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Isospin and the Tau Hadronic Decay Modes

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Abstract

Isospin constraints on τ decay modes are revisited with a special interest for the low branching ratio channels. It is shown that dominance of the vector current in the $K\bar{K}\pi$ channel is suggested by the present data and that some discrepancies exist between τ partial width and e^+e^- cross-section measurements in the 6π channel.

1 Introduction

The existence of isospin restrictions on the τ decay modes is well known [1] and isospin relations have been used to put limits on unobserved channels and relate vector current contributions to e^+e^- annihilation cross-sections. Now that precise and almost complete measurements of the branching ratios are available [3–22], it is worthwhile to compare them with the most complete isospin constraints in order not only to check their consistency but also and mainly to get some insight into the dynamics of the final states. This allows to improve the evaluation of axial and vector current contributions which is important for the estimation of non perturbative QCD effects in τ decays [23].

A helpful feature for such study is that, for all the relevant channels, the amplitudes can be put in classes with the following properties: *i)* Amplitudes in a same class yield the same charge configuration proportions. *ii)* Interferences of amplitudes in different classes do not contribute to the partial width.

We will not review here the high branching ratio channels where the experimental errors are now of the size of isospin violating effects like radiative decays and kinematical corrections for the $\pi^0/\pi^\pm$ mass difference [24]. On the contrary we will concentrate on the low branching ratio channels with three possible charge configurations where the complete use of the isospin constraints may improve the available information.

2 $\tau \rightarrow \nu_\tau + K + \bar{K} + n\pi$

In a Cabibbo allowed $K\bar{K}n\pi$ final state, both vector and axial currents can contribute. They lead respectively to G-parities: $G = +1$ (V) and $G = -1$ (A) for the hadronic system. Since the J^P quantum numbers are also different, there is no interference term in the partial width and it can be written: $\Gamma = \Gamma_V + \Gamma_A$.

The G-parity of the $K\bar{K}$ system is $G_{K\bar{K}} = (-1)^n G$: $G_{K\bar{K}} = (-1)^n$ for a vector contribution and $G_{K\bar{K}} = (-1)^{(n+1)}$ for an axial one. The isospin $I_{K\bar{K}}$ of the $K\bar{K}$ system and its orbital momentum $l_{K\bar{K}}$ are related to the G-parity by: $G_{K\bar{K}}(-1)^{I_{K\bar{K}}}(-1)^{l_{K\bar{K}}} = 1$. For a given (V or A) current, the values of $l_{K\bar{K}}$ associated to $I_{K\bar{K}} = 0$ and $I_{K\bar{K}} = 1$ are different. As a result the interferences of $I_{K\bar{K}} = 0$ and $I_{K\bar{K}} = 1$ amplitudes vanish in the partial width¹. It can be written $\Gamma = \Gamma_V^0 + \Gamma_V^1 + \Gamma_A^0 + \Gamma_A^1$ where the upper index refers to the $K\bar{K}$ isospin. In the only relevant channel $\tau \rightarrow \nu_\tau K\bar{K}\pi$:

$$\begin{aligned}\Gamma_{K^0\bar{K}^0\pi^-} &= \Gamma_{K^+K^-\pi^-} = \frac{\Gamma_V^0 + \Gamma_A^0}{2} + \frac{\Gamma_V^1 + \Gamma_A^1}{4} \\ \Gamma_{K^0K^-\pi^0} &= \frac{\Gamma_V^1 + \Gamma_A^1}{2} \leq \Gamma_{K^0\bar{K}^0\pi^-} + \Gamma_{K^+K^-\pi^-}\end{aligned}\tag{1}$$

In the $K^0\bar{K}^0n\pi$ final state, odd values of $l_{K\bar{K}}$ correspond to K_1K_2 (K_SK_L) systems and even values to K_1K_1 (K_SK_S) and K_2K_2 (K_LK_L), the proportions of K_SK_S and K_LK_L being equal.

¹With second-class currents, interferences are possible. Their maximum value corresponds to the limit on the charge configuration fractions given in [1].

