A note on Majorana neutrinos, leptonic CKM and electron electric dipole moment

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Abstract

The electric dipole moment of the electron, $d_e$, is known to vanish up to three-loops in the standard model with massless neutrinos. However, if neutrinos are massive Majorana particles, we obtain the result that $d_e$ induced by leptonic CKM mechanism is non-vanishing at two-loop order, and it applies to all massive Majorana neutrino models.
The experimental searches for the electric dipole moments of the neutron, \( d_n \), and the electron, \( d_e \), have reached unprecedented accuracies. At present, their values \([1,2]\) are given by

\[
d_n = (30 \pm 50) \times 10^{-27} \text{e-cm},
\]

and

\[
d_e = (1.8 \pm 1.2 \pm 1.0) \times 10^{-27} \text{e-cm}.
\]

These values put important constraints on CP violating theories beyond the standard model (SM).

Within the SM, CP violation is encoded in the complex elements of the CKM unitary matrix, \( V \), in the quark sector and observable effects are proportional to \([3]\) the quantity

\[
J = \text{Im}(V_{ij}V^*_{kl}V^*_{il})
\]

where the indices are not summed. Obviously, \( J \) is non-vanishing only if not all the elements \( V_{ij} \) can be made real and this implies the existence of at least three generations of non-degenerate massive quarks. It is clear from Eq.(3) that \( J \) is antisymmetric about \( i \) and \( k \) as well as \( j \) and \( l \). This leads to \( J = 0 \) for degenerate quarks. Using these properties, Shabalin \([4]\) showed that electric dipole moment of quarks vanishes at two-loop level and hence implies a very small \( d_n \).

Within the SM, the suppression of \( d_e \) is even more severe than that of \( d_n \). This is due in part to the masslessness of the three neutrinos and hence there is no CKM type mixing in the lepton sector. Any CP violation effects for leptons will have to be induced from the quark sector. For the case of \( d_e \), the lowest non-vanishing contribution arises from the electric dipole moment of the \( W \)-boson, \( d_W \). The authors of Ref. \([5]\) showed that \( d_W \) vanishes at the two-loop level and this implies that \( d_e \) vanishes at the three-loop level. Hence, the SM \( d_e \) is estimated at the four-loop level to be \([6]\)

\[
d_e \sim \frac{eG_F}{\pi^2} \left( \frac{\alpha}{2\pi} \right)^3 m_e J \leq 4 \times 10^{-38},
\]

where \( J \leq 2 \times 10^{-4} \) for the CKM elements is used. One factor of \( \alpha \) can be replaced by \( \alpha_s \), but this will not be a sufficient enhancement to be experimentally interesting. We can see that it is beyond experimental capabilities in the foreseeable future. At the same time if offers the opportunity that \( d_e \) is a clean test of CP violation beyond the SM.

One can contemplate extending the SM by simply making the neutrinos massive. For instance, one can add a right-handed Dirac neutrino to each family. Then the neutrinos can have arbitrary but very small masses. Although this is unappealing, it is a possibility that is not ruled out. Now the leptonic sector will exhibit the CKM type of mixing as the analogue of the CKM matrix in the quark sector. In complete parallel to the case of quarks, \( d_e \) can only be induced at the three-loop level and is totally negligible since \( d_e \) will be proportional to the difference of masses squared of the neutrinos. For the heaviest allowed mass of 30 MeV for \( \nu_\tau \), we anticipate a suppression factor of \( \frac{M_{\nu_\tau}}{M_W^2} \leq 3 \times 10^{-7} \).

In this letter, we consider the situation when the neutrinos are massive Majorana particles. The simplest manifestation is the addition of one or more \([7,8]\) right-handed neutrinos which then allows us to construct Majorana mass terms; however, for seesaw models of neutrino masses, more than one right-handed neutrino is required to generate a physical phase.
Such Majorana neutrinos are present in many grand unified theories such as SO(10), E(6), etc. They can also exist in left-right symmetric models. As opposed to the Dirac neutrinos where there is only one CP violating phase for three generations and other phases can be transformed to the non-observable right-handed sector, there are many observable phases associated with Majorana neutrinos. For example even with two generations of Majorana neutrinos there exists one CKM phase [7]. For definiteness, we consider $d_e$ as induced by a leptonic CKM mechanism which results from such massive Majorana neutrinos mixing. Interestingly, this case has not been studied in the literature and our main result is that $d_e$ is non-vanishing at the two-loop level for Majorana neutrinos.

