At the first stage of reheating after inflation, parametric resonance may rapidly transfer most of the energy of an inflaton field \( \phi \) to the energy of other bosons. We show that quantum fluctuations of scalar and vector fields produced at this stage are much greater than they would be in a state of thermal equilibrium. This leads to cosmological phase transitions of a new type, which may result in a copious production of topological defects and in a secondary stage of inflation after reheating.

The theory of reheating is one of the most important parts of inflationary cosmology. Elementary theory of this process was developed many years ago [1,2]. Some important steps toward a complete theory have been made in [3]. However, the real progress in understanding of this process was achieved only recently when the new theory of reheating was developed. According to this theory [4], reheating typically consists of three different stages. At the first stage, a classical oscillating scalar field \( \phi \) (the inflaton field) decays into massive bosons due to parametric resonance. In many models the resonance is very broad, and the process occurs extremely rapidly. To distinguish this stage of explosive reheating from the stage of particle decay and thermalization, we called it preheating. Bosons produced at that stage are far away from thermal equilibrium and have enormously large occupation numbers. The second stage is the decay of previously produced particles. This stage typically can be described by methods developed in [1]. However, these methods should be applied not to the decay of the original homogeneous inflaton field, but to the decay of particles produced at the stage of preheating. This changes many features of the process including the final value of the reheating temperature. The third stage of reheating is thermalization.

Different aspects of the theory of explosive reheating have been studied by many authors [5]–[8]. In our presentation we will follow the original approach of ref. [4], where the theory of reheating was investigated with an account taken both of the expansion of the universe and of the backreaction of created particles.

One should note that there exist such models where this first stage of reheating is absent; e.g., there is no parametric resonance in the theories where the field \( \phi \) decays into fermions. However, in the theories where preheating is possible one may expect many unusual phenomena. One of the most interesting effects is the possibility of specific non-thermal post-inflationary phase transitions which occur after preheating. As we will see, these phase transitions in certain cases can be much more pronounced that the standard high temperature cosmological phase transitions [9,10]. They may lead to copious production of topological defects and to a secondary stage of inflation after reheating.

Let us first remember the theory of phase transitions in theories with spontaneous symmetry breaking in the theory of scalar fields \( \phi \) and \( \chi \) with the effective potential

\[
V(\phi,\chi) = \frac{\lambda}{2}(\phi^2 - \phi_0^2)^2 + \frac{1}{2}g^2\phi^2\chi^2.
\]

Here \( \lambda, g \ll 1 \) are coupling constants. \( V(\phi,\chi) \) has a minimum at \( \phi = \phi_0, \chi = 0 \) and a maximum at \( \phi = \chi = 0 \) with the curvature \( V_{\phi\phi} = -m^2 = -\lambda\phi_0^2 \). This effective potential acquires corrections due to quantum (or thermal) fluctuations of the scalar fields [9,10], \( \Delta V = \frac{3}{2}\lambda\langle(\delta\phi)^2\rangle\phi^2 + \frac{g^2}{2}\langle(\delta\chi)^2\rangle\phi^2 + \frac{g^2}{2}\langle(\delta\phi)^2\rangle\chi^2 + ... \), where the quantum field operator is decomposed as \( \phi = \phi + \delta\phi \) with \( \phi \equiv \langle\phi\rangle \), and we have written only leading terms depending on \( \phi \) and \( \chi \equiv \langle\chi\rangle \). In the large temperature limit \( \langle(\delta\phi)^2\rangle = \langle(\delta\chi)^2\rangle = \frac{T^4}{12} \). The effective mass squared of the field \( \phi \)

\[
m_{\phi,eff}^2 = -m^2 + 3\lambda\phi^2 + 3\lambda\langle(\delta\phi)^2\rangle + g^2\langle(\delta\chi)^2\rangle
\]

becomes positive and symmetry is restored (i.e. \( \phi = 0 \) becomes the stable equilibrium point) for \( T > T_c \), where
$T_c^2 = \frac{12m^2}{4\pi^2 f^2} \gg m^2$. At this temperature the energy density of the gas of ultrarelativistic particles is given by