Figure 1: Charge configuration fractions for the $K\bar{K}\pi$ final state. The solid line is the isospin prediction. The experimental point is shown with the 1σ (39% probability) contour. The dotted ellipse is the limit [1] when second class currents are allowed. The $\star$ is the naïve expectation for K^* production.

For the $K\bar{K}\pi$ channel:

$$\frac{\Gamma_{K_S K_S \pi^-} + \Gamma_{K_L K_L \pi^-}}{\Gamma_{K_S K_L \pi^-}} = \frac{2\Gamma_A^0 + \Gamma_V^1}{2\Gamma_V^0 + \Gamma_A^1} \quad (2)$$

Comparing this ratio to the charge branching ratios may permit to put limits on V and A contributions. The extreme situations lead to very simple predictions: for a pure axial current we expect:

$$\frac{\Gamma_{K_S K_S \pi^-} + \Gamma_{K_L K_L \pi^-}}{\Gamma_{K_S K_L \pi^-}} = \frac{2\Gamma_A^0}{\Gamma_A^1} = \frac{2\Gamma^0}{\Gamma^1} = \frac{\Gamma_{K^0 \bar{K}^0 \pi^-} + \Gamma_{K^+ K^- \pi^-} - \Gamma_{K^0 K^- \pi^0}}{\Gamma_{K^0 K^- \pi^0}} \quad (3)$$

and for a pure vector current:

$$\frac{\Gamma_{K_S K_S \pi^-} + \Gamma_{K_L K_L \pi^-}}{\Gamma_{K_S K_L \pi^-}} = \frac{\Gamma_V^1}{2\Gamma_V^0} = \frac{\Gamma^1}{2\Gamma^0} = \frac{\Gamma_{K^0 K^- \pi^0}}{\Gamma_{K^0 \bar{K}^0 \pi^-} + \Gamma_{K^+ K^- \pi^-} - \Gamma_{K^0 K^- \pi^0}} \quad (4)$$

so dominance of the axial or vector current can be checked if the ratio $\Gamma^1/2\Gamma^0$ is not 1. For K^* production, the naïve (zero width K^*) prediction is $\Gamma_{K^0 \bar{K}^0 \pi^-} = \Gamma_{K^+ K^- \pi^-} = \Gamma_{K^0 K^- \pi^0}$ implying $\Gamma^1/2\Gamma^0 = 1$ and hence $(\Gamma_{K_S K_S \pi^-} + \Gamma_{K_L K_L \pi^-})/\Gamma_{K_S K_L \pi^-} = 1$. Nevertheless with other contributions than K^* and/or interference terms the ratio can be different and the present data seem to point to such situation.

Several measurements are available for the $B_{K^0 K^- \pi^0}$ [13,15] and $B_{K^+ K^- \pi^-}$ [10,18–20] branching ratios. Their averages are $B_{K^0 K^- \pi^0} = (0.115 \pm 0.041)\%$ and $B_{K^+ K^- \pi^-} =$

$(0.202 \pm 0.052)\%$ respectively. The $B_{K_S K_S \pi^-}$ branching ratio measured by the CLEO collaboration [13] is $B_{K_S K_S \pi^-} = (0.021 \pm 0.0057)\%$ and the $B_{K_S K_L \pi^-}$ branching ratio, measured by ALEPH [17] $B_{K_S K_L \pi^-} = (0.130 \pm 0.045)\%$. They yield a ratio:

$$(\Gamma_{K_S K_S \pi^-} + \Gamma_{K_L K_L \pi^-})/\Gamma_{K_S K_L \pi^-} = 2\Gamma_{K_S K_S \pi^-}/\Gamma_{K_S K_L \pi^-} = 0.32^{+0.21}_{-0.11}$$

and a branching fraction $B_{K^0 \bar{K}^0 \pi^-} = B_{K_S K_L \pi^-} + 2B_{K_S K_S \pi^-} = (0.172 \pm 0.046)\%$

A direct measurement of the branching ratio $B_{K^0 \bar{K}^0 \pi^-} = (0.310 \pm 0.126)\%$ has been performed by the L3 collaboration [16] leading to an average $B_{K^0 \bar{K}^0 \pi^-} = (0.188 \pm 0.046)\%$. The fractions of the three charge configurations shown in Fig. 1 are in good agreement with the isospin relation $B_{K^+ K^- \pi^-} = B_{K^0 \bar{K}^0 \pi^-}$ and not very far from the naïve prediction for K^* production. The most significant discrepancy is the value of the ratio $2\Gamma_{K_S K_S \pi^-}/\Gamma_{K_S K_L \pi^-}$ which, using all the data and constraints, is at three standard deviations from 1.

