

$$\begin{aligned}
A_e &= \Delta A_{e,\psi} + \Delta A_{e,W} \\
A_m &= \Delta A_{m,\psi} + \Delta A_{m,W},
\end{aligned} \tag{4.6}$$

where the subscripts ψ and W refer to these two hopping terms. Below, the higher order terms in the expansion shall not be discussed explicitly, but one can apply the same arguments as in Ref.[2] claiming that these terms favor the existence of CD phase transition, so the qualitative results obtained below need no modifications.

$\Delta A_{e,\psi}$ is calculated as

$$\begin{aligned}
\Delta A_{e,\psi} &= \left(\frac{V_0}{2m\gamma}\right)^2 \sum_{xj} \int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \bar{U}_{xj}(\tau_1) U_{xj}(\tau_2) \\
&\quad \times \langle \bar{\psi}_x(\tau_1) \psi_{x+\hat{j}}(\tau_1) \bar{\psi}_{x+\hat{j}}(\tau_2) \psi_x(\tau_2) \rangle \\
&= \left(\frac{V_0}{2m\gamma}\right)^2 \rho(1-\rho) \sum_{xj} \int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \bar{U}_{xj}(\tau_1) U_{xj}(\tau_2) \\
&= \left(\frac{V_0}{2m\gamma}\right)^2 \rho(1-\rho) \beta^2 \sum_{xj} \bar{U}_{xj,0} U_{xj,0},
\end{aligned} \tag{4.7}$$

where we have used the Wick's theorem with (3.2), and introduced the Fourier components $U_{xj,n}$ through

$$U_{xj}(\tau) = \sum_n e^{i\omega_n \tau} U_{xj,n}, \tag{4.8}$$

⁷The form of A_m is different from the usual magnetic term considered in lattice gauge theory, $\int d\tau \sum_x U_{x2} U_{x+\hat{2},1} \bar{U}_{x+\hat{1},2} \bar{U}_{x1}(\tau) + \text{H.c.}$, but one can apply the Polyakov-Susskind theory also to $A[U]$ equally well to study its CD transition.

$$\sum_n \bar{U}_{xj,n} U_{xj,n+m} = \delta_{m,0} \quad (4.9)$$

with $\omega_n = 2\pi n/\beta$ ($n = 0, \pm 1, \pm 2, \dots$).

From (4.8), it is obvious that $U_{xj,n}$ corresponds to the amplitude that the link variable $U_{xj}(\tau)$ changes its winding number along the imaginary time axis by n as τ runs from 0 to β . Therefore, if all the non-static amplitudes $U_{xj,n}$ ($n \neq 0$) are suppressed, $A_{xj}(\tau)$ has no fluctuations in τ at all, leading to $\langle U_{xj} \rangle \neq 0$, so the gauge dynamics is in the deconfinement phase. More detailed discussion on this point was given in the previous papers [2], referring to the Polyakov-Susskind CD phase transition at finite T [12].

The contribution from the term $\bar{U}_{xj} W_{x+j} \bar{W}_x$ is calculated in a similar way;

$$\begin{aligned} \Delta A_{e,W} &= \gamma^2 V_0^2 \sum_{xj} \int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \bar{U}_{xj}(\tau_1) U_{xj}(\tau_2) \\ &\quad \times \langle W_{x+j} \bar{W}_x(\tau_1) W_x \bar{W}_{x+j}(\tau_2) \rangle \\ &= \gamma^2 V_0^2 \sum_{xj} \int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \bar{U}_{xj}(\tau_1) U_{xj}(\tau_2) \\ &\quad \times \langle \exp[2\pi qi \sum_{r'} \epsilon_{jl} \Delta_l G(r, r') (\bar{\psi}_{x'} \psi_{x'}(\tau_1) - \bar{\psi}_{x'} \psi_{x'}(\tau_2))] \rangle. \end{aligned} \quad (4.10)$$

From (3.8), in the leading order of the hopping expansion, we have

$$\Delta A_{e,W} = \gamma^2 V_0^2 \beta^2 \sum_{xj} \bar{U}_{xj,0} U_{xj,0}, \quad (4.11)$$

which is of the same form with $\Delta A_{e,\psi}$.

