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Dissipative Gauge Theory of Fermions and Its Marginal-Fermi-Liquid-Like Behavior

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Abstract

A non-relativistic fermion system interacting with a gauge field is studied by Wilsonian renormalization-group analysis. This model is motivated by the work of Halperin, Lee and Read, which discussed a two-dimensional electron system in a magnetic field at the Landau filling factor $\nu = 1/2$. A dissipative term of the gauge field (the Landau damping factor) is generated under the renormalization-group transformations, and it plays a very important role in the low-energy behaviors of the system. Renormalization constants are calculated in the one-loop level. In some parameter region, the renormalized gauge coupling constant has a non-trivial infrared fixed point, and the low-energy renormalized fermion propagator has a branch cut rather than a pole, just like the Luttinger liquid in one dimension. At a special value of the parameter, the weight of quasiparticles decreases at low energy as $1/\ln\omega$, where ω is the energy of fermion. This behavior is nothing but that of the marginal Fermi liquid, which is proposed to describe the anomalous metallic phase of high- T_C cuprates.

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1 Introduction

Systems of non-relativistic fermions interacting with gauge field are of current interest. For the fractional quantum Hall effect (FQHE), a Chern-Simons gauge theory coupled with bosonized electron has played an essentially important role[1]. Being motivated by this approach to the FQHE, Halperin, Lee and Read[2] studied a model of two-dimensional electron system in a perpendicular magnetic field at Landau filling factor $\nu = 1/2$. It is a Chern-Simons gauge theory coupled with so-called composite fermions: a composite fermion is a compound of an electron and two flux quanta attaching to it[3]. In the mean-field-type approximation, an external magnetic field is cancelled by the averaged Chern-Simons gauge field, and the model is reduced to a system of fermions with Fermi surface (or line) interacting with residual fluctuations of the gauge field. In the random-phase approximation (RPA), they showed that anomalous behaviors of fermion propagator appear because of the interaction with the gauge field and the Coulomb interaction among electrons. Some experiments support their results[4].

It was suggested also for the high- T_C superconductivity that dynamical gauge bosons, which appear as a result of the strong-correlation among electrons, may explain anomalous properties of high- T_C cuprates[5]. It was shown that the dynamics of composite gauge bosons is closely related with the phenomenon of so-called charge-spin separation[6].

Most studies on gauge theories of non-relativistic fermions given so far employ the straightforward loop expansion or the RPA. Recently, it was suggested that the renormalization-group (RG) study is very efficient in investigating low-energy behaviors of non-relativistic fermion systems[7]. Especially, Nayak and Wilczek[8] performed a RG study of certain gauge theory of non-relativistic fermions. The kinetic term of gauge field in their action is given by

$$\int d^2\mathbf{q} A_i(\mathbf{q}, \epsilon) q^{2-b} A_i(-\mathbf{q}, -\epsilon), \quad (1.1)$$

where $\mathbf{q}$ and ϵ are the momentum and the energy of gauge boson, respectively, and b is a parameter of the model. Actually, this generalization was first introduced in Ref.[2], and the parameter b describes interaction between electrons as

$$V(\rho) = \int d\mathbf{k} \rho(\mathbf{k}) \frac{1}{k^b} \rho(-\mathbf{k}), \quad (1.2)$$

where $\rho(\mathbf{k})$ is the density of electrons. So the Coulomb interaction corresponds to the case $b = 1$. In the Chern-Simons gauge theory of fermions, the density of fermion is connected with the field strength (the magnetic field) of the Chern-Simons gauge field (the Chern-Simons constraint). Therefore, the interaction (1.2) controls fluctuations of the gauge field. The kinetic term (1.1) appears from (1.2) by substituting the Chern-Simons constraint, or, in the RPA-type calculation, by summing bubble diagrams. In the RPA or the RG study, the Lagrange multiplier of the Chern-Simons constraint, i.e., $A_0(\mathbf{q}, \epsilon)$, acquires a mass. So it seems correct to assume that the Chern-Simons constraint tends to irrelevant at low energies, and the mean-field approximation is justified. This consideration guarantees the existence of a Fermi surface in the present model.

A careful study of this gauge model of non-relativistic fermions by using the RG analysis was given in our previous paper[9]. There, we used the $(1 - b)$ -expansion in a spirit similar to the usual ϵ -expansion[8]. We employed both the straightforward loop expansion and the $1/N$ -RPA, where N is the number of species of fermions. It was shown that the dissipative term of the gauge boson (the Landau damping factor) plays a very important role. Our calculations and conclusions differ from Nayak and Wilczek [8] in some important points.

In the present paper, we shall revisit the same model. This time, we shall employ the Wilsonian RG study with a momentum cutoff. The low-energy effective model is obtained recursively by integrating out high-momentum modes gradually. Practical calculations show that the dissipative term of the gauge field is generated by RG transformations, and its coefficient scales non-trivially. This situation is quite different from the $(1 - b)$ -expansion in Ref.[9]. In the $(1 - b)$ -expansion, the coefficient of

the dissipative term does not change under RG transformations both in the loop expansion and in the $1/N$ -RPA, since no singularities at $b = 1$ appear. So we treated it as a fixed constant there. Here we shall calculate one-loop corrections coming from a shell in the momentum space. They are nonvanishing and one obtains the β -function of gauge-coupling constant, coefficient of the dissipative term, anomalous dimension of fermion, renormalization of Fermi velocity, etc. At one-loop level, the β -function of gauge coupling has a non-trivial infrared (IR) fixed point for $b < 1$. On the other hand, for $b > 1$, there exists only the trivial Fermi-liquid fixed point. The low-energy behavior of the fermion propagator is explicitly determined by solving recursion equations of RG. For $b < 1$, the fermion propagator has a branch cut rather than a pole just like the Luttinger liquid in one dimension[10]. For $b = 1$, the quasiparticle weight decreases logarithmically at low energy. This behavior is nothing but that of the marginal Fermi liquid[11]. For $b > 1$, low-energy excitations are described by the Fermi-liquid theory.

This paper is organized as follows. In Sect.2, the model is given. We shall employ a specific momentum cutoff to incorporate the effects of gauge field correctly. In Sect.3, we shall explain the basic steps of RG transformations. It is remarked that there is an ambiguity in assigning scaling dimensions to each field. However, later calculations show that final results for certain physical quantities do not depend on which scaling law is employed. In Sect.4, the results of one-loop calculation are given. Renormalizations of wave function, the Fermi velocity, etc., are obtained. In Sect.5, recursion equations of RG and their solutions are presented. According to the value of parameter b , the low-energy behavior of the model drastically changes, as explained above. Section 6 is devoted to discussion, where we point out the sensitivity of the results on the choice of renormalization prescription and on the scaling assignment in non-relativistic systems in contrast with the conventional renormalization-group theory in relativistic field theory.

2 Model

In this section, we shall introduce a model which describes the (2+1)-dimensional system of non-relativistic fermions coupled with a gauge field. The partition function of the model in the imaginary-time formalism is given by

$$\begin{aligned}
Z &= \int [d\psi][dA_i] \Pi \delta(\partial_i A_i) \exp\left(\int d\tau A\right) \\
&= \lim_{\alpha \rightarrow \infty} \int [d\psi][dA_i] \exp\left(\int d\tau A_\alpha\right), \\
A &= - \int d^2x \bar{\psi}(\mathbf{x}, \tau) (\partial_\tau - \mu - \frac{1}{2m} D_i D_i) \psi(\mathbf{x}, \tau) \\
&\quad - \frac{1}{2} \int d^2x \int d^2y B(\mathbf{x}, \tau) V_B(\mathbf{x} - \mathbf{y}) B(\mathbf{y}, \tau), \\
A_\alpha &= A - \int d^2x \frac{\alpha}{2} (\partial_i A_i(\mathbf{x}, \tau))^2, \\
D_i &= \partial_i + ig A_i, \\
B &= \partial_1 A_2 - \partial_2 A_1, \\
V_B(\mathbf{x}) &= v_B \int \underline{dq} \frac{\exp(i\mathbf{q} \cdot \mathbf{x})}{q^b}, \\
\underline{dq} &\equiv (2\pi)^{-2} dq d\phi, \\
\mu &= \frac{K_F^2}{2m},
\end{aligned} \tag{2.1}$$

where $\psi(\mathbf{x}, \tau)$ represents a non-relativistic spinless fermion and $A_i(\mathbf{x}, \tau)$ ($i = 1, 2$) is a gauge field. As explained in Introduction, b is a parameter of the model and we are interested in the region $b \sim 1$. $\alpha (\rightarrow \infty)$ is the gauge parameter selecting the Coulomb gauge. μ is the chemical potential for fermions.