The ratio $R_{1/0} = \Gamma^1/2\Gamma^0 = \Gamma_{K^0 K^- \pi^0}/(\Gamma_{K^0 \bar{K}^0 \pi^-} + \Gamma_{K^- K^+ \pi^-} - \Gamma_{K^0 K^- \pi^0})$ is

$$R_{1/0} = 0.41^{+0.25}_{-0.19} \quad 1/R_{1/0} = 2.4^{+1.9}_{-0.9}$$

In spite of the modest precision, the comparison with the ratio $K_S K_S / K_L K_S$ can be considered as supporting the dominance of the vector current in agreement with the model of [25] but in disagreement with [26] and in a lesser extent [27]. To be more explicit, the data certainly allow a large class of models but all the models with only axial current are excluded while models with only vector current are not.

3 $\tau \rightarrow \nu_\tau + n\pi$

The general construction of isospin amplitudes for a $n\pi$ system taking into account Bose statistics has been given by Pais [28] a long time ago but was only recently used for τ physics [29]. Let us briefly recall the main results of Pais.

A basis of states can be found for a n pion system whose isospin functions belong to irreducible representations of the symmetric group $\mathcal{S}_n$ and are characterized by three row Young diagrams, i. e. 3 numbers $n_1 \geq n_2 \geq n_3 \geq 0$, $n_1 + n_2 + n_3 = n$. If the total isospin is $I \leq 1$, a given symmetry class corresponds either to $I = 0$ or $I = 1$ and all the states in a class (n_1, n_2, n_3) give the same charge configuration fractions $f_{n^+ n^- n^0} = \Gamma_{n^+ n^- n^0}/\Gamma_{n\pi}$. The symmetry properties of the momentum and isospin amplitudes are the same due to the Pauli principle. Thus integration over the momentum configurations kills the interferences between amplitudes belonging to different classes.

For a τ decay into $n\pi$, the total isospin is 1 and we can write for the $n\pi$ partial width: $\Gamma = \sum_{sc} \Gamma^{sc}$ where sc is the symmetry class. For each charge configuration cc we have: $\Gamma^{cc} = \sum_{sc} C_{sc}^{cc} \Gamma^{sc}$. The coefficients C_{sc}^{cc} which are the branching fractions in each class are tabulated in the appendix. Defining the weights $W^{sc} = \Gamma^{sc}/\Gamma$, the charge configuration fractions satisfy:

$$f_{cc} = \sum_{sc} C_{sc}^{cc} W^{sc} \\ W^{sc} \geq 0, \quad \sum_{sc} W^{sc} = 1 \quad (5)$$

For a given decay channel, the allowed values of the fractions f_{cc} are represented by points inside the convex polyhedron whose vertices correspond to all the W but one equal to zero. Let us stress that the allowed domain is smaller than the region defined by the inequalities $f_{cc}^{min} \leq f_{cc} \leq f_{cc}^{max}$.

Resonance production can be related to the classes using their symmetry properties; for instance ω production requires a three row Young diagram and ρ production is forbidden for one row diagrams. More generally a physical hypothesis that excludes a class of amplitudes reduces in a simple way the allowed region. Conversely experimental results at the boundary of the domain are of special interest since they can exclude whole classes of amplitudes.

In the case of a decay into $2n\pi$ the amplitudes are related by an isospin rotation to the amplitudes of e^+e^- annihilation and the same weights W^{sc} that describe the τ branching fractions allow the computation of the probabilities of the charge configurations in e^+e^- annihilation at an energy equal to the $2n\pi$ mass.