We begin our study by first noting the interaction lagrangian involving the electric dipole moment is given by

$$\mathcal{L} = -\frac{1}{2}d_e F_{\mu\nu} \bar{e} i \sigma^{\mu\nu} \gamma_5 e,$$

and the charged leptonic current interaction is

$$\frac{g}{\sqrt{2}} W^\mu U_{ij} \bar{e}_i \gamma_\mu \frac{1 - \gamma_5}{2} e_j + h.c.,$$

where $U_{ij}$ is the charged current mixing matrix analogue to the CKM matrix in the quark sector. Here, the number of neutrinos are unrestricted. The detailed structure of $U$ is model-dependent but is not needed for our discussion. What is of importance is that the mixing matrix $U$ is unitary and the elements are in general complex. This is certainly true for the realistic case of three light neutrinos. It is also obvious that the one-loop contribution to $d_e$ is vanishing.

We now proceed to the two-loop calculation. As was argued before, the diagrams that are common to Dirac neutrinos need not be considered since we know that they do not contribute to $d_e$; hence we need only concentrate on those that are specific to Majorana neutrinos. The two-loop $W$ exchange diagrams, depicted in Fig. 1a and 1b, give important contributions and illustrate the essential physics at play here. Notice that the diagrams do not exist in the SM nor for massive Dirac neutrinos, since lepton number conservation is not respected by the internal neutrino lines. An interesting remark can be made here. If one cuts across two $W$-bosons and the internal lepton line in Fig. 1a, and takes the latter to be the electron, one gets the process $e^- e^- \rightarrow W^- W^-$ which is characteristic of Majorana neutrinos [9,10]. After some standard manipulation, $d_e$ can be written in terms of Feynman integrals given as

$$d_e(a) = -\frac{\alpha^2 m_e}{256 \pi^2 s_W^4} m_i m_j J_{ij} \int_0^1 dx \int_0^{1-x} dy \int_0^1 ds \int_0^{1-s} dt \int_0^{1-s-t} du \frac{x(1-x)^2[(1-s)^2-(t+u)^2] + xy^2u(1-u) - x(1-x)y(1+3s+t+u-2tu-2u^2)}{[m_i^2x(1-x)(1-s-t-u) + m_j^2(x-1-y)u + m_W^2 \bar{W}(1-x)(s+t) + yu] + m_i^2xu},$$

and

$$d_e(b) = \frac{\alpha^2 m_e}{256 \pi^2 s_W^4} m_i m_j J_{ij} \int_0^1 dx \int_0^{1-x} dy \int_0^1 ds \int_0^{1-s} dt \int_0^{1-s-t} du \frac{x(1-x)^2[(1-s)^2-(t+u)^2] + xy^2u(1-u) - x(1-x)y(1+3s+t+u-2tu-2u^2)}{[m_i^2x(1-x)(1-s-t-u) + m_j^2(x-1-y)u + m_W^2 \bar{W}(1-x)(s+t) + yu] + m_i^2xu}.$$
where the CP violating factor $J_{ij}^l$ is given by

$$J_{ij}^l = \text{Im}(U_{le}^* U_{lj}^* U_{ii} U_{ie}) ,$$

(9)

with $l$ being the internal charged lepton in the diagrams, where $l = e, \mu, \tau$ for the SM. This is a variant of the quark phase invariant factor. Note that $d_e(a)$ and $d_e(b)$ are antisymmetric to each other by interchanging $m_i$ and $m_j$, and this leads to nonvanishing EDM of an electron. It is important to notice some interesting properties of $J_{ij}^l$. $J_{ij}^l$ is antisymmetric about $i$ and $j$, namely $J_{ij}^l = -J_{ji}^l$, but symmetric about $e$ and $l$. In addition, $J_{ii}^l = 0$ which implies at least two different massive Majorana neutrinos are needed to generate two-loop non-vanishing $d_e$.