$$\rho = N(T_c)^2 T_c^4 = \frac{12m^2 N(T_c)^2}{4\pi^2 f^2}.$$  

Here $N(T)$ is the effective number of degrees of freedom at large temperature, which in realistic situations may vary from $10^3$ to $10^6$. Note that for $g^4 \approx \frac{96N^2\pi^2}{\xi}$ this energy density is greater than the vacuum energy density $V(0) = \frac{m^4}{4\lambda}$. Meanwhile, for $g^4 \approx \lambda$ radiative corrections are important, and they lead to creation of a local minimum of $V(\phi, \chi)$, and the phase transition occurs from a strongly supercooled state [9]. That is why the first models of inflation required supercooling at the moment of the phase transition.

An exception from this rule is given by supersymmetric theories, where one may have $g^4 \gg \lambda$ and still have a potential which is flat near the origin due to cancellation of quantum corrections of bosons and fermions [11]. In such cases thermal energy becomes smaller than the vacuum energy at $T < T_0$, where $T_0 = \frac{15}{\pi^2} m^2 \phi_0$. Then one may even have a short stage of inflation which begins at $T \sim T_0$ and ends at $T = T_c$. During this time the universe may inflate by the factor

$$a_c = \frac{T_c}{a_0} \sim 10^{-1} \left(\frac{g^4}{\lambda}\right)^{1/4} \approx 10^{-1} g \sqrt{\phi_0/\lambda}.$$  

(3)

In supersymmetric theories with flat directions $\Phi$ it may be more natural to consider potentials of the so-called “flaton” fields $\Phi$ without the term $\frac{1}{4} \Phi^4$ [11]:

$$V(\Phi, \chi) = -\frac{m^2 \Phi^2}{2} + \frac{\lambda_1 \Phi^4}{6M^2} + \frac{m^2 \Phi_0^2}{3} + \frac{1}{2} g^2 \Phi^2 \chi^2,$$  

(4)

where $\Phi_0 = \frac{1}{\lambda^{1/4}} \sqrt{mM_p}$ corresponds to the minimum of this potential. The critical temperature in this theory for $\Phi_0 \ll g^2 M^2$ is the same as in the theory (1) for $\lambda \ll g^2$, and expansion of the universe during thermal inflation is given by $10^{-1} g \sqrt{\phi_0/\lambda}$, as in eq. (3). This short additional stage of “thermal inflation” may be very useful; in particular, it may provide a solution to the Polonyi field problem [11].

The theory of cosmological phase transitions is an important part of the theory of the evolution of the universe, and during the last twenty years it was investigated in a very detailed way. However, typically it was assumed that the phase transitions occur in the state of thermal equilibrium. Now we are going to show that similar phase transitions may occur even much more efficiently prior to thermalization, due to the anomalously large expectation values $\langle \delta \phi \rangle^2$ and $\langle \delta \chi \rangle^2$ produced during the first stage of reheating after inflation.

We will first consider the model (1) without the scalar field $\chi$ and with the amplitude of spontaneous symmetry breaking $\phi_0 \ll M_p$. In this model inflation occurs during the slow rolling of the scalar field $\phi$ from its very large values until it becomes of the order $M_p$. Then it oscillates with the frequency $\sim \sqrt{\lambda/M_p}$, and within few oscillations it transfers most of its energy $\sim \lambda M_p^4$ to its long-wave fluctuations $\langle \delta \phi \rangle^2 \sim M_p^2$ in the regime of broad parametric resonance.