Since we shall consider excitations around given Fermi points $\mathbf{K}_{F,n}$ (see Fig.1), it is useful to describe the above system in terms of the Fourier transformed fields. In condensed matter physics, there is generally a natural cutoff on all momenta. (e.g., inverse of lattice constant.) We introduce momentum cutoff for our later purposes. In a non-relativistic system, there is a four-point term in the fermion-gauge field interaction, but this term will be neglected. This is justified, since this term contributes only to the renormalization of chemical potential of fermion and the effective mass of

gauge field. In the field theoretical approach[9], it is shown that the effective mass of gauge field is vanishing because the contribution from the four-point interaction cancels that from the three-point interaction. Thus, in the following discussion, we shall not discuss the contribution from the three-point interaction to the renormalization of chemical potential and the mass of gauge field.

In term of the Fourier transformed fields, Z is expressed as

$$\begin{aligned}
Z &= \lim_{\alpha \rightarrow \infty} \int [d\psi][dA_i] \exp(S_\alpha), \\
S_\alpha &= S_\psi + S_{A,\alpha} + S_{int,3}, \\
S_\psi &= - \int \underline{d\omega} \sum_{\mathbf{e}_n} \int^\Lambda \underline{dk} \bar{\psi}(n; \mathbf{k}, \omega) \{i\omega + v_F(\mathbf{e}_n \cdot \mathbf{k})\} \psi(n; \mathbf{k}, \omega), \\
S_{A,\alpha} &= -\frac{1}{2} \int \underline{d\epsilon} \int^\Lambda \underline{dq} A_i(\mathbf{q}, \epsilon) \left\{ \frac{v_B}{q^{b-2}} (\delta_{ij} - \frac{q_i q_j}{q^2}) + \alpha q_i q_j \right\} A_j(-\mathbf{q}, -\epsilon), \\
S_{int,3} &= - \int \underline{d\omega} \underline{d\omega'} \underline{d\epsilon} \sum_{\mathbf{e}_n} \int^\Lambda \underline{dk} \underline{dk'} \underline{dq} \bar{\delta}(\omega - \omega' + \epsilon) \bar{\delta}(\mathbf{k} - \mathbf{k}' + \mathbf{q}) \\
&\quad \times \Lambda^{\frac{1-b}{2}} g v_F \bar{\psi}(n; \mathbf{k}', \omega') \{ \mathbf{A}(\mathbf{q}, \epsilon) \cdot \mathbf{e}_n \} \psi(n; \mathbf{k}, \omega),
\end{aligned} \tag{2.3}$$

where

$$\begin{aligned}
\mathbf{k} &\equiv \mathbf{K}_{F,n} - \mathbf{K}, \\
\mathbf{e}_n &\equiv \frac{\mathbf{K}_{F,n}}{K_F}, \\
\underline{d\omega} &\equiv (2\pi)^{-1} d\omega, \quad \underline{dk} \equiv (2\pi)^{-2} k dk d\phi, \\
\bar{\delta}(\omega) &\equiv (2\pi) \delta(\omega), \quad \bar{\delta}(\mathbf{k}) \equiv (2\pi)^2 \delta(\mathbf{k}).
\end{aligned} \tag{2.4}$$

We have introduced somewhat unusual momentum cutoff as shown in Figs.1 and 2. We divided the momentum region near the Fermi surface into $N = \pi K_F / \Lambda$ parts, each of which is numbered by $n = 1, \dots, N$.¹ $\mathbf{K}$ is the fermion momentum and $\mathbf{k}$ measures its deviation from the nearby Fermi momentum $\mathbf{K}_{F,n}$. The unit vector $\mathbf{e}_n$ points towards the center of the n -th part. The momentum cutoff Λ is introduced as $|\mathbf{k}| = |\mathbf{K}_{F,n} - \mathbf{K}| < \Lambda$. In order to incorporate the effect of gauge field correctly, one

¹We simply assume that the Fermi surface has the shape of a circle.

needs to use this type of momentum-cutoff prescription. In $S_{int,3}$, the factor $\Lambda^{\frac{1-b}{2}}$ has been introduced to make the coupling constant g dimensionless. See the scaling law (3.2) and (3.4).

From (2.3), the tree-level fermion propagator is given by

$$G(n; \mathbf{k}, \omega) = \frac{e^{-i\omega\eta_0}}{i\omega + v_F \mathbf{e}_n \cdot \mathbf{k}}, \quad (2.5)$$

where the factor $e^{-i\omega\eta_0}$, with η_0 being an infinitesimally small positive constant, comes from the ordering of path-integral quantization.

Here, we generalize the model by introducing a dissipative term in the gauge field action;

$$S_{A,\alpha} = -\frac{1}{2} \int d\epsilon \int^\Lambda d\mathbf{q} A_i(\mathbf{q}, \epsilon) \left\{ F(q, \epsilon) \left(\delta_{ij} - \frac{q_i q_j}{q^2} \right) + \alpha q_i q_j \right\} A_j(-\mathbf{q}, -\epsilon), \quad (2.6)$$

where

$$\begin{aligned} F(q, \epsilon) &= v_B (q^{2-b} + \Lambda^{2-b} \lambda \frac{|\epsilon|}{q}) \\ &= \gamma(q) \{ |\epsilon| + B(q) \}, \\ \gamma(q) &= \frac{\Lambda^{2-b} v_B \lambda}{q}, \quad B(q) = \gamma^{-1}(q) v_B q^{2-b}. \end{aligned} \quad (2.7)$$

and $\lambda > 0$. As it is shown in the following calculation, RG transformation generates a dissipative term and the parameter λ behaves non-trivially under RG. The tree-level gauge propagator is

$$\Delta_{ij}(\mathbf{q}, \epsilon) = \frac{1}{F(q, \epsilon)} \left(\delta_{ij} - \frac{q_i q_j}{q^2} \right). \quad (2.8)$$

3 RG transformations

In this section, we shall define RG transformations. This program of RG transformations essentially follows the work of Shankar[7].

In most of the problems in condensed-matter physics, long-wave length excitations have low excitation energies, hence important. In the present model, we are interested

in low-energy excitations near the Fermi surface. Therefore, we need an effective theory describing the low-momentum modes, which includes effects of high-momentum modes. To obtain the effective theory of low-energy excitations, a Wilson-type RG study is very efficient.

Let us now explain the procedure of the RG transformation. It consists of three steps (1,2,3).