The properties of the momentum amplitudes are also constrained by their symmetry class but, except for three pion systems [30], few general properties are known².

The “barrier factors” L , i. e. the minimum powers of 3-momenta that appear in the wave-functions at the non relativistic limit are given in [31]. In the case of a τ decay into pions where the total angular momentum of the hadrons is 1 due to CVC or PCAC the “barrier factor” L is either 1 or 3. For high multiplicity decays the $L = 3$ amplitudes could be suppressed. However all the decay channels are dominated by the production of intermediate resonances and the interpretation of L as a barrier factor in this case is somewhat dubious. For instance ω production amount to 2 in the L factor.

3.1 $\tau \rightarrow \nu_\tau + (2n + 1)\pi$

There exist only two symmetry classes, (300) and (210), for an $I = 1$ three pion system and, with the above notations, $f_{\pi^-\pi^-\pi^+} - f_{\pi^-\pi^0\pi^0} = (3/5)W^{(300)}$. Amplitudes in the completely symmetric class (300) cannot be associated with resonance production and have four kinematical zeros including one at the center of the 3π Dalitz plot [30]. Hence they are very small compared to the (210) amplitudes enhanced by ρ production. This explains the observed ratio $B_{\pi^-\pi^0\pi^0}/B_{\pi^-\pi^-\pi^+} = 1$ [24,15].

For an $I = 1$ five pion system four symmetry classes are possible. Production of ω is allowed in the class (221) only and neither ρ nor ω resonance can be produced in the completely symmetric class (500) which is expected to be relatively suppressed.

The allowed region in the $f_{3\pi^\pm 2\pi^0}/f_{5\pi^\pm}$ plane is shown in Fig. 2. If the class (500) is neglected the triangle XYZ must also be excluded. Cabibbo forbidden $K^-4\pi$ final states are also suppressed by phase space. The isospin limits for them are represented by a dotted line in Fig. 2. It is clear that a small fraction of $K^-4\pi$ in the 5π final states does not affect the isospin tests.

²The increase in complexity for more than three pions is clearly shown in [32].

Figure 2: The decay fractions for $\tau \rightarrow \nu 5\pi$. The points corresponding to the symmetry classes are: X (500), Y (410), Z (320) and T (221). The experimental point is shown with the 1σ (39 % probability) contour.

Measurements of $B_{5\pi^\pm}$ [7-9,15], $B_{3\pi^\pm 2\pi^0}$ [5,15] and $B_{\pi^- 4\pi^0}$ [4,15] are available. Their averages are: $B_{5\pi^\pm} = (0.073 \pm 0.008)\%$, $B_{3\pi^\pm 2\pi^0} = (0.485 \pm 0.049)\%$ and $B_{\pi^- 4\pi^0} = (0.160 \pm 0.0570)\%$. Subtracting the contributions of η estimated from [3] we get the charge configuration fractions presented in Fig. 2.

Within the experimental errors the values are in agreement with the isospin requirements but it is interesting to remark that the data point lies outside the allowed domain while each of the three charge configuration fractions independently satisfies the isospin inequalities.

Since ω production is possible in the class (221) only, independent information can be gained from the ω fraction f_ω in the $3\pi^\pm 2\pi^0$ final state. It is represented graphically by scaling the allowed domain by a factor $(1 - f_\omega)$. The domain reduced by the measured ω fraction is displayed as a grey area in Fig. 2 (limits for plus or minus one standard deviation on f_ω are also shown) using the average of the observed ω fractions from [5] and [21].

The ω fraction is reasonably consistent with the branching ratios and the comparison shows that the class (221) is saturated by the ω production.

With the presently available data, the error ellipse is of the same size as the region allowed by the ω fraction, so the use of the complete information allows to refine the estimation of $B_{5\pi}$.

3.2 $\tau \rightarrow \nu_\tau + 2n\pi$

Since an even number of pions in a tau decay is produced by the vector current the partial decay widths of the τ for $2n\pi$ systems in the symmetry class (sc) are related

by an isospin rotation to the annihilation cross sections for $2n\pi$ production in the same symmetry class. Normalizing the decay widths to $\Gamma_{\tau \rightarrow \nu \bar{\nu} e}$ we can write [33,1]:

$$\frac{1}{\Gamma_{\tau \rightarrow \nu \bar{\nu} e}} \frac{d\Gamma_{\tau \rightarrow \nu 2n\pi}^{sc}}{dm^2} = \frac{3 \cos \theta_c^2}{2\pi \alpha^2 m_\tau^8} m^2 (m_\tau^2 - m^2)^2 (m_\tau^2 + 2m^2) \sigma_{e^+e^- \rightarrow 2n\pi}^{sc}(m^2) \quad (6)$$

Both branching ratios and mass spectra can in principle be computed from the data of e^+e^- annihilation [1,34,23].

For four pion systems, there are two symmetry classes and two measured e^+e^- reactions: $e^+e^- \rightarrow 2\pi^+2\pi^-$ and $e^+e^- \rightarrow \pi^+\pi^-2\pi^0$. The two branching ratios $B_{\pi^+2\pi^-\pi^0}$ and $B_{\pi^-3\pi^0}$ can be independently predicted [1] by the relation (6) using the $\sigma_{\pi^+\pi^-\pi^0} + \frac{1}{2}\sigma_{2\pi^+2\pi^-}$ and $\frac{1}{2}\sigma_{2\pi^+2\pi^-}$ combinations whose coefficients can be obtained from table 2.

The agreement between τ decays and e^+e^- annihilations is good [34,24] though the precision of the test is limited by the quality of the e^+e^- data.

Figure 3: The decay fractions for $\tau \rightarrow \nu 6\pi$ (a) and the similar fractions for $e^+e^- \rightarrow 6\pi$ (b). The points corresponding to the symmetry classes are: X (510), Y (411), Z (330) and T (321). On (a), the grey region corresponds to one standard deviation from the measured $B_{3\pi^+3\pi^0}/B_{5\pi^+\pi^0}$ ratio. The hatched region is the 95% C.L. region from the $B_{5\pi^+\pi^0}$ and $\sigma_{e^+e^- \rightarrow 6\pi^\pm} + \sigma_{e^+e^- \rightarrow 4\pi^\pm 2\pi^0}$ measurements. Limits of the allowed regions for $\sigma_{e^+e^- \rightarrow 6\pi^\pm}/\sigma_{e^+e^- \rightarrow 4\pi^\pm 2\pi^0}$ equal to 10% and 20% are also shown.

Four classes can contribute to six pion production. Production of ω is allowed in the (411) and (321) classes; $\omega\rho$ production in the (321) class.

A consequence of the existence of four classes is that even if the three different cross sections in e^+e^- annihilation were measured it would not be possible to predict individual branching ratios and mass distributions in τ decays.

On the other hand, with two measurements performed in e^+e^- annihilations [35–38] ($\sigma_{6\pi^\pm}$ and $\sigma_{4\pi^\pm 2\pi^0}$) and two measurements in τ decays [7,8,15] ($B_{5\pi^\pm\pi^0}$ and $B_{3\pi^\pm 3\pi^0}$) it is in principle possible to determine the contributions of the four classes. Unfortunately we will see that inconsistencies in the data prevent such analysis so that we can only draw rather qualitative conclusions.

Fig. 3 (a) displays the allowed region in the plane of charge configuration fractions for τ decays into 6π as does Fig. 3 (b) for e^+e^- annihilations. The ratio of the measurements of $B_{5\pi^\pm\pi^0}$ and $B_{3\pi^\pm 3\pi^0}$ seems to favour the (330) class but, due to the error on $B_{3\pi^\pm 3\pi^0}$, all the classes are allowed at less than two standard deviations. A more precise information comes from comparing the branching ratio $B_{5\pi^\pm\pi^0} = (0.021 \pm 0.005)\%$ (average of [7,8,15]) to the estimation of $B_{6\pi}$ from e^+e^- annihilation data.

Using $\sigma_{e^+e^- \rightarrow 6\pi^\pm} + \sigma_{e^+e^- \rightarrow 4\pi^\pm 2\pi^0}$ as an estimate of $\sigma_{e^+e^- \rightarrow (6\pi)^\pm}$, a value $B_{6\pi} = (0.134 \pm 0.023)\%$ is found [34]. This value is in fact a lower limit and the corresponding upper limit is 2.5 times greater as can be seen from Fig. 3 (b). From the above numbers a 95% C.L. interval $0.03 < f_{5\pi^\pm\pi^0} < 0.27$ is deduced which is in agreement with the $B_{3\pi^\pm 3\pi^0}/B_{5\pi^\pm\pi^0}$ ratio.

Troubles come from the fact that in e^+e^- annihilations the final state $4\pi^\pm 2\pi^0$ is dominant [35–38]. The final state $6\pi^\pm$ is only of the order of 10 to 20% of it. This corresponds (see Fig. 3 (b)) to the dominance of amplitudes in classes (411) and/or (321) and could be explained by ω production. As shown in Fig. 3 (a) a given $\sigma_{e^+e^- \rightarrow 6\pi^\pm}/\sigma_{e^+e^- \rightarrow 4\pi^\pm 2\pi^0}$ ratio reduces the allowed domain to a smaller triangular region and the observed ratio is in disagreement with the above limits on $f_{5\pi^\pm\pi^0}$. Quantitatively the inequality $f_{5\pi^\pm\pi^0} < 0.27$ implies $\sigma_{e^+e^- \rightarrow 6\pi^\pm}/\sigma_{e^+e^- \rightarrow 4\pi^\pm 2\pi^0} > 0.84$ in definite contradiction with data.

A very different variation with energy for $\sigma_{e^+e^- \rightarrow 6\pi^\pm}$ and $\sigma_{e^+e^- \rightarrow 4\pi^\pm 2\pi^0}$ could help but is not sufficient to solve the contradiction. Hence caution must be exercised against estimations of $B_{6\pi}$. A reasonable but not completely safe conclusion is that $B_{\pi^- 5\pi^0}$ and $\sigma_{e^+e^- \rightarrow \pi^+ \pi^- 4\pi^0}$ are negligible (i. e. dominant (321) and (330) classes) and that $B_{6\pi}$ lies somewhere between twice the observed $B_{5\pi^\pm\pi^0}$ and the CVC prediction [34]. With large samples of τ , observation of ω in the final state $5\pi^\pm\pi^0$ would be an interesting information on the problem.

4 $\tau \rightarrow \nu_\tau + K + n\pi$

For a decay into $Kn\pi$, the total isospin is 1/2 and the isospin of the $n\pi$ system 0 or 1. Therefore the amplitudes can be labelled by the symmetry class of the pion system which implies its isospin. The charge multiplicity coefficients can be deduced from the coefficients for $I = 0$ and $I = 1$ $n\pi$ systems; they are given in table 3.

The three branching ratios $B_{K^-\pi^+\pi^-}$, $B_{K^-\pi^0\pi^0}$ and $B_{K^0\pi^-\pi^0}$ have been measured [10–16,18]. The averages of the results are presented in Fig. 4. There are only two classes of amplitudes for a $K\pi\pi$ decay corresponding respectively to isospin 1 and 0 for the two

Figure 4: The decay fractions for $\tau \rightarrow \nu K \pi \pi$. The experimental point is shown with the 1σ (39% probability) contour. The solid line is the isospin constraint.

pion system. This implies the relation:

$$B_{K^-\pi^+\pi^-} = \frac{1}{2}B_{K^0\pi^-\pi^0} + 2B_{K^-\pi^0\pi^0} \quad (7)$$

The measurements shown in Fig. 4 are in good agreement with the constraint and also with the $K^*\pi$ dominance of the final state $K^-\pi^+\pi^-$ observed in [10] and used by some experiments [19,22] to directly measure $K^*\pi$ branching ratios.

5 Summary

We have reviewed all the τ hadronic decay modes with three possible charge configurations. The complete isospin constraints and their relation to resonance production have been presented.

It is shown that, in the $K\bar{K}\pi$ channel, the present data, though statistically limited, point to a situation where model independent information on the dynamics, favouring vector current dominance, can be extracted from the branching ratios.

In the 6π channel, discrepancies between τ decays and e^+e^- annihilations appear when the complete information is used.

A Appendix: Charge configuration coefficients

The symmetry class (sc) of an $I \leq 1$ n pion system is characterized by the triplet (n_1, n_2, n_3) , ($n_1 \geq n_2 \geq n_3 \geq 0$, $n_1 + n_2 + n_3 = n$).

If $n_1 - n_2$ and $n_1 - n_3$ are both even the isospin of the system is 0, otherwise it is 1.

The charge configuration (cc) coefficients [28] given below are normalized by:

$$\sum_{cc} C_{sc}^{cc} = 1 \quad (8)$$

The “barrier factor” L of [31] is given in the last column of the table.

Class	$3\pi^\pm$	$\pi^\pm 2\pi^0$	L
(300)	4/5	1/5	3
(210)	1/2	1/2	1

Class	$5\pi^\pm$	$3\pi^\pm 2\pi^0$	$\pi^\pm 4\pi^0$	L
(500)	24/35	8/35	3/35	3
(410)	2/5	3/10	3/10	1
(320)	2/5	3/5	0	3
(221)	0	1	0	3

Table 1: Charge configuration coefficients for multipion final states ($G = -1$).

Class	τ		$e^+ e^-$		L
	$3\pi^\pm \pi^0$	$\pi^\pm 3\pi^0$	$4\pi^\pm$	$2\pi^\pm 2\pi^0$	
(310)	3/5	2/5	4/5	1/5	1
(211)	1	0	0	1	3

Class	τ			$e^+ e^-$			L
	$5\pi^\pm \pi^0$	$3\pi^\pm 3\pi^0$	$\pi^\pm 5\pi^0$	$6\pi^\pm$	$4\pi^\pm 2\pi^0$	$2\pi^\pm 4\pi^0$	
(510)	16/35	2/7	9/35	24/35	8/35	3/35	1
(411)	4/5	1/5	0	0	2/5	3/5	3
(330)	1/5	4/5	0	3/5	2/5	0	3
(321)	1/2	1/2	0	0	1	0	3

Table 2: Charge configuration coefficients for multipion final states ($G = 1$) in τ decays and $e^+ e^-$ annihilation.

Class	$K^- 2\pi^\pm$	$K^- 2\pi^0$	$\bar{K}^0 \pi^\pm \pi^0$
$I = 1$	1/3	0	2/3
$I = 0$	2/3	1/3	0

Class	$K^- 2\pi^\pm \pi^0$	$K^- 3\pi^0$	$\bar{K}^0 3\pi^\pm$	$\bar{K}^0 \pi^\pm 2\pi^0$
$I = 1 (300)$	2/15	1/5	8/15	2/15
$I = 1 (210)$	1/3	0	1/3	1/3
$I = 0 (111)$	1	0	0	0

Class	$K^- 4\pi^\pm$	$K^- 2\pi^\pm 2\pi^0$	$K^- 4\pi^0$	$\bar{K}^0 3\pi^\pm \pi^0$	$\bar{K}^0 \pi^\pm 3\pi^0$
$I = 1 (310)$	4/15	1/15	0	2/5	4/15
$I = 1 (211)$	0	1/3	0	2/3	0
$I = 0 (400)$	8/15	4/15	1/5	0	0
$I = 0 (220)$	1/3	2/3	0	0	0

Table 3: Charge configuration coefficients for strangeness 1 final states. Symmetry class and isospin refer to the pion system.

References

- [1] F.J. Gilman and S.H. Rhie Phys. Rev. D **31**, 1066 (1985).
- [2] Proceedings of the Third Workshop on Tau Lepton Physics, Montreux, Switzerland 19-22 September 1994, L. Rolandi Ed., Nucl. Phys. B (Proc. Supp.) **40** (1995).
- [3] CLEO Collab., M. Artuso et al., Phys. Rev. Lett. **69** (1992) 3278.
- [4] CLEO Collab., M. Procaro et al., Phys. Rev. Lett. **70** (1993) 1207.
- [5] CLEO Collab., D. Bortoletto et al., Phys. Rev. Lett. **71** (1993) 1791.
- [6] CLEO Collab., M. Artuso et al., Phys. Rev. Lett. **72** (1994) 3762.
- [7] CLEO Collab., D. Gibaut et al., Phys. Rev. Lett. **73** (1994) 934.
- [8] HRS Collab., B.G. Bylsma et al., Phys. Rev. D **35** (1987) 2269.
- [9] ARGUS Collab., H. Albrecht et al., Phys. Lett. B **202** (1988) 149.
- [10] TPC/Two-Gamma Collab., D.A. Bauer et al., Phys. Rev. D **50** (1994) R13.
- [11] ALEPH Collab., D. Buskulic et al., Phys. Lett. B **332** (1994) 209.
- [12] ALEPH Collab., D. Buskulic et al., Phys. Lett. B **332** (1994) 219.
- [13] CLEO Collab., J.G. Smith, Studies of tau decay modes with K^0 mesons from CLEO, in [2] p. 351.
- [14] CLEO Collab., M. Battle et al., Phys. Rev. Lett. **73** (1994) 1079.

- [15] ALEPH Collab., Tau hadronic branching ratios Contribution to the EPS-HEP Conference, Brussels, Belgium, 27 Jul.-2 Aug. 1995.
- [16] L3 Collab., M. Acciari et al. Phys. Lett. B 352 (1995) 487.
- [17] ALEPH Collab. Tau lepton decays into neutral kaons. Contribution to the EPS-HEP Conference, Brussels, Belgium, 27 Jul.-2 Aug. 1995.
- [18] DELCO Collab. G. B. Mills et al., Phys. Rev. Lett. 54 (1985) 624.
- [19] BES Collab., N. Qi, Tau Lepton Experiment at BES Detector, in [2] p. 387.
- [20] DELPHI Collab. A measurement of τ decay into two charged kaons. DELPHI 95-66 Contribution to the EPS-HEP Conference, Brussels, Belgium, 27 Jul.-2 Aug. 1995.
- [21] ALEPH Collab., P. Bourdon, Branching ratio and resonances of $\tau \rightarrow 3\pi\pi^0$ in ALEPH, in [2] p. 203.
- [22] ARGUS Collaboration, H. Albrecht et al., Tau Decays into K^* Mesons, DESY 95-087.
- [23] S. Narison and A. Pich, Phys. Lett. B 304 (1993) 359.
- [24] B.K. Heltsley, Hadronic decay modes of the tau lepton in [2] p. 413.
- [25] J.J. Gomez-Cadenas, M.C. Gonzalez-Garcia and A. Pich, Phys. Rev. D 42 (1990) 3093.
- [26] R. Decker, E. Mirkes, R. Sauer and Z. Wąs, Z. Phys. C 58 (1993) 445.
- [27] M. Finkemeier and E. Mirkes, LNF-95/05(P) MAD/PH/882 (1995).
- [28] A. Pais, Ann. Phys. 9 (1960) 548; Ann. Phys. 22 (1963) 274.
- [29] A. Donnachie and A.B. Clegg, Phys. Rev. D 51 (1995) 4979.
- [30] C. Zemach, Phys. Rev. B 133 (1964) 1201.
- [31] E.M. Henley and B.A. Jacobsohn, Phys. Rev. 128 (1962) 1394.
- [32] Č. Zupančič, Ann. Phys. 235 (1994) 245.
- [33] Y.S. Tsai, Phys. Rev. D 4 (1971) 2821.
H.B. Thacker and J.J. Sakurai, Phys. Lett. B 36 (1971) 103.
- [34] S.I. Eidelman and V.N. Ivanchenko, Phys. Lett. B 257 (1991) 437.
S.I. Eidelman and V.N. Ivanchenko, Tau decays and CVC in [2] p. 131.
- [35] G. Cosme et al., Nucl. Phys. B 152 (1979) 215.
- [36] C. Bacci et al., Nucl. Phys. B 184 (1981) 31.
- [37] D. Bisello et al., Phys. Lett. B 107 (1981) 145.
- [38] M. Schioppa, Thesis Roma 1986.