The crucial observation is the following. If the initial energy density $\sim \lambda M_p^4$ were instantaneously thermalized, the reheating temperature $T_r \sim \lambda^{1/4} M_p$ would be much greater than the typical particle energy after preheating $\langle E \rangle \sim \sqrt{\lambda M_p}$, and the magnitude of fluctuations $\langle \delta \phi \rangle^2 \sim T_r^2/12 \sim \sqrt{\lambda M_p}$ would be much smaller than the magnitude of non-thermalized fluctuations $\langle \delta \phi \rangle^2 \sim M_p^2$. Thus after the first stage of reheating, the non-thermalized fluctuations of the scalar field $\phi$ are much greater than the thermalized ones. Thermal fluctuations would lead to symmetry restoration in our model only for $\phi_0 \lesssim T_r \sim \lambda^{1/4} M_p$. Meanwhile, according to eq. (2), the non-thermalized fluctuations $\langle \delta \phi \rangle^2 \sim M_p^2$ may lead to symmetry restoration in our model even if the symmetry breaking parameter $\phi_0$ is as large as $M_p$. Thus, the non-thermal symmetry restoration occurs even in those theories where the symmetry restoration due to high temperature effects would be impossible. Later on, $\langle \delta \phi \rangle^2 \propto a^{-2}$, $E \propto a^{-1}$ because of the expansion of the universe (as far as $E \propto \lambda$). This leads to the phase transition with symmetry breaking at the moment $t = t_c \sim \sqrt{\lambda M_p} \sim m^2$ when $m_{\text{eff}} = 0$, $\langle \delta \phi \rangle^2 = \phi_0^2/3$, $E \sim m$. Note that the homogeneous component $\phi(t)$ at this moment is significantly less than $\sqrt{\langle \delta \phi \rangle^2}$ due to its decay in the regime of the narrow parametric resonance after preheating [4]: $\phi(t) \sim t^{-7/6} \propto t^{-1/6} \langle \delta \phi \rangle^2$; bar means averaging over oscillations.

The mechanism of symmetry restoration described above is very general; in particular, it explains a surprising behavior of oscillations of the scalar field found numerically in the $O(N)$-symmetric model of ref. [6]. It is important that during the interval between preheating and the establishment of thermal equilibrium the universe could experience a series of phase transitions which we did not anticipate before. For example, cosmic strings and textures, which could be an additional source for the formation of the large scale structure of the universe, should have $\phi_0 \sim 10^{16}$ GeV [12]. To produce them by thermal phase transitions in our model we should have the temperature after reheating greater than $10^{16}$ GeV, which is extremely hard to obtain [13]. Meanwhile, as we see now, fluctuations produced during the first stage of reheating are more than sufficient to restore the symmetry and to produce topological defects at the moment when the symmetry breaks down again. In other words, production of superheavy topological defects can be easily compatible with inflation.

On the other hand, the topological defect production can be quite dangerous. For example, the model (1) of a one-component real scalar field $\phi$ has a discrete symmetry $\phi \rightarrow -\phi$ as a result, after the phase transition induced by fluctuations $\langle \delta \phi \rangle^2$ the universe may become filled with domain walls separating phases $\phi = +\phi_0$ and $\phi = -\phi_0$. This is expected to lead to a cosmological disaster.
This question requires a more detailed analysis. Even though the point $\phi = 0$ after preheating becomes a minimum of the effective potential, the field $\phi$ continues oscillating around this minimum. Therefore, at the moment $t_c$ it may happen to be either to the right of the maximum of $V(\phi)$ or to the left of it everywhere in the universe. In this case the symmetry breaking will occur in one preferable direction, and no domain walls will be produced. A similar mechanism may suppress production of other topological defects.

However, this would be correct only if the magnitude of fluctuations $\langle (\delta \phi)^2 \rangle$ were smaller than the average amplitude of the oscillations $\delta^2$. In our case fluctuations $(\delta \phi)^2$ are greater than $\delta^2$, and they can have considerable local deviations from their average value $\langle (\delta \phi)^2 \rangle$. Investigation of this question shows that in the theory (1) with $\phi_0 \ll M_p$, fluctuations destroy the coherent distribution of the oscillating field $\phi$ and divide the universe into equal number of domains with $\phi = \pm \phi_0$. It leads to the domain wall problem and may imply that consistent inflationary models of the type of (1) should not have a discrete symmetry breaking. To have a complete reheating in models with $\phi_0 = 0$, the scalar field $\phi$ should interact with fermions, since the usual boson interaction $g^2 \phi^2 \chi^2$ does not lead to a complete reheating [4].