The situation is different in Fig. 1c. The amplitude is given as

$$-e \left( \frac{g}{\sqrt{2}} \right)^4 J_{ij}^l m_i m_j \int \frac{dk_1^4}{(2\pi)^4} \int \frac{dk_2^4}{(2\pi)^4} \left[ 4(\not{p}_1 - \not{k}_1 - \not{k}_2)\gamma^\mu(\not{p}_2 - \not{k}_1 - \not{k}_2) \right] \frac{1}{k_1^2 - m_i^2} \frac{1}{k_2^2 - m_j^2} \frac{1}{(k_1 - p_1)^2 - m_W^2} \frac{1}{(k_2 - p_2)^2 - m_W^2} \frac{1}{(p_2 - k_1 - k_2)^2 - m_i^2} \frac{1}{(p_1 - k_1 - k_2)^2 - m_j^2} .$$

(10)

We can see from Eq.(10) that aside from the factor $J_{ij}^l$, the intergrals in Eq.(10) are symmetrical about $m_i$ and $m_j$. Thus Fig. 1c does not contribute to $d_e$ when the sum over $i$ and $j$ is taken.

In addition to the $W$-boson exchange diagrams, one has to include the exchanges of would-be Goldstone-bosons. These diagrams are multiplied by factors of $(m_i/m_W^2)^2$ or $(m_j/m_W^2)^2$ depending on whether one or two Goldstone-bosons are exchanged. For $m_i < m_W$, Goldstone-boson exchange diagrams are less important. We can now give a semi-quantitative estimate of $d_e$ and obtain

$$d_e \sim \frac{\alpha^2 m_e m_i m_j (m_i^2 - m_j^2)}{256 \pi^2 s_W^4 m_W^6} J_{ij}^l F(m_i^2/m_W^2, m_i^2/m_W^2, m_j^2/m_W^2)(1.97 \times 10^{-16})\text{e-cm} ,$$

(11)

where all masses are taken in units of GeV.

The GIM factors $(m_i^2 - m_j^2)/m_W^2$ for neutrinos are explicit and the fact that $d_e$ vanishes when the neutrinos are massless is also obvious. Less obvious is the limit of degenerate charged lepton masses where $d_e$ must also vanish. For $m_i < m_W$, as the case in the SM, this factor can be obtained by Taylor expanding the denominator in Eqs.(7,8) and a further factor of $m_i^2/m_W^2$ arises. We can now give numerical estimation for some cases. If there exist Majorana neutrinos with masses, $m_i^2 \leq m_W^2$, and $|U_{ei}|^2 \leq 10^{-2}$ obtained from the charged current universality constraints [8], we get $d_e \leq 10^{-32}$ e–cm assuming that the integral function $F$ in Eqs.(7,8) is of the order unity. In general the function $F$ can only be evaluated numerically; however, for the special limit of taken all masses to be equal within the integral only it can be evaluated and we found it to be 0.05.

Our calculation can be straightforwardly applied to $\mu$ and $\tau$. There are enhancement factors in these cases. For example, for $d_\mu/d_e \sim m_\mu/m_e \sim 200$; and $d_\tau \sim 10^{-26}$ e–cm without mixing suppression. Unfortunately, the measurements of $d_\mu$ and $d_\tau$ are far more difficult.

In conclusion, we find that in general Majorana neutrinos can induce electric dipole moment at the two-loop level. This is to be compared to the Dirac neutrinos case in which contribution comes in at the three-loop level. It is a necessary condition to have two or more non-degenerate Majorana neutrinos in order to contribute to $d_e$ at this level. However, the numerical estimates for $d_e \leq 10^{-32}$ e–cm even for Majorana neutrinos of masses in the 100
GeV range. This value is several orders of magnitude lower than the current experiments can reach in the near future. The suppression arises from two sources, the first being the loop factor of $\sim \frac{\alpha}{2\pi}$; the second one arises from the mixing of heavy neutrinos with the light ones which is constrained to be very small. This suppression factor $J^l_{ij}$ can be overcome if there exist heavy charged leptons that couple to the heavy neutrinos with full strength. It is also less severe for the $\tau$-lepton. This can enhance our result by up to two orders of magnitude.

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REFERENCES


FIGURES

FIG. 1. Two-\(W\) exchange Feynman diagrams