The total electric term reads

$$A_e = \left(\gamma^2 + \frac{\rho(1-\rho)}{(2m\gamma)^2} \right) V_0^2 \beta^2 \sum_{xj} \bar{U}_{xj,0} U_{xj,0}. \quad (4.12)$$

From (4.12) and (4.9), we see that, at high T , i.e., at small β , A_e makes every component $U_{xj,n}$ to fluctuate randomly, so the dynamics of $U_{xj}(\tau)$ is realized in the confinement phase, $\langle U_{xj} \rangle = 0$. ψ_x and W_x interact via a strong attractive force, so ψ_x and U_{xj} cannot be quasiparticles separately. On the other hand, at low T , i.e., at large β , only the static component $U_{xj,0}$ is enhanced as $|U_{xj,0}| \sim 1$ as read off from the sum rule (4.9), and all the non-static components $U_{xj,n}$ ($n \neq 0$) are suppressed. This indicates that, at low T , the gauge dynamics is realized in the deconfinement phase, where $\langle U_{xj}(\tau) \rangle \simeq U_{xj,0} \neq 0$. Both ψ_x and A_{xj} are the quasiparticles there, and the perturbative treatment of the gauge interaction, assuming fluctuations of U_{xj}

around $\langle U_{x_j} \rangle$ are small (but, of course, nontrivial), should lead us to the correct picture. From these considerations for high and low T , we conclude that there is a CD phase transition at some critical temperature T_{CD} . From (4.12), T_{CD} is roughly estimated as

$$T_{CD}^2 \sim \left(\gamma^2 + \frac{\rho(1-\rho)}{(2m\gamma)^2} \right) V_0^2. \quad (4.13)$$

As mentioned, these conclusions can be rederived on a more solid basis by using the Polyakov-Susskind theory [12] of CD transition. In this theory, an effective lattice gauge system is to be mapped to the classical XY spin model in the same spatial lattice under question, and the established phase transition in spin dynamics is connected to the CD transition. Therefore, we predict that the CD transition in our model has a nature of the Kosterlitz-Thouless phase transition in the 2D XY model. It is known to be very smooth, actually of infinite order. We stress that the confinement and deconfinement regions are *not* connected through a crossover, but are separated by a phase transition, whatever smooth it is.

Precisely speaking, the amplitude V_0 itself depends on T as discussed in Sect.3. Therefore, detailed numerical study of V_0 is required to determine T_{CD} .⁸ For the case of CSS, the corresponding problem is under study and the result will be reported in the forthcoming paper.

The above result of the gauge dynamics, i.e., appearance of the deconfinement phase at *low* T , is in a strong contrast with the finite-temperature properties of QCD. In QCD, a deconfinement phase exists at *high* T , i.e., above T_{CD} . However this is not so surprising because the electric and magnetic terms of U_{x_j} are generated by integrating out the fermion field, so the "effective gauge-coupling constant" is not a constant at all but has a strong T dependence. Actually, from A_e of (4.12) and the completeness condition (4.9), we rewrite A_e as

$$\begin{aligned} A_e &= -\left(\gamma^2 + \frac{\rho(1-\rho)}{(2m\gamma)^2} \right) V_0^2 \beta^2 \sum_{x_j} \left(\sum_{n \neq 0} \bar{U}_{x_j, n} U_{x_j, n} - 1 \right) \\ &\simeq -\left(\gamma^2 + \frac{\rho(1-\rho)}{(2m\gamma)^2} \right) V_0^2 \beta^3 \sum_{x_j} \int d\tau \partial_\tau \bar{U}_{x_j} \partial_\tau U_{x_j}, \end{aligned} \quad (4.14)$$

which is exact for $n = 0, \pm 1$ modes and qualitatively correct also for other modes. Therefore the effective gauge coupling g_{eff}^2 behaves as

$$g_{eff}^2 \propto V_0^{-2} T^3. \quad (4.15)$$

⁸As V_0 is a decreasing function of T , there is certainly a unique solution to (4.13).

The magnetic term A_m in the effective action $A[U]$ is calculated in a similar way. A_m consists of the products of four U_{xj} 's around the four links of each plaquette. From (4.3) and (3.2), the contribution from the hopping term $\bar{\psi}V\psi$ is calculated as

$$\begin{aligned}\Delta A_{m,\psi} &= -\left(\frac{V_0}{2m\gamma}\right)^4 \sum_x \prod_{i=1}^4 \int d\tau_i \left(U_{x2}(\tau_4) U_{x+\hat{2},1}(\tau_3) \bar{U}_{x+\hat{1},2}(\tau_2) \bar{U}_{x1}(\tau_1) + \text{H.c.} \right) \\ &\quad \times \prod_{i=1}^4 G_\psi(\tau_i - \tau_{i+1}) \\ &= -C_\psi \left(\frac{V_0\beta}{2m\gamma}\right)^4 \sum_x \left(U_{x2,0} U_{x+\hat{2},1,0} \bar{U}_{x+\hat{1},2,0} \bar{U}_{x1,0} + \text{H.c.} \right),\end{aligned}\tag{4.16}$$

where $\tau_5 = \tau_1$, and C_ψ is given by (3.13). In the last line of (4.16), we have retained only the terms of products of four zero modes $U_{xj,0}$. The remaining terms containing nonzero modes ($n \neq 0$) have a suppression factor $\propto 1/n^2$. These terms bring no modifications to the results obtained below, since we have used the second-order electric term A_e to discuss the CD transition, or the time dependence of U_{xj} 's, and shall use the fourth-order magnetic term A_m to discuss the relevant spatial configuration of U_{xj} 's. In other words, we are interested mostly in the spatial configuration of zero modes, after identifying that, from A_e , the relevant modes are the zero modes.

From (4.16), it is obvious that, at high T , the coefficient in the magnetic term is small and there are no correlations among neighboring U_{xj} 's. On the other hand, at low T , the coefficient is getting large and the above magnetic term (4.16) favors certain static configuration of $U_{xj}(\tau)$ for all links.⁹ Therefore, it is concluded that both the magnetic term (4.16) and the electric term (4.12) indicate the CD phase transition. As mentioned, one can argue that the higher-order terms of the hopping expansion of ψ_x , which start with $O(V_0^6)$, support the confinement phase at high T and the deconfinement phase at low T .

Next, let us examine the contribution $\Delta A_{m,W}$ from $\bar{U}_{xj} W_{x+j} \bar{W}_x$. It is calculated in the leading order as follows,

$$\begin{aligned}\Delta A_{m,W} &= \gamma^4 V_0^4 \sum_x \prod_{i=1}^4 \int d\tau_i \left(U_{x2}(\tau_4) U_{x+\hat{2},1}(\tau_3) \bar{U}_{x+\hat{1},2}(\tau_2) \bar{U}_{x1}(\tau_1) \right) \\ &\quad \times \langle \bar{W}_x W_{x+\hat{1}} \bar{W}_{x+\hat{1}} W_{x+\hat{1}+\hat{2}} \bar{W}_{x+\hat{1}+\hat{2}} W_{x+\hat{2}} \bar{W}_{x+\hat{2}} W_x \rangle + \text{H.c.}\end{aligned}$$

⁹More precisely, $\Delta A_{m,\psi}$ favors the uniform (fluxless) configuration of U_{xj} for negative C_ψ 's and the configurations with a half flux quantum ($UU\bar{U}\bar{U} = -1$) for positive C_ψ 's.

$$\begin{aligned}
&= \gamma^4 V_0^4 \sum_x \prod_{i=1}^4 \int d\tau_i \left(U_{x2}(\tau_4) U_{x+\hat{2},1}(\tau_3) \bar{U}_{x+\hat{1},2}(\tau_2) \bar{U}_{x1}(\tau_1) \right) e^{-2\pi q i \rho} \\
&\quad \times \prod_{r'} \left(e^{-2\pi q i \Delta_2 G(r, r') \bar{\psi}_{r'} \psi_{r'}(\tau_1)} \cdot e^{2\pi q i \Delta_1 G(r+\hat{2}, r') \bar{\psi}_{r'} \psi_{r'}(\tau_2)} \cdot \right. \\
&\quad \left. e^{-2\pi q i \Delta_2 G(r+\hat{1}, r') \bar{\psi}_{r'} \psi_{r'}(\tau_3)} \cdot e^{2\pi q i \Delta_1 G(r, r') \bar{\psi}_{r'} \psi_{r'}(\tau_4)} \right) + \text{H.c.} \\
&= \gamma^4 V_0^4 \sum_x \prod_{i=1}^4 \int d\tau_i \left(U_{x2}(\tau_4) U_{x+\hat{2},1}(\tau_3) \bar{U}_{x+\hat{1},2}(\tau_2) \bar{U}_{x1}(\tau_1) \right) \\
&\quad \times e^{-2\pi q i \rho} \langle e^{2\pi q i \psi_x^\dagger \psi_x} \rangle + \text{H.c.} \\
&= \gamma^4 V_0^4 \sum_x \beta^4 \left(U_{x2,0} U_{x+\hat{2},1,0} \bar{U}_{x+\hat{1},2,0} \bar{U}_{x1,0} \right) \\
&\quad \times e^{-2\pi i q \rho} [1 - \rho + e^{2\pi i q} \rho] + \text{H.c.}, \tag{4.17}
\end{aligned}$$

where we used (3.6) and $\Delta_i \Delta_i G(r, r') = \delta_{r, r'}$. The average over the four exponentials may be evaluated using the same technique as in (3.16) and (3.17). This completes the calculation of $A[U]$: it is obtained by collecting the results (4.12), (4.16) and (4.17). The above term (4.17) indicates that the sign of $\Delta A_{m,w}$ depends on the parameter q and the density ρ (at least near the CD transition region of $\langle U_{xj} \rangle \simeq 0$). Therefore, we shall study two interesting cases separately in the following subsections.

4.2 Composite fermions in the half-filled Landau level

Let us start by briefly reviewing the CS gauge theory of electrons in the half-filled Landau level. The Hamiltonian of the system (in the continuum) is given by

$$H_{hf} = \frac{1}{2m} \int d^2x |(i\partial_j + eA_j^{ex})\psi_e|^2 + V(\psi_e^\dagger \psi_e), \tag{4.18}$$

where ψ_e is a spinless fermion field describing electrons. A_j^{ex} ($j = 1, 2$) is the vector potential for a uniform magnetic field B^{ex} , and $V(\psi_e^\dagger \psi_e)$ is a potential term describing interactions among electrons. As the system is half-filled, there holds the relation,

$$\nu \equiv \frac{2\pi\rho}{eB^{ex}} = \frac{1}{2}. \tag{4.19}$$

There is a natural cutoff in the system: the magnetic length $l_B = (eB^{ex})^{-1/2}$.¹⁰

The composite fermion (CF) operator $\psi_{cf}(x)$ is introduced by

$$\psi_{cf} = \psi_e(x) \exp \left[-2i \int d^2y \theta(x-y) \hat{\rho}(y) \right], \tag{4.20}$$

¹⁰We are using the units $\hbar/(2\pi) = c = 1$.

where

$$\hat{\rho}(x) = \psi_e^\dagger(x)\psi_e(x) = \psi_{cf}^\dagger(x)\psi_{cf}(x), \quad (4.21)$$

and

$$\theta(x) = \arctan \frac{x_2}{x_1}. \quad (4.22)$$

In terms of ψ_{cf} , the Hamiltonian H_{hf} is written as

$$H_{hf} = \frac{1}{2m} \int d^2x |(i\partial_j + eA_j^{ex} - A_j^{CS})\psi_{cf}|^2 + V(\psi_{cf}^\dagger\psi_{cf}), \quad (4.23)$$

with

$$\begin{aligned} A_j^{CS} - eA_j^{ex} &= 2 \int d^2y \partial_j \theta(x-y) (\psi_{cf}^\dagger(y)\psi_{cf}(y) - \rho) \\ &= 4\pi\epsilon_{jl} \int d^2y \partial_l G(x-y) (\psi_{cf}^\dagger(y)\psi_{cf}(y) - \rho), \\ G(x) &= \int \frac{d^2k}{(2\pi)^2} \frac{e^{ik \cdot x}}{k^2}. \end{aligned} \quad (4.24)$$

Now let us consider the lattice model. By comparing the lattice model (2.1) with the continuum model (4.23) and (4.24), it is obvious that $q = 2$ in our lattice model. It is also natural to identify the lattice spacing a which we have set as $a = 1$ to be the same order with the magnetic length l_B . To be explicit, we set $2\pi l_B^2 = 1$. Then, one gets the relations $\nu = 2\pi\rho/eB^{ex} = 2\pi\rho l_B^2 = \rho$, so the average fermion density in the lattice model is fixed as $\rho = 1/2$. The conclusion in Sect.4.1 that there exists T_{CD} holds with this assignment of the parameters q and ρ . However, the relevant pattern of U_{xj} in the deconfinement phase is not trivial.

The magnetic term $\Delta A_{m,W}$ in this case reads

$$\begin{aligned} \Delta A_{m,W} &= \gamma^4 V_0^4 \sum_x \prod_{i=1}^4 \int d\tau_i (U_{x2}(\tau_4) U_{x+\hat{2},1}(\tau_3) \bar{U}_{x+\hat{1},2}(\tau_2) \bar{U}_{x1}(\tau_1)) + \text{H.c.} \\ &= \gamma^4 U_0^4 \beta^4 \sum_x (U_{x2,0} U_{x+\hat{2},1,0} \bar{U}_{x+\hat{1},2,0} \bar{U}_{x1,0} + \text{H.c.}). \end{aligned} \quad (4.25)$$

Then, $\Delta A_{m,W}$ prefers the spatially uniform and temporally static configurations of $U_{xj}(\tau)$, i.e., $U_{xj}(\tau) \sim 1$ (up to time-independent gauge rotation). On the other hand, the other magnetic term $\Delta A_{m,\psi}$ of (4.16) prefers the configurations of U_{xj} with a half flux quantum per every plaquette because of the fact $C_\psi = 1/96 > 0$ at $\rho = 1/2$. Therefore, near T_{CD} , the spatial

configuration of U_{xj} 's depends on the signature of the coefficient C_m of the total magnetic term A_m ,

$$C_m \equiv \beta^4 \gamma^4 V_0^4 \left[1 - \frac{1}{6(4m\gamma^2)^4} \right], \quad (4.26)$$

or, in other words, on the values of $m\gamma^2$ (measured in the unit of $2\pi l_B^2 = 1$).

At sufficiently low T , the assumption of uniform fermion distribution should work well, as explained in Sect.3.2. This implies the replacement $\hat{\rho}_x \rightarrow \rho = 1/2$ in W_x , or $\bar{W}_{x+j}W_x = \exp(i\Delta_j\chi_x)$, hence gives rise to the fluxless configuration of U_{xj} 's.

The consideration of the dynamics of U_{xj} 's in the effective gauge theory given so far demonstrates that the CS gauge theory of CF's at $\nu = 1/2$ is in the *deconfinement* phase and the PFS takes place at *low* T , $T < T_{CD}$, where the quasiparticles are the CF's and the gauge bosons. Near $T = 0$, fluctuations of A_{xj} around zero are small and can be treated by the conventional loopwise perturbation theory. Thus, the RPA or the RGE employing such perturbative calculations are legitimately applied for investigating low-energy behavior of the system. Not only showing validity of the perturbative calculations at low T , our study also predicts a phase transition at finite T into the confinement phase, where the CF's, compounds of electrons and CS fluxes, are unstable and the quasiparticles are nothing but electrons themselves.

There is a comment on this point. Precisely speaking, the model of the half-filled Landau level considered in Refs.[5, 7] is slightly different from the present one of (4.3). The former is obtained by replacing the flux hopping term $\bar{W}VW$ in (4.3) by a nonlocal kinetic term of gauge field. Although the phenomenon of PFS is universal in this system and so the calculations in Refs.[5, 7] that are done assuming a PFS are legitimate, it is still interesting to clarify the relation between these two models.

4.3 The 2D quantum XY spin model

The Hamiltonian of the spin-one-half 2D quantum XY model is given by

$$H_{XY} = -J \sum_{a=1}^2 \sum_j S_x^a S_{x+j}^a, \quad (4.27)$$

where S_x^a ($a = 1, 2, 3$) are $s = 1/2$ spin operators and J is the exchange parameter. A Jordan-Wigner-type transformation for $s = 1/2$ spins is discussed in Ref.[9]. The spin-rising and

lowering operators S_x^+ and S_x^- , and the third component S_x^3 are expressed in terms of the fermion operator ψ_x as¹¹

$$S_x^+ = W_x^\dagger \psi_x, \quad S_x^- = \psi_x^\dagger W_x, \quad S_x^3 = \frac{1}{2} - \psi_x^\dagger \psi_x. \quad (4.28)$$

As S_x^a commutes with S_y^b ($y \neq x$), the parameter q in W_x (2.9) must be an *odd integer*. For the ground state, it is expected that the expectation value of S_x^3 vanishes at nonzero T , $\langle S_x^3 \rangle = 0$, therefore the (average) density of the fermions should be set $\rho = 1/2$.¹² Then the Hamiltonian H_{XY} is represented in terms of ψ_x as

$$H_{XY} = -J \sum \psi_{x+j}^\dagger W_{x+j} W_x^\dagger \psi_x + \text{H.c.} \quad (4.29)$$

By comparing this Hamiltonian with eq.(2.8), we identify $J = -1/2m$.

The discussion given in Sect.4.1 is applicable to the above XY model in the fermionic representation. Below T_{CD} , the total electric term $A_e = \Delta A_{e,\psi} + \Delta A_{e,W}$ prefers the static configuration of U_{xj} and the term $\Delta A_{m,\psi}$ prefers a half flux quantum per plaquette since its coefficient $C_\psi = 1/96 > 0$. In order to obtain $\Delta A_{m,W}$, we have to evaluate $\langle e^{2\pi q i(\hat{\rho}_x - \rho)} \rangle$ as in (4.17). In the leading order of the hopping expansion, we get $\langle e^{2\pi q i(\hat{\rho}_x - \rho)} \rangle = -1$ as q is an odd integer and $\rho = 1/2$. Therefore the term $\Delta A_{m,W}$ also prefers configurations with a half flux quantum per plaquette. This conclusion is correct at temperature near T_{CD} , where the leading-order result of the hopping expansion is reliable.

On the other hand, at very low T , fluctuations of U_{xj} are much more suppressed, and so the almost uniform distribution of *fermions* is realized. In this case, we can set $\hat{\rho}_x \rightarrow \rho = 1/2$ as done in Sect.3.2. Then the term $\Delta A_{m,W}$ prefers the uniform configuration without magnetic fluxes since its coefficient becomes positive. Let us mention the effect of the term $\Delta A_{m,\psi}$ at low T . The straightforward mean-field study for the system of $\bar{\psi}V\psi$ hopping term alone (i.e., without the $\bar{W}VW$ term) is known to predict that the most favorable configuration is those

¹¹It should be remarked that the Jordan-Wigner-type transformation (4.28) is slightly different from that in Ref.[9]. In the definition of W_x of (2.9), there is an extra c-number term that depends on ρ , with which the summation in W_x is convergent for the uniform distribution of fermions. There is another difference: the χ_x term in the phase, with which the transformation property under gauge transformation becomes transparent.

¹²Just on $T = 0$, the system is known to have an off-diagonal long-range order, so separate analyses are necessary.

having a fraction ρ of a flux per each plaquette. From this fact, we think that $\Delta A_{m,\psi}$ still favors configurations with a half flux quantum at very low T . Then competition between $\Delta A_{m,\psi}$ and $\Delta A_{m,W}$ arises. We expect that the term $\Delta A_{m,W}$ wins to select out the uniform configuration. This expectation is consistent with the existing mean-field-type calculations for the 2D XY model [13] which assume the perturbative treatment of gauge fluctuations around a uniform configuration in terms of our ψ_x and W_x .¹³

From these considerations, it is anticipated that the average flux per plaquette changes from a half flux to no flux as T is lowered. The nature of this change, a phase transition or a cross over, is interesting but has not been studied yet. We note that the condition $\langle B^{CS} \rangle = eB^{ex}$ of (2.7), used to establish the relation $\nu = 1/q$, leads to fluxless uniform state. So it is applicable near $T = 0$ with the uniform state, but not compatible with the half-flux state near $T = T_{CD}$.

5 Discussion

In this paper, we studied the phase structure of the CS gauge theory of fermions at finite T . This work is motivated by the recent studies on the composite fermions for the half-filled Landau level. We stress that the hopping expansion we used is an ideal tool to study a possible CD transition, as long as it is of second or higher order or of weak first order, since the expansion assumes smallness of the order parameter, U_{xj} in the present case. The situation goes parallel as in the usual GL theory of second order transitions. Only but essential difference is that the order parameter is now a gauge field sitting on links, so gauge-theoretical interpretation is most natural and powerful. For the region near $T = 0$, we employed a straightforward mean field theory assuming fermions distribute homogeneously.

We showed that there is a CD phase transition at finite temperature, T_{CD} , below which the PFS takes place. This result means that the composite fermions in the half-filled Landau level are the stable quasiparticles at low T . But they become unstable at high T and the

¹³In the mean-field theory of Ref.[13], the ground state is described by fermions moving in the comensurate magnetic field, $B^{ex} = 2\pi\rho$. This indicates that, for $\rho = 1/2$, a *half flux quanta* exists for each plaquette. However, the definition of fermions there is different from the present one by a c-number term $A_x^{ex} = q \sum_{r'} \theta(x, r') \rho$ in the exponent of W_x of (2.9). It is straightforward to check that the *flux state* in Ref.[13] corresponds to the *fluxless* uniform state in the present model.

compounds of fermions and fluxes, which are nothing but the original electrons, become new quasiparticles. This main conclusion validates the assumption $\langle A^{CS} \rangle = eA^{ex}$ in the CS gauge theory with fermions near $T = 0$, or equivalently, $\langle B^{CS} \rangle = eB^{ex}$ of (2.7). This assumption is employed in Jain's approach to quantum Hall states [4] itself and in the analyses [5, 7] based on the mean-field theory plus loop corrections.

As we have seen in Sect.4.2 and Sect.4.3, the relevant configurations of gauge field near T_{CD} may not be the uniform one that is realized near $T = 0$. The nature of the change of average fluxes as T changes should be studied for a coherent understanding. Concerning to this point, we mention its relation to a simpler model: a lattice system of nonrelativistic fermions coupled with U(1) gauge link variables which possess no own kinetic term [2]. This model also exhibits a CD transition and the ground state is characterized by a commensurate flux $B/(2\pi) = \rho$ per each plaquette. Our present model reduces to it if one ignores the flux hopping term, $V\bar{W}W$. Therefore, any difference in the physical behavior, like the value of average fluxes near $T = 0$, is to be attributed to this flux hopping term.

Let us briefly comment on the four-Fermi interaction in (2.11) which appears as a result of introducing the auxiliary gauge field V_{xj} . In the confinement phase at $T > T_{CD}$, V_{xj} generates strong attraction (repulsion) in the $\bar{\psi} - \psi$ ($\bar{\psi} - \bar{\psi}$ and $\psi - \psi$) channel, besides attraction between ψ and $\bar{W}$. The effect of the four-Fermi interaction is canceled out by this gauge interaction. On the other hand, in the deconfinement phase at $T < T_{CD}$, the gauge interaction is not so strong, and the four-Fermi interaction may generate a condensation of fermion pairs $\langle \bar{\psi}_{x+j} \bar{\psi}_x \rangle \neq 0$. This indicates a superconducting phase of the CS gauge theory. To investigate this possibility, one may introduce another auxiliary field on links which describe superconducting fermion pairs. The situation resembles to the CSS case of the t-J model in which two different kinds of U(1) gauge fields are introduced both in the normal and super channels. A condensation of one of the gauge link variables may generate a mass term for the other gauge field, and so the gauge dynamics of the system has a possibility to be realized in a Higgs phase. A Higgs phase is distinguished from a Coulomb phase due to a finite mass of gauge boson. Therefore, if $\langle \bar{\psi}_{x+j} \bar{\psi}_x \rangle \neq 0$ in some region, the deconfinement phase of the present model may be further classified into a Coulomb phase and a Higgs phase. This possibility needs to be studied.

Finally, we comment on the parameter γ . As explained, this important parameter determines

the ratio of velocities of fermions and fluxes. However, a preliminary calculation shows that the four-Fermi term discussed above plays a crucial role to determine γ . (Note that its coefficient depends on γ .) So we plan to calculate its value together with the analysis of the four-Fermi term in a future publication.

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FIGURE CAPTION

Fig.1. The value of V_0 is plotted as a function of T . The curve starting from $T = T_{MF}$ is the result (3.21), and the curve starting from $T = 0$ is the solution of the mean-field equation (3.25). The latter approaches as $V_0 \rightarrow \gamma$ as $T \rightarrow \infty$.

Fig. 1