Now we will consider models where the symmetry breaking occurs for fields other than the inflaton field $\phi$. The simplest model has an effective potential

$$V(\phi, \chi) = \frac{\lambda}{4} \phi^4 + \frac{\alpha}{4} \left( \chi^2 - \frac{M^2}{\alpha} \right)^2 + \frac{1}{2} g^2 \phi^2 \chi^2. \quad (5)$$

The models of such type have been studied in [13,14]. We will assume here that $\lambda \ll \alpha, g^2$, so that at large $\phi$ the curvature of the potential in the $\chi$-direction is much greater than in the $\phi$-direction. In this case at large $\phi$ the field $\chi$ rapidly rolls toward $\chi = 0$. An interesting feature of such models is the symmetry restoration for the field $\chi$ for $\phi > \phi_c = M/g$, and symmetry breaking when the inflaton field $\phi$ becomes smaller than $\phi_c$. As was emphasized in [13], such phase transitions may lead to formation of topological defects without any need for high-temperature effects.

Now we would like to point out some other specific features of such models. If the phase transition discussed above happens during inflation [13] (i.e. if $\phi_c > M_p$ in our model), then no new phase transitions occur in this model after reheating. However, for $\phi_c \ll M_p$ the situation is much more complicated. First of all, in this case the field $\phi$ oscillates with the initial amplitude $\sim M_p$ (if $M^4 < \alpha \lambda M_p^4$). This means that each time when the absolute value of the field becomes smaller than $\phi_c$, the phase transition with symmetry breaking occurs and topological defects are produced. Then the absolute value of the oscillating field $\phi$ again becomes greater than $\phi_c$, and symmetry restores again. However, this regime does not continue for a too long time. Within a few oscillations, quantum fluctuations of the field $\chi$ will be generated with the dispersion $\langle (\delta \chi)^2 \rangle \sim g^{-1} \sqrt{\lambda} M_p^2$ [4]. For $M^2 < g^{-1} \sqrt{\lambda} M_p^2$, these fluctuations will keep the symmetry restored. The symmetry breaking will be finally completed when $\langle (\delta \chi)^2 \rangle$ will become small enough.

One may imagine even more complicated scenario when oscillations of the scalar field $\phi$ create large fluctuations of the field $\chi$, which in their turn interact with the scalar fields $\Phi$ breaking symmetry in GUTs. Then we would have phase transitions in GUTs induced by the fluctuations of the field $\chi$. Note that in the models considered in this section the field $\chi$ does not oscillate near $\chi = 0$ prior to the phase transition, since such oscillations are damped out during the long stage of inflation prior to the phase transition. Thus oscillations of the field $\chi$ in the theory (5) definitely do not suppress the topological defect production. This means that no longer can the absence of primordial monopoles be considered as an automatic consequence of inflation. To avoid the monopole production one should use the theories where quantum fluctuations produced during preheating are small or decoupled from the GUT sector. This condition imposes strong constraints on realistic inflationary models. On the other hand, parametric resonance and non-thermal phase transitions allow a much more efficient baryogenesis, since superheavy particles responsible for baryogenesis can be produced even in the models where the reheating temperature is extremely small.

Now let us return to the theory (1) including the field $\chi$ for $g^2 \gg \lambda$. In this case the main fraction of the potential energy density $\sim \lambda M_p^4$ of the field $\phi$ predominantly transfers to the energy of fluctuations of the field $\chi$ due to the explosive $\chi$-particles creation in the broad parametric resonance. The dispersion of fluctuations after preheating, is $\langle (\delta \chi)^2 \rangle \sim g^{-1} \sqrt{\lambda} M_p^2$, the characteristic energy of created particles $\rho_\chi \sim (g \lambda)^{1/4} M_p$ [4]. As we see again, this dispersion is greater by the factor $g^{-1}$ than the dispersion of thermal fluctuations of the field $\chi$ which would be produced by an instantaneous reheating and thermalization. These fluctuations lead to the symmetry restoration in the theory (1) with $\phi_0 < M_p$.

