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Are Giant Resonances Harmonic Vibrations?

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Are Giant Resonances Harmonic Vibrations?

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We investigate the non-linear response of a quantum anharmonic oscillator as a model for the excitation of giant resonances in heavy ion collisions. We show that the introduction of small anharmonicities and non-linearities can double the predicted cross section for the excitation of the two-phonon states.

1. Introduction

Giant resonances, known since more than 50 years [1], are understood as the first quantum of collective vibrations. However, the first experimental indication of a possible double-excitation of giant resonances (i.e. of the second vibration quantum) only dates back to 1977 when some bumps have been observed in the cross-section of heavy ion inelastic scattering. Further evidence has been accumulated in heavy ion inelastic scattering at intermediate and relativistic energies and also in double charge exchange reactions [2,3].

Most theoretical calculations aimed at the interpretation of such processes involve a harmonic picture for the collective vibrations and assume a linear excitation process. This implies that the only elementary process considered is the excitation or deexcitation of one vibration quantum. A common feature of such calculations is that the resulting cross sections of the two phonon states appear 2 to 4 times smaller than the experimental ones[3]. From the observed discrepancy between experimental and theoretical cross-section of double excitations, it was concluded that either the experimental results over-estimate the real cross-section (see contributions on this subject) or the phonon picture of giant resonances might be inadequate [4].

From the theoretical point of view we would like to propose that the observed discrepancy might partly be a consequence of the anharmonic nature of the considered vibrations and of the non-linear components of the external field.

It has been argued that in a 3D calculation the presence of spin and parity quantum numbers may strongly reduce the possibility of introducing couplings between multiphonon states, but one has to take into account the large number of quasi-degenerate collective states with different quantum numbers present in the nucleus from the monopole (GMR), the dipole (GDR), the quadrupole (GQR) to the hexadecapole vibrations.

Many estimations of the anharmonicities of giant resonances can be found in the literature [3]. They all consistently yield to the same order of magnitude: the interaction between one and two phonons or among two-phonon states is about 0.5-1 MeV. Experimentally the observations indicate a small anharmonicity in the multiphonon energies at maximum of the order of 10%.

Another important ingredient to be considered is that the external field is in principle non-linear. In particular, in an already excited nucleus an external one-body field may induce transitions between particle (or hole) states and therefore induce direct transitions between two giant resonances.

2. A Simple Schematic Model

In order to investigate the consequences of the anharmonic terms in the hamiltonian of the target nucleus and of the non-linear terms in the external field describing the action of the projectile, we will study an over-simplified model.

Let us then consider a single harmonic oscillator of frequency ω and of mass m ($\hbar = 1$) and let us add anharmonic terms so that the total hamiltonian reads

$$\hat{H}_T = \frac{1}{2m}\hat{P}^2 + \frac{1}{2}m\omega^2\hat{X}^2 + \frac{1}{3}\alpha\hat{X}^3 + \frac{1}{4}\beta\hat{X}^4 \quad (1)$$

Since we want to mimic the excitation of the GDR, the mass is the reduced mass of neutrons and protons in the nucleus and the frequency ω is such to reproduce the GDR energy. The sign and strength of the anharmonic terms can be fixed by imposing that their matrix elements in the one and two phonon space have values of the order found in microscopic calculations (see ref.[5] for more details).

Diagonalizing the internal hamiltonian $\hat{H}_T$ in the basis $|n\rangle$ of the harmonic oscillator, we can obtain the excitation energies E_α and their corresponding eigenstates $|\phi_\alpha\rangle$. It should be noticed that the introduction of anharmonicities modifies the excitation energy E_1 of the first phonon state. In order to avoid the trivial consequences of this modification which would have nothing to do with anharmonic effects, the energy E_1 has been kept equal to that of the considered GR by a suitable overall renormalization of the hamiltonian.

In a relativistic reaction between heavy ions the strong transverse electric field $E_\perp(t)$ dominates. Therefore, the excitation of the transverse GDR degree of freedom in a nucleus of mass A_T , charge Z_T and neutron number N_T , in the linear response approximation, can be simulated by the linear external field

$$\hat{W}(t) = \frac{Z_T N_T e}{A_T} E_\perp(t) \hat{X} = \frac{Z_T N_T e}{A_T} E_\perp(t) \sqrt{\frac{\hbar}{2m\omega}} (O^\dagger + O) = w_T E_\perp(t) (O^\dagger + O) \quad (2)$$

which corresponds to the excitation of the GDR with 100 % of the energy weighted sum rule. The next step is the inclusion of non-linear terms in this external field. In an RPA based calculation of the various matrix elements of the external field we have seen that the dipole matrix elements between one phonon states (the GDR and the GQR) is about 1/6 of the matrix transition from the ground state. Therefore we have added to the external field a term in $O^\dagger O$ with such strength.

We denote by $|\Psi(t) \rangle$ the solution of the time dependent Schroedinger equation in presence of the external field. The probability to excite an internal state of the target will then be equal to $P_\alpha = | \langle \phi_\alpha | \Psi(t = \infty) \rangle |^2 \equiv |A_\alpha|^2$ and the cross sections σ_α can be calculated by integrating this probability over the appropriate range of impact parameters, starting at $b_0 = r_0(A_P^{1/3} + A_T^{1/3})$.

Table 1

Cross-section of the Coulomb excitation of one- and two- phonon states, σ_1 and σ_2 respectively, calculated for a parameter $r_0 = 1.5$ fm and $r_0 = 1.2$ fm (in parenthesis). The first row is the reference calculation of a linear harmonic oscillator. In the second row we present the results of the calculations including anharmonicities while the last row combines the effects of both non-linearities and anharmonicities.

	$\sigma_1(\text{mb})$	$\sigma_2(\text{mb})$	σ_2/σ_1	σ_1/σ_1^{ref}	σ_2/σ_2^