(1) Integrate over fast modes [modes with the momenta $\Lambda/\epsilon^t < k < \Lambda$; $t(> 0)$ is the scaling parameter]. Then the action will be modified as

$$\begin{aligned}
\tilde{S}_\psi &= - \int \frac{d\omega}{\epsilon} \sum_{\mathbf{e}_n} \int^{\Lambda/\epsilon^t} \frac{dk}{\epsilon} \bar{\psi}(n; \mathbf{k}, \omega) R(t) \{i\omega + R_{v_F}(t) v_F(\mathbf{e}_n \cdot \mathbf{k})\} \psi(n; \mathbf{k}, \omega), \\
\tilde{S}_{A,\alpha} &= -\frac{1}{2} \int \frac{d\epsilon}{\epsilon} \int^{\Lambda/\epsilon^t} \frac{dq}{\epsilon} A_i(\mathbf{q}, \epsilon) A_j(-\mathbf{q}, -\epsilon) \\
&\quad \times \left\{ v_B(q^{2-b} + \Lambda^{2-b} R_\lambda(t) \lambda \frac{|\epsilon|}{q}) (\delta_{ij} - \frac{q_i q_j}{q^2}) + R_\alpha(t) \alpha q_i q_j \right\}, \\
\tilde{S}_{int,3} &= - \int \frac{d\omega}{\epsilon} \frac{d\omega'}{\epsilon} \frac{d\epsilon}{\epsilon} \sum_{\mathbf{e}_n} \int^{\Lambda/\epsilon^t} \frac{dk}{\epsilon} \frac{dk'}{\epsilon} \frac{dq}{\epsilon} \bar{\delta}(\omega - \omega' + \epsilon) \bar{\delta}(\mathbf{k} - \mathbf{k}' + \mathbf{q}) \\
&\quad \times \Lambda^{\frac{1-b}{2}} R_g(t) g v_F \bar{\psi}(n; \mathbf{k}', \omega') \{ \mathbf{A}(\mathbf{q}, \epsilon) \cdot \mathbf{e}_n \} \psi(n; \mathbf{k}, \omega), \tag{3.1}
\end{aligned}$$

where $R(t)$, etc., are renormalization constants. We have neglected remaining terms which are higher-order in momenta or fields.

(2) To compare the original model and the partly-integrated effective model (3.1), the cutoff must be returned to the original value by rescaling momenta. Similarly, by rescaling fields, we require that certain quadratic terms in the effective action have the same coefficients. Thus we define new momenta and new fields as

$$\begin{aligned}
\omega &\rightarrow \tilde{\omega} = \epsilon^t \omega, \quad \mathbf{k} \rightarrow \tilde{\mathbf{k}} = \epsilon^t \mathbf{k}, \\
\psi(n; \mathbf{k}, \omega) &\rightarrow \tilde{\psi}(n; \tilde{\mathbf{k}}, \tilde{\omega}) = \epsilon^{-2t} \{R(t)\}^{\frac{1}{2}} \psi(n; \mathbf{k}, \omega), \\
A(\mathbf{k}, \omega) &\rightarrow \tilde{A}(\tilde{\mathbf{k}}, \tilde{\omega}) = \epsilon^{\frac{b-s}{2}t} A(\mathbf{k}, \omega), \tag{3.2}
\end{aligned}$$

where the wave function renormalization $R(t)$ is obtained below explicitly as a func-

tion of t . The effective action will be rewritten in terms of these variables as

$$\begin{aligned}
\tilde{S}_\psi &= - \int \underline{d\omega} \sum_{\mathbf{e}_n} \int^\Lambda \underline{dk} \bar{\psi}(n; \mathbf{k}, \omega) \{i\omega + R_{v_F}(t) v_F(\mathbf{e}_n \cdot \mathbf{k})\} \psi(n; \mathbf{k}, \omega), \\
\tilde{S}_{A,\alpha} &= -\frac{1}{2} \int \underline{d\epsilon} \int^\Lambda \underline{dq} A_i(\mathbf{q}, \epsilon) A_j(-\mathbf{q}, -\epsilon) \\
&\quad \times \left\{ v_B(q^{2-b} + \Lambda^{2-b} e^{(2-b)t} R_\lambda(t) \lambda \frac{|\epsilon|}{q}) (\delta_{ij} - \frac{q_i q_j}{q^2}) + e^{-bt} R_\alpha(t) \alpha q_i q_j \right\}, \\
\tilde{S}_{int,3} &= - \int \underline{d\omega} \underline{d\omega'} \underline{d\epsilon} \sum_{\mathbf{e}_n} \int^\Lambda \underline{dk} \underline{dk'} \underline{dq} \bar{\psi}(\omega - \omega' + \epsilon) \bar{\delta}(\mathbf{k} - \mathbf{k}' + \mathbf{q}) \\
&\quad \times \Lambda^{\frac{1-b}{2}} e^{\frac{1-b}{2}t} \frac{R_g(t)}{R(t)} g v_F \bar{\psi}(n; \mathbf{k}', \omega') \{ \mathbf{A}(\mathbf{q}, \epsilon) \cdot \mathbf{e}_n \} \psi(n; \mathbf{k}, \omega), \tag{3.3}
\end{aligned}$$

where tildes in the field variables have been omitted.

(3) The effective coupling constant $g(t)$, the effective Fermi velocity $v_F(t)$, the effective coefficient of the dissipative term $\lambda(t)$, and the anomalous dimension of fermion $\eta(t)$ are defined as follows,

$$\begin{aligned}
g(t) &= e^{\frac{1-b}{2}t} \frac{R_g(t)}{R(t) R_{v_F}(t)} g, \\
v_F(t) &= R_{v_F}(t) v_F, \\
\lambda(t) &= e^{(2-b)t} R_\lambda(t) \lambda, \\
\eta(t) &= \frac{d}{dt} \ln R(t). \tag{3.4}
\end{aligned}$$

Then the effective action will be given by the same form as the starting action except that the coefficients are replaced by the t -dependent effective ones;

$$\begin{aligned}
\tilde{S}_\psi &= - \int \underline{d\omega} \sum_{\mathbf{e}_n} \int^\Lambda \underline{dk} \bar{\psi}(n; \mathbf{k}, \omega) \{i\omega + v_F(t) (\mathbf{e}_n \cdot \mathbf{k})\} \psi(n; \mathbf{k}, \omega), \\
\tilde{S}_{A,\alpha} &= -\frac{1}{2} \int \underline{d\epsilon} \int^\Lambda \underline{dq} A_i(\mathbf{q}, \epsilon) A_j(-\mathbf{q}, -\epsilon) \\
&\quad \times \left\{ v_B(q^{2-b} + \Lambda^{2-b} \lambda(t) \frac{|\epsilon|}{q}) (\delta_{ij} - \frac{q_i q_j}{q^2}) + e^{-bt} R_\alpha(t) \alpha q_i q_j \right\}, \\
\tilde{S}_{int,3} &= - \int \underline{d\omega} \underline{d\omega'} \underline{d\epsilon} \sum_{\mathbf{e}_n} \int^\Lambda \underline{dk} \underline{dk'} \underline{dq} \bar{\psi}(\omega - \omega' + \epsilon) \bar{\delta}(\mathbf{k} - \mathbf{k}' + \mathbf{q}) \\
&\quad \times \Lambda^{\frac{1-b}{2}} g(t) v_F(t) \bar{\psi}(n; \mathbf{k}', \omega') \{ \mathbf{A}(\mathbf{q}, \epsilon) \cdot \mathbf{e}_n \} \psi(n; \mathbf{k}, \omega). \tag{3.5}
\end{aligned}$$

Let us remark that there is an ambiguity in assigning scaling dimensions of energy and fields. The set of scaling assignment (3.2) and (3.4) is not the unique possibility to obtain the effective action (3.5). Actually, one may introduce a general scaling law parametrized by a free parameter δ that leads to the effective action (3.5). It is given by

$$\begin{aligned}\omega &\rightarrow \tilde{\omega} = e^{\delta t} \omega, & \mathbf{k} &\rightarrow \tilde{\mathbf{k}} = e^t \mathbf{k}, \\ \psi(n; \mathbf{k}, \omega) &\rightarrow \tilde{\psi}(n; \tilde{\mathbf{k}}, \tilde{\omega}) = e^{-(\delta+1)t} \{R(t)\}^{\frac{1}{2}} \psi(n; \mathbf{k}, \omega), \\ A(\mathbf{k}, \omega) &\rightarrow \tilde{A}(\tilde{\mathbf{k}}, \tilde{\omega}) = e^{\frac{b-4-\delta}{2}t} A(\mathbf{k}, \omega).\end{aligned}\tag{3.6}$$

In this case, effective constants are defined as

$$\begin{aligned}g(t) &= e^{\frac{2-b-\delta}{2}t} \frac{R_g(t)}{R(t)R_{v_F}(t)} g, \\ v_F(t) &= e^{(\delta-1)t} R_{v_F}(t) v_F, \\ \lambda(t) &= e^{(3-b-\delta)t} R_\lambda(t) \lambda, \\ \eta(t) &= \frac{d}{dt} \ln R(t).\end{aligned}\tag{3.7}$$

Eq.(3.2) obviously corresponds to $\delta = 1$, and we shall call the case $\delta = 1$ the first way of scaling. Here the tree level scaling of v_F vanishes. On the other hand, for $\delta = 3 - b$, the tree level scaling of λ is vanishing. We shall call this case the second way of scaling.

As explained, the effective action in terms of these effective constants does not have explicit δ dependences. However, practical calculations show that behaviors of these variables themselves under RG transformations depend on the value of δ . Therefore, the low-energy behavior of the model seems to depend on the way of scaling. However, we shall see that some physically interesting quantities are again δ independent. This point will be discussed later after practical calculations.

4 One-Loop Calculations

In this section, we shall perform an infinitesimal RG transformation ($e^{dt} \simeq 1 + dt$) at one-loop level, and calculate the renormalization constants R , R_{v_F} , R_g and R_λ .

4.1 Renormalization of fermion part

By integrating high-momentum modes of $A_i(\mathbf{q}, \epsilon)$ and $\psi(n; \mathbf{k}, \omega)$ ($\Lambda(1-dt) < q, k < \Lambda$) in the one-loop diagrams, one obtains the effective action. The correction to the free action of fermion is given by

$$\begin{aligned}
\tilde{S}_\psi|_{1\text{-loop}} &= \Lambda^{1-b} g^2 v_F^2 \int \underline{d\omega} \sum_{\mathbf{e}_n} \int^{\Lambda(1-dt)} \underline{dk} \underline{dk}' \bar{\psi}(n; \mathbf{k}', \omega) \psi(n; \mathbf{k}, \omega) \\
&\quad \times \int \underline{d\epsilon} \int_{\Lambda(1-dt)}^{\Lambda} \underline{dk}'' \underline{dq} \bar{\delta}(k - q - k'') \bar{\delta}(k'' + q - k') \\
&\quad \times e_{n,i} \Delta_{ij}(\mathbf{q}, \epsilon) e_{n,j} G(n; \mathbf{k}'', \omega - \epsilon) \\
&= \Lambda^{1-b} g^2 v_F^2 \int \underline{d\omega} \sum_{\mathbf{e}_n} \int_{k < \Lambda(1-dt)} \underline{dk} \bar{\psi}(n; \mathbf{k}, \omega) \psi(n; \mathbf{k}, \omega) \\
&\quad \times \int \underline{d\epsilon} \int_{\Lambda(1-dt)}^{\Lambda} \underline{dq} \frac{\sin^2 \phi}{F(q, \epsilon)} \frac{e^{-i(\omega-\epsilon)\eta_0}}{i(\omega - \epsilon) + v_F \mathbf{e}_n \cdot (\mathbf{k} - \mathbf{q})} \\
&= g^2 v_F^2 \int \underline{d\omega} \sum_{\mathbf{e}_n} \int_{k < \Lambda(1-dt)} \underline{dk} \bar{\psi}(n; \mathbf{k}, \omega) \Gamma(n; \mathbf{k}, \omega) \psi(n; \mathbf{k}, \omega), \quad (4.1)
\end{aligned}$$

where

$$\begin{aligned}
\Gamma(n; \mathbf{k}, \omega) &= \Lambda^{1-b} \int_{\Lambda(1-dt)}^{\Lambda} \underline{dq} \frac{\sin^2 \phi}{\gamma(q)} J(v_F \mathbf{e}_n \cdot (\mathbf{k} - \mathbf{q}), B(q), \omega), \\
J(E, B, \epsilon) &= \int \underline{d\epsilon} \frac{e^{-i(\omega-\epsilon)\eta_0}}{i(\omega - \epsilon) + E} \frac{1}{|\epsilon| + B} \\
&= \frac{1}{2\pi i} \int_0^\infty dx \frac{-2(\omega - iE)}{x^2 - (\omega - iE)^2} \frac{1}{x + B} \\
&= \frac{1}{2\pi i} \left\{ 2iE - 2\omega + 2iE \cdot i\omega \frac{\partial}{\partial E} \right\} K(E, B) + O(\omega^2), \\
K(E, B) &= \int_0^\infty dx \frac{1}{x^2 + E^2} \frac{1}{x + B} \\
&= \frac{1}{2(E^2 + B^2)} \left\{ -\ln \frac{B^2}{E^2} + \frac{\pi B}{|E|} \right\}. \quad (4.2)
\end{aligned}$$

In the above integrals over $\mathbf{q}$, the integral region is restricted as $\Lambda(1-dt) < |\mathbf{k}-\mathbf{q}| < \Lambda$. The calculations here and below go in a way parallel to the case of $(1-b)$ -expansion [9], where one can find details.

We are interested in the linear terms with respect to $\mathbf{k}$ and ω , and thus make an expansion as follows:²

$$\begin{aligned}\Gamma(n; \mathbf{k}, \omega) &= \Gamma_m + \Gamma_{\mathbf{k}} v_F (\mathbf{e}_n \cdot \mathbf{k}) + \Gamma_{\omega} i\omega + O(k^2, \omega^2, k\omega), \\ \Gamma_m &= \frac{\Lambda^{1-b}}{2\pi i} \int_{\Lambda(1-dt)}^{\Lambda} \frac{dq}{\gamma(q)} \frac{\sin^2 \phi}{\gamma(q)} 2iEK(E, B), \\ \Gamma_{\mathbf{k}} &= \frac{\Lambda^{1-b}}{2\pi i} \int_{\Lambda(1-dt)}^{\Lambda} \frac{dq}{\gamma(q)} \frac{\sin^2 \phi}{\gamma(q)} \frac{\partial}{\partial E} \{2iEK(E, B)\}, \\ \Gamma_{\omega} &= \frac{\Lambda^{1-b}}{2\pi i} \int_{\Lambda(1-dt)}^{\Lambda} \frac{dq}{\gamma(q)} \frac{\sin^2 \phi}{\gamma(q)} \{2i + 2iE \frac{\partial}{\partial E}\} K(E, B).\end{aligned}\quad (4.3)$$

where

$$E = -v_F (\mathbf{e}_n \cdot \mathbf{q}). \quad (4.4)$$

By changing variables from (q, ϕ) to (q, y) , $y \equiv E/B$, we have an expression,

$$\begin{aligned}\Gamma_m &= \frac{\Lambda^{2-b} dt}{4\pi^3 v_F \gamma(\Lambda)} \int_{-Y}^Y dy |\sin \phi(y)| \left\{ \frac{y \ln y^2}{1+y^2} + \pi \operatorname{sgn}(y) \right\}, \\ Y &= \frac{v_F \Lambda}{B(\Lambda)} = v_F \lambda.\end{aligned}\quad (4.5)$$

Since the integrand is an odd function of y , Γ_m vanishes;

$$\Gamma_m = 0. \quad (4.6)$$

Next, we calculate Γ_{ω} , again by changing variables;

$$\begin{aligned}\Gamma_{\omega} &= \frac{\Lambda^{2-b} dt}{2\pi^3 v_F \gamma(\Lambda) B(\Lambda)} \int_0^Y dy |\sin \phi(y)| \left\{ \frac{1-y^2}{(1+y^2)^2} \ln y^2 + \frac{2}{1+y^2} - \frac{2\pi y}{(1+y^2)^2} \right\} \\ &= -\frac{dt}{2\pi^2 v_F v_B} I(Y, 0),\end{aligned}\quad (4.7)$$

²It is well-known that RG transformation using the sharp momentum cutoff sometimes produces non-local effective action. There is a more elaborate method [12] which cures this difficulty. In the present paper, we are using the sharp momentum cutoff, assuming that a similar technique also works for non-relativistic fermion systems, and concentrate on the renormalizations of wave function, Fermi velocity, etc.

where

$$I(Y, Y') = -\frac{1}{\pi} \int_{Y'}^Y dy |\sin \phi(y)| \left\{ \frac{1-y^2}{(1+y^2)^2} \ln y^2 + \frac{2}{1+y^2} - \frac{2\pi y}{(1+y^2)^2} \right\}. \quad (4.8)$$

Due to the structure of the integrand, the integral is dominated by the region $y \simeq 0$.

Thus we can approximate $|\sin \phi(y)| \simeq 1$, and we get

$$I(\infty, 0) \simeq 1 \quad (4.9)$$

Finally, we calculate $\Gamma_{\mathbf{k}}$. First we note the relation,

$$\frac{\partial}{\partial E} \{EK(E, B)\} = K(E, B) + E \frac{\partial}{\partial E} K(E, B) + \frac{\pi}{B} \delta(E). \quad (4.10)$$

Thus, $\Gamma_{\mathbf{k}}$ is given as

$$\begin{aligned} \Gamma_{\mathbf{k}} &= \Gamma_{\omega} + \frac{\Lambda^{3-b} dt}{4\pi^2 \gamma(\Lambda) B(\Lambda)} \int_{-\pi}^{\pi} d\phi \sin^2 \phi \delta(-v_F \Lambda \cos \phi), \\ &= \frac{dt}{2\pi^2 v_F v_B} \{-I(Y, 0) + I(\infty, 0)\}, \\ &= \frac{dt}{2\pi^2 v_F v_B} I(\infty, Y). \end{aligned} \quad (4.11)$$

Therefore, the one-loop correction to the free fermion part of the action is calculated as

$$\begin{aligned} \tilde{S}_{\psi}|_{1\text{-loop}} &= - \int \frac{d\omega}{2\pi} \sum_{\mathbf{e}_n} \int_{k < \Lambda(1-dt)} \frac{dk}{2\pi} \bar{\psi}(n; \mathbf{k}, \omega) \psi(n; \mathbf{k}, \omega) \\ &\quad \times \frac{g^2 v_F}{2\pi^2 v_B} dt \{I(Y, 0)(i\omega) - I(\infty, Y) v_F (\mathbf{e}_n \cdot \mathbf{k})\}. \end{aligned} \quad (4.12)$$

From the above formula, we get the renormalization constants,

$$\begin{aligned} R(dt) &= 1 + \frac{g^2 v_F}{2\pi^2 v_B} dt I(Y, 0), \\ R(dt) R_{v_F}(dt) &= 1 - \frac{g^2 v_F}{2\pi^2 v_B} dt I(\infty, Y). \end{aligned} \quad (4.13)$$

Thus $R_{v_F}(dt)$ dose not depend on Y ;

$$R_{v_F}(dt) = 1 - \frac{g^2 v_F}{2\pi^2 v_B} dt. \quad (4.14)$$

4.2 Renormalization of gauge field part

The one-loop correction to the free part of the gauge field action is given by

$$\begin{aligned}
\tilde{S}_{A,\alpha}|_{1\text{-loop}} &= -\frac{1}{2}\Lambda^{1-b}g^2v_F^2 \int \underline{d\epsilon} \int^{\Lambda(1-dt)} \underline{dq} \underline{dq}' A_i(\mathbf{q}, \epsilon) A_j(\mathbf{q}', -\epsilon) \\
&\quad \times \int \underline{d\omega} \sum_{\mathbf{e}_n} \epsilon_{n,i} \epsilon_{n,j} \int_{\Lambda(1-dt)}^{\Lambda} \underline{dk} \underline{dk}' \bar{\delta}(\mathbf{k} - \mathbf{k}' + \mathbf{q}) \bar{\delta}(\mathbf{k} - \mathbf{k}' - \mathbf{q}') \\
&\quad \times G(n; \mathbf{k}', \omega + \epsilon) G(n; \mathbf{k}, \omega) \\
&= \frac{1}{2} \int \underline{d\epsilon} \int_{q < \Lambda(1-dt)} \underline{dq} A_i(\mathbf{q}, \epsilon) \Pi_{ij}(\mathbf{q}, \epsilon) A_j(-\mathbf{q}, -\epsilon), \tag{4.15}
\end{aligned}$$

where

$$\begin{aligned}
\Pi_{ij}(\mathbf{q}, \epsilon) &= -\Lambda^{1-b}g^2v_F^2 \sum_{\mathbf{e}_n} \frac{\epsilon_{n,i} \epsilon_{n,j}}{i\epsilon + v_F(\mathbf{e}_n \cdot \mathbf{q})} \\
&\quad \times \int_{\Lambda(1-dt)}^{\Lambda} \underline{dk} \{ \theta(-v_F \mathbf{e}_n \cdot (\mathbf{k} + \mathbf{q})) - \theta(-v_F \mathbf{e}_n \cdot \mathbf{k}) \}. \tag{4.16}
\end{aligned}$$

We are interested in marginal and relevant terms. Therefore we shall neglect higher order terms in q and ϵ . Furthermore we shall assume $\epsilon \ll q$, because these modes dominate the quantum corrections to the fermion propagator and the vertex. Expanding θ function with respect to $\mathbf{q}$, the first term can be evaluated as

$$\begin{aligned}
\Pi_{ij}(\mathbf{q}, \epsilon) &= \frac{\Lambda^{2-b}g^2v_F dt}{4\pi^2} \delta_{ij} \sum_{\mathbf{e}_n} \frac{v_F(\mathbf{e}_n \cdot \mathbf{q})}{i\epsilon + v_F(\mathbf{e}_n \cdot \mathbf{q})} \\
&\simeq \frac{\Lambda^{2-b}g^2v_F dt}{8\pi^2} \left(\frac{K_F}{\Lambda}\right) \delta_{ij} \int_{-\pi}^{\pi} d\varphi \frac{v_F q \cos \varphi}{i\epsilon + v_F q \cos \varphi} \\
&\simeq \frac{\Lambda^{2-b}g^2v_F dt}{4\pi} \left(\frac{K_F}{\Lambda}\right) \left(1 - \frac{|\epsilon|}{v_F q}\right) \delta_{ij}, \tag{4.17}
\end{aligned}$$

where we have used the relation $\sum_{\mathbf{e}_n} (\frac{2\Lambda}{K_F}) \simeq \int_{-\pi}^{\pi} d\varphi$ that comes from Fig.1. Therefore we get

$$\begin{aligned}
\tilde{S}_{A,\alpha}|_{1\text{-loop}} &= -\frac{1}{2} \int \underline{d\epsilon} \int_{q < \Lambda(1-dt)} \underline{dq} A_i(\mathbf{q}, \epsilon) A_j(-\mathbf{q}, -\epsilon) \\
&\quad \times \frac{\Lambda^{2-b}g^2v_F dt}{2\pi} \left(\frac{K_F}{\Lambda}\right) \left(1 - \frac{|\epsilon|}{v_F q}\right) \delta_{ij}. \tag{4.18}
\end{aligned}$$

This result gives the renormalization of λ ,

$$R_\lambda(dt) = 1 + \frac{g^2}{2\pi v_B \lambda} \left(\frac{K_F}{\Lambda}\right) dt. \tag{4.19}$$

4.3 Vertex Correction

The one-loop correction to the three-point interaction is given by

$$\begin{aligned}
\tilde{S}_{int,3}|_{1\text{-loop}} &= \Lambda^{\frac{1-b}{2}} \int \frac{d\omega}{\omega} \frac{d\omega'}{\omega'} \frac{d\epsilon}{\epsilon} \sum_{\mathbf{e}_n} \int_{\mathbf{k}, \mathbf{k}', q < \Lambda(1-dt)} \frac{d\mathbf{k}}{k} \frac{d\mathbf{k}'}{k'} \frac{dq}{q} \bar{\psi}(\omega - \omega' + \epsilon) \bar{\delta}(\mathbf{k} - \mathbf{k}' + \mathbf{q}) \\
&\quad \times \bar{\psi}(n; \mathbf{k}', \omega') A_i(\mathbf{q}, \epsilon) \psi(n; \mathbf{k}, \omega) (-gv_F)^3 S_{n,i}, \\
S_{n,i} &= \Lambda^{1-b} \int \frac{d\epsilon'}{\epsilon'} \int_{\Lambda(1-dt)}^{\Lambda} \frac{dq'}{q'} \\
&\quad \times e_{n,i} e_{n,l} e_{n,m} G(n; \mathbf{k} - \mathbf{q}', \omega - \epsilon') G(n; \mathbf{k}' - \mathbf{q}', \omega' - \epsilon') \Delta_{lm}(\mathbf{q}', \epsilon').
\end{aligned} \tag{4.20}$$

Let us calculate $S_{n,i}$ first;

$$\begin{aligned}
S_{n,i} &= e_{n,i} \Lambda^{1-b} \int_{\Lambda(1-dt)}^{\Lambda} \frac{dq'}{q'} \frac{\sin^2 \phi}{\gamma(q')} U(n; \mathbf{k}, \omega, \mathbf{k}', \omega'), \\
U(n; \mathbf{k}, \omega, \mathbf{k}', \omega') &= \int \frac{d\epsilon'}{\epsilon'} G(n; \mathbf{k} - \mathbf{q}', \omega - \epsilon') G(n; \mathbf{k}' - \mathbf{q}', \omega' - \epsilon') \frac{1}{|\epsilon'| + B} \\
&= \frac{1}{2\pi i} \frac{1}{i(\omega' - \omega) + (E' - E)} \left[\{2iE - 2\omega + 2iE \cdot i\omega \frac{\partial}{\partial E}\} K(E, B) \right. \\
&\quad \left. - \{2iE' - 2\omega' + 2iE' \cdot i\omega' \frac{\partial}{\partial E'}\} K(E', B) \right],
\end{aligned} \tag{4.21}$$

where

$$E = v_F \mathbf{e}_n \cdot (\mathbf{k} - \mathbf{q}'), \quad E' = v_F \mathbf{e}_n \cdot (\mathbf{k}' - \mathbf{q}'). \tag{4.22}$$

We are not interested in irrelevant terms. Thus we shall consider only the "on-shell" limit, but there are two possible ways to take this limit:

$$\begin{aligned}
\lim_{\omega' \rightarrow \omega} \lim_{\mathbf{k}' \rightarrow \mathbf{k}} U &= -\frac{1}{2\pi i} \{2i + 2iE \frac{\partial}{\partial E}\} K(E, B), \\
\lim_{\mathbf{k}' \rightarrow \mathbf{k}} \lim_{\omega' \rightarrow \omega} U &= -\frac{1}{2\pi i} \frac{\partial}{\partial E} \{2iE K(E, B)\}.
\end{aligned} \tag{4.23}$$

As shown in the previous paper[9], the correction depends on how the "on-shell" limit is taken;

$$\begin{aligned}
\lim_{\omega' \rightarrow \omega} \lim_{\mathbf{k}' \rightarrow \mathbf{k}} S_{n,i} &= -e_{n,i} \Gamma_{\omega}, \\
\lim_{\mathbf{k}' \rightarrow \mathbf{k}} \lim_{\omega' \rightarrow \omega} S_{n,i} &= -e_{n,i} \Gamma_{\mathbf{k}}.
\end{aligned} \tag{4.24}$$

The above equations express the Ward-Takahashi (W-T) identities[9] that are derived from the gauge invariance of the model. If we require manifest gauge invariance of the *effective* action (i.e., $R(t)R_{v_F}(t) = R_g(t)$), the limit $\omega' \rightarrow \omega$ should be taken first before the limit $\mathbf{k}' \rightarrow \mathbf{k}$ [9]. Thus we get

$$\begin{aligned} \tilde{S}_{int,3}|_{1\text{-loop}} &= \Lambda^{\frac{1-b}{2}} g v_F \int \underline{d\omega} \underline{d\omega'} \underline{d\epsilon} \sum_{\mathbf{e}_n} \int_{\mathbf{k}, \mathbf{k}', \mathbf{q} < \Lambda(1-dt)} \underline{dk} \underline{dk'} \underline{dq} \bar{\delta}(\omega - \omega' + \epsilon) \bar{\delta}(\mathbf{k} - \mathbf{k}' + \mathbf{q}) \\ &\quad \times \bar{\psi}(n; \mathbf{k}', \omega') \{ \mathbf{A}(\mathbf{q}, \epsilon) \cdot \mathbf{e}_n \} \psi(n; \mathbf{k}, \omega) \left\{ \frac{g^2 v_F}{2\pi^2 v_B} dt I(\infty, Y) \right\}. \end{aligned} \quad (4.25)$$

From this result we get

$$R_g(dt) = 1 - \frac{g^2 v_F}{2\pi^2 v_B} dt I(\infty, Y). \quad (4.26)$$

5 RG equations and the Low-Energy Behaviors

5.1 RG recursion equations

Now we can derive the recursion equations of $g(t)$, $v_F(t)$ and $\lambda(t)$ by using the results obtained in the previous section. Let us consider first the first way of scaling [the assignment (3.4)]. It is straightforward to obtain the following RG functions;

$$\begin{aligned} \frac{dg}{dt}(t) &= \frac{1-b}{2} g(t), \\ \frac{dv_F}{dt}(t) &= -\frac{g^2(t) v_F(t)}{2\pi^2 v_B} v_F(t), \\ \frac{d\lambda}{dt}(t) &= (2-b)\lambda(t) + \frac{g^2(t)}{2\pi^2 v_B} \left(\frac{2\pi K_F}{2\Lambda} \right). \end{aligned} \quad (5.1)$$

From (5.1), we get recursion equations for the following combinations;

$$\begin{aligned} Y(t) &= v_F(t) \lambda(t), \\ \alpha(t) &= \frac{g^2(t) v_F(t)}{2\pi^2 v_B}. \end{aligned} \quad (5.2)$$

$\alpha(t)$ is nothing but an effective expansion parameter of the perturbative calculation that replaces $g^2(t)$. We obtain

$$\begin{aligned}\frac{dY}{dt}(t) &= \{(2-b) - \alpha(t)\}Y(t) + \alpha(t)\left(\frac{2\pi K_F}{2\Lambda}\right), \\ \frac{d\alpha}{dt}(t) &= (1-b)\alpha(t) - \alpha^2(t).\end{aligned}\tag{5.3}$$

The anomalous dimension $\eta(t)$ is expressed by $Y(t)$ and $\alpha(t)$ as

$$\eta(t) \equiv \frac{d}{dt} \ln R(t) = \alpha(t)I(Y(t), 0),\tag{5.4}$$

where we used eq.(4.13).

Therefore, if there were no dissipative term, i.e., $\lambda = 0$, anomalous dimension would be always vanishing $\eta = 0$, since $I(0, 0) = 0$. This result is consistent with the calculation in Ref.[9]. From (5.3), it is obvious that there is a non-trivial IR fixed point for $b < 1$ at $\alpha_{IR}^* = 1 - b$. (Note that the definition of usual β function is $\beta(\alpha) = -d\alpha/dt$). On the other hand, for $b > 1$, there is a trivial IR fixed point $\alpha = 0$ that corresponds to the Landau's Fermi-liquid theory.

Next we consider the second way of scaling, in which the tree level scaling of the dissipative term is vanishing. In this case, the RG equations are given as

$$\begin{aligned}\frac{dg}{dt}(t) &= -\frac{1}{2}g(t), \\ \frac{dv_F}{dt}(t) &= \left\{(2-b) - \frac{g^2(t)v_F(t)}{2\pi^2v_B}\right\}v_F(t), \\ \frac{d\lambda}{dt}(t) &= \frac{g^2(t)}{2\pi^2v_B}\left(\frac{2\pi K_F}{2\Lambda}\right).\end{aligned}\tag{5.5}$$

These equations are different from those obtained in the previous case (5.1). However, the equations of $Y(t)$ and $\alpha(t)$ coincide with those of (5.3) in the previous case. It is verified that this holds for an arbitrary value of δ . Therefore, $Y(t)$, $\alpha(t)$ and η behave independently of the value of δ . In short, for $0 \leq b < 1$, $\alpha(\infty) = \eta(\infty) = 1 - b$. On the other hand, for $b \geq 1$, $\alpha(\infty) = \eta(\infty) = 0$. This suggests that only two constants α and η (not g^2 itself) have physical meanings at low energies. Especially, it should be remarked that behavior of the effective Fermi velocity depends on the value of δ . However, physical conclusions should not be affected by the choice of δ .

5.2 Low-energy behavior of fermion propagator

Next let us consider the behaviors of the full fermion propagator under a RG transformation with the generalized scaling assignment (parametrized by δ);

$$\begin{aligned}\tilde{\omega} &= e^{\delta t}\omega, \quad \tilde{\mathbf{k}} = e^t\mathbf{k}, \\ \tilde{\psi}(n; \tilde{\mathbf{k}}, \tilde{\omega}) &= e^{-(\delta+1)t}\{R(t)\}^{\frac{1}{2}}\psi(n; \mathbf{k}, \omega).\end{aligned}\tag{5.6}$$

We begin with the definition;

$$\langle \psi(n; \mathbf{k}, \omega) \bar{\psi}(m; \mathbf{k}', \omega') \rangle \equiv \bar{\delta}(\omega - \omega') \bar{\delta}(\mathbf{k} - \mathbf{k}') \delta_{mn} \hat{G}(v_F(\mathbf{e}_n \cdot \mathbf{k}), \omega; \alpha, \Lambda). \tag{5.7}$$

By substituting the last line of (5.6) into the LHS of (5.7) and then expressing it by the propagator at $\tilde{\mathbf{k}}$, it is written as

$$\begin{aligned}\langle \psi(n; \mathbf{k}, \omega) \bar{\psi}(m; \mathbf{k}', \omega') \rangle &= e^{2(\delta+1)t} \{R(t)\}^{-1} \langle \tilde{\psi}(n; \tilde{\mathbf{k}}, \tilde{\omega}) \bar{\tilde{\psi}}(m; \tilde{\mathbf{k}}', \tilde{\omega}') \rangle \\ &= e^{2(\delta+1)t} \{R(t)\}^{-1} \bar{\delta}(\tilde{\omega} - \tilde{\omega}') \bar{\delta}(\tilde{\mathbf{k}} - \tilde{\mathbf{k}}') \delta_{mn} \hat{G}(v_F(t)(\mathbf{e}_n \cdot \tilde{\mathbf{k}}), \tilde{\omega}; \alpha(t), \Lambda).\end{aligned}\tag{5.8}$$

Next, from a straightforward dimensional analysis, the propagator satisfies

$$\hat{G}(v_F(\mathbf{e}_n \cdot \mu \mathbf{k}), \mu \omega; \alpha, \mu \Lambda) = \mu^{-1} \hat{G}(v_F(\mathbf{e}_n \cdot \mathbf{k}), \omega; \alpha, \Lambda). \tag{5.9}$$

By equating (5.7) and (5.8) and using (5.9) with $\mu = \exp(\delta t)$, we get

$$\begin{aligned}\hat{G}(v_F(\mathbf{e}_n \cdot \mathbf{k}), \omega; \alpha, \Lambda) &= e^{\delta t} \{R(t)\}^{-1} \hat{G}(v_F(t)(\mathbf{e}_n \cdot \tilde{\mathbf{k}}), \tilde{\omega}; \alpha(t), \Lambda) \\ &= \{R(t)\}^{-1} \hat{G}(v_{F1}(t)(\mathbf{e}_n \cdot \mathbf{k}), \omega; \alpha(t), e^{-\delta t} \Lambda),\end{aligned}\tag{5.10}$$

where we have used, in the last line, the relationship between two solutions of the effective Fermi velocity;

$$v_F(t) = e^{(\delta-1)t} v_{F1}(t). \tag{5.11}$$

which comes from (3.7). [Here $v_F(t)$ is the solution for general δ and $v_{F1}(t)$ is the solution in the first way of scaling ($\delta = 1$).]

To obtain the low-energy behavior, we set $\omega = \Lambda \exp(-t)$ or $t = \ln \frac{\Lambda}{\omega}$ in eq.(5.10). Thus, at sufficiently large t or low ω , the coefficient of Λ is negligible for $\delta > 0$, so eq.(5.10) tells us that each scaling gives the *same* (δ -independent) result as follows;

$$\hat{G}(v_F(\mathbf{e}_n \cdot \mathbf{k}), \omega; \alpha, \Lambda) \simeq \{R(\ln \frac{\Lambda}{\omega})\}^{-1} \hat{G}(v_{F1}(\ln \frac{\Lambda}{\omega})(\mathbf{e}_n \cdot \mathbf{k}), \omega; \alpha(\ln \frac{\Lambda}{\omega}), 0). \quad (5.12)$$

To get an explicit expression for the propagator, we need the solutions of $R(t)$ and $v_{F1}(t)$ at large t . So let us consider them. The recursion equations (5.3) and (5.4) for $\delta = 1$ can be solved for large t by elementary calculations. So we give only the results. At the leading order, they are given as follows;

(1) $b < 1$

$$\begin{aligned} \alpha(\infty) &= \eta(\infty) = 1 - b, \\ v_{F1}(t) &\simeq e^{-\eta(\infty)t} v_F, \\ R(t) &\simeq e^{\eta(\infty)t} \end{aligned} \quad (5.13)$$

(2) $b = 1$

$$\begin{aligned} \alpha(t) &\simeq \eta(t) \simeq \frac{\alpha}{1 + \alpha t}, \\ v_{F1}(t) &\simeq \frac{v_F}{1 + \alpha t}, \\ R(t) &\simeq 1 + \alpha t. \end{aligned} \quad (5.14)$$

(3) $1 < b$

$$\begin{aligned} \alpha(t) &\simeq \eta(t) \simeq e^{(1-b)t} \alpha, \\ v_{F1}(t) &\simeq \frac{v_F}{1 + \frac{\alpha}{b-1}}, \\ R(t) &\simeq \text{constant}, \end{aligned} \quad (5.15)$$

where $Y(\infty) = \infty$ for all the cases.

Now we can evaluate the full fermion propagator simply by substituting these expressions to the RHS of eq.(5.12) as follows;

(1) $b < 1$

$$\hat{G}(v_F(\mathbf{e}_n \cdot \mathbf{k}), \omega; \alpha, \Lambda) \simeq \frac{1}{\left(\frac{\omega}{\Lambda}\right)^{b-1}(i\omega) + v_F(\mathbf{e}_n \cdot \mathbf{k})}. \quad (5.16)$$

(2) $b = 1$

$$\hat{G}(v_F(\mathbf{e}_n \cdot \mathbf{k}), \omega; \alpha, \Lambda) \simeq \frac{1}{-\alpha \ln\left(\frac{\omega}{\Lambda}\right)(i\omega) + v_F(\mathbf{e}_n \cdot \mathbf{k})}. \quad (5.17)$$

(3) $1 < b$

$$\hat{G}(v_F(\mathbf{e}_n \cdot \mathbf{k}), \omega; \alpha, \Lambda) \simeq \frac{1}{\left(1 + \frac{\alpha}{b-1}\right)(i\omega) + v_F(\mathbf{e}_n \cdot \mathbf{k})}. \quad (5.18)$$

For (1) $b < 1$, we have assumed b is close to 1 or equivalently a small $\alpha(\infty)$ in order to apply the free form of the propagator. These are the main conclusions of this subsection.

These results show that for $b < 1$, the fermion propagator has a branch cut. The Luttinger liquid in one dimension[10] has a similar propagator. In that case, appearance of the branch cut stems from the fact that the charge-spin separation occurs. In the present case, however, it is not yet clear what kind of physical phenomenon (like charge-spin separation) generates the branch cut of propagator. For $b = 1$, the weight of quasiparticles becomes logarithmically vanishing at low energy $\omega \rightarrow 0$. This behaviour is nothing but that of the marginal-Fermi liquid[11], which was proposed for describing anomalous properties of the high- T_C cuprate. It is interesting, therefore, to see how gauge theory similar to the present model with $b = 1$ appears from models of strongly-correlated electrons, like the t-J model. Finally for $b > 1$, the low-energy behavior of the model is described by the usual Landau's Fermi liquid theory.

From (5.12), (5.13) and (5.14), for $b \leq 1$ the effective Fermi velocity $v_{F1}(\ln \frac{\Lambda}{\omega})$ becomes vanishing at low energy $\omega \rightarrow 0$. This means that the effective mass of the quasiparticle becomes divergent. This result was first observed by Halperin, Lee and Read[2] by the RPA. Recent experiments support this mass-enhancement phenomenon[13].

6 Discussion

In this paper, we studied a system of fermions interacting with dissipative gauge field by Wilsonian RG analysis. Fluctuations of gauge field are controlled by the parameter b as (2.3). We also employed certain specific momentum cutoff in order to incorporate the interaction with the gauge field correctly. RG study determines the low-energy behaviors of the coupling constant, the anomalous dimension of fermion, and the fermion propagator. For $b \leq 1$, the model has non-Fermi liquid behavior. The dissipative term of the gauge field plays an essentially important role for this result. Especially it scales non-trivially under RG transformations as in (3.4). In contrast to the Wilsonian RG, in the $(1 - b)$ -expansion RG, which was given in the previous paper[9], the coefficient of the dissipative term is finite in the loop calculation, and so no renormalization term of the dissipative term has been included in the action. Because of this difference, the study in the present paper gives different results for the β -function, etc. from those obtained by the $(1 - b)$ -loop expansion and also the $1/N$ -RPA.

Let us consider this point a little bit more. Usual strategy of renormalization group consists of three steps; (i) assignment of tree-level scaling dimensions to energy, momentum, each field, and each "coupling constant". From the last one, one extracts "dimensionless" coupling constant, which is regarded as a physically relevant parameter; (ii) calculation of renormalization constants (the Z 's in Refs.[8, 9] and the R 's in the present paper) according to each prescription employed ($(1 - b)$ -expansion, Wilsonian decimation, etc.); (iii) calculation of β functions for "dimensionless" couplings to discuss their scaling behaviors.

In relativistic field theory, the step (i) is unique since each coupling has a definite physical mass dimension. For non-relativistic fermions, existence of a Fermi surface produces ambiguity in the assignment. In Ref.[8] and in the loop expansion in Ref.[9], the scaling assignment for momentum and energy was done so that the energy term and the momentum term in the bare fermion propagator balance upon scaling when

the Fermi velocity v_F is fixed. This assignment leads to the result that the momentum term dominates the energy term in the *gauge field propagator*, and it has been explicitly verified that the appearance of an energy term of ω^2 , a higher-order term in our calculation, does not change any results obtained by the loop expansion. In the RPA in [9], on the other hands, assignment was done so that the dissipative term and the momentum term in the RPA gauge propagator should balance, since we have witnessed that the dissipative term plays an essentially important role. This requirement leads to the unbalancing of the two terms in fermion propagator (which does not mean that the assignment is wrong at all, of course), and also gives rise to a $\beta(\alpha)$ different from that obtained by the loop expansion for $b < 1$.

In summary, we have two possible origins of potential difference in behaviors of "dimensionless" couplings; (1) scaling assignment; (2) prescription of renormalization. Accordingly, one may ask two open questions; (A) are there sufficient principles to determine tree-level scaling dimensions for all quantities uniquely, and (B) do the two origins above be correlated each other. Concerning to these points, we think that the result of the present paper poses a possibility to bypass these problems to some extent. First, we found out that there is a one-parameter sequence of assignments of tree-level scaling dimensions (parametrized by δ), which leaves the effective action unchanged in its form. Furthermore there are certain special combinations of coupling constants that are independent of δ . Since they are free from ambiguity, physical conclusions derived from them should be convincing. Second, the Wilson-type decimation prescription generates an effective action that involves all the possible renormalization constants on an equal footing, while the $(1 - b)$ -expansion, by rules, picks up only such renormalization constants that contain singularities. When there are no strong physical reasons to treat certain renormalization constants as special ones, it is natural to employ a prescription of decimation type.

Finally, let us go back to the physics of the present model. It is expected that this model describes an electron system in a magnetic field at Landau filling factor

$\nu = 1/2$. It is suggested that FQHE at filling factor closed to $\nu = 1/2$, e.g., $\nu = p/(2p \pm 1)$, should be explained as the Landau quantization of fermions of the system at $\nu = 1/2$ [3]. Therefore, the non-Fermi liquid like behavior of fermions presented in the present model is capable to be observed through experimental studies on FQHE at $\nu = p/(2p \pm 1)$. Theoretical investigation on this problem is under study and results will be reported in a future publication.

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Figure Captions

Fig.1

Partition of the two-dimensional space of fermion momentum $\vec{K}$. The big arc $C - C'$ shows a part of Fermi circle of radius K_F around the origin O . There are $N = 2\pi K_F/(2\Lambda)$ Fermi wave vectors, $\vec{K}_{F,n} (n = 1, \dots, N)$, being separated each other by equal distances 2Λ along the circle. Here Λ is the cutoff momentum. Under the renormalization-group transformations, we integrate gradually the modes with the momenta which sit "near" the Fermi circle within the distance $\simeq \Lambda$. To be explicit, we integrate the momenta which locate within one of the N small circles of radius Λ , each circle being around the end point of $\vec{K}_{F,n}$ as illustrated. One may enlarge the region of relevant momenta, e.g., to those that locate within the union of small circles of radius 2Λ , to avoid the momenta which sit very near to the Fermi circle but never be integrated. However, the main conclusions of the text do not depend on such detail.

Fig.2

The two-dimensional space of momentum $\vec{k} = (k_x, k_y)$. $\vec{k} = \vec{K} - \vec{K}_{F,n}$ is a deviation of fermion momentum $\vec{K}$ near the Fermi circle measured from the nearest Fermi momentum $\vec{K}_{F,n}$. In the step of renormalization-group transformation from t to $t + dt$, the modes with such momenta that sit in the shaded area are to be integrated. This area is nothing but a shell surrounded by two circles of radii Λe^{-t} and $\Lambda e^{-(t+dt)}$.

Fig. 1

Fig. 2