Later the process of decay of the field $\phi$ continues, but, just as in the model described in the previous section, one may say with a good accuracy that the fluctuations $\langle (\delta \chi)^2 \rangle$ decrease as $g^{-1} \sqrt{\lambda} M_p^2 (a_i/a_0)^2$ and their energy density $\rho$ decreases as the energy density of ultra-relativistic matter, $\rho(t) \sim M_p^4 (a_i/a_0)^4$, where $a_i$ is the scale factor at the end of inflation. This energy density becomes equal to the vacuum energy density $m_4^2/4\pi$ at $a_0 \sim a_i \sqrt{\lambda} M_p/m$, $t \sim \sqrt{\lambda} M_p m^{-2}$. Since that time and until the time of the phase transition with symmetry breaking the vacuum energy dominates, and the universe enters secondary stage of inflation.

The phase transition with spontaneous symmetry breaking occurs when $m_{\phi, eff} = 0$, $\langle (\delta \chi)^2 \rangle = g^{-2} m^2$. This happens at $a_i = a_i \lambda^{1/4} g^{1/2} M_p/m$. Thus, during this additional period of inflation the universe expands $a/a_i \sim \sqrt{g} \sqrt{\phi_0/m} = (g/\lambda)^{1/4}$. Times. This is greater.
than expansion during thermal inflation (3) by the factor $O(g^{-1/2})$, and in our case inflation occurs even if $g^4 \ll \lambda$.

In this example we considered the second stage of inflation driven by the inflaton field $\phi$. However, the same effect can occur in theories where other scalar fields are coupled to the field $\chi$. For example, in the theories of the type of (4) fluctuations $(\delta \chi)^2$ produced at the first stage of reheating by the oscillating inflaton field $\phi$ lead to a secondary inflation driven by the potential energy of the "flaton" field $\Phi$. During this stage the universe expands $\sim \sqrt{g} \sqrt{\Phi_0/m}$ times. To have a long enough inflation one may consider, e.g., supersymmetric theories with $m \sim 10^2$ GeV and $\Phi_0 \sim 10^{12}$ [11]. This gives a relatively long stage of inflation with $\sqrt{g} \Phi_0 \sim \sqrt{g} 10^5$, which may be enough to solve the Polonyi field problem if the constant $g$ is not too small.

If the coupling constant $g$ is sufficiently large, fluctuations of the field $\chi$ will thermalize during this inflationary stage. Then the end of this stage will be determined by the standard theory of high temperature phase transition, and the degree of expansion during this stage will be given by $g \sqrt{\Phi_0/m}$ (3). It is important, however, that the inflationary stage may begin even if the field $\chi$ has not been thermalized at that time.

The stage of inflation described above occurs in the theory with a potential which is not particularly flat near the origin. But what happens in the models which have flat potentials, like the original new inflation model in the Coleman-Weinberg theory [15]? One of the main problems of inflation in such models was to understand why should the scalar field $\phi$ jump onto the top of its effective potential, since this field in realistic inflationary model is extremely weakly interacting and, therefore, it could not be in the state of thermal equilibrium in the very early universe. Thus, it is much more natural for inflation in the Coleman-Weinberg theory to begin at very large $\phi$, as in the simplest version of chaotic inflation in the theory $\lambda \phi^4$. However, during the first few oscillations of the scalar field $\phi$ at the end of inflation in this model, it produces large non-thermal perturbations of vector fields $(\delta A_\mu)^2 \sim g^{-1} \Lambda M_p^2$. This leads to symmetry restoration and initiates the second stage of inflation beginning at $\phi = 0$. It suggests that in many models inflation most naturally begins at large $\phi$ as in the simplest version of the chaotic inflation scenario [16]. But then, after the stage of explosive reheating, the second stage of inflation may begin like in the new inflationary scenario. In other words, the new theory of reheating after chaotic inflation may rejuvenate the new inflation scenario!

The main conclusion of this paper is the following. In addition to the standard high temperature phase transition, there exists a new class of phase transitions which may occur at the intermediate stage between the end of inflation and the establishing of thermal equilibrium. These phase transitions take place even in the theories where the scale of spontaneous symmetry breaking is comparable to $M_p$ and where the reheating temperature is very small. Therefore, phase transitions of the new type may have dramatic consequences for inflationary models and the theory of physical processes in the very early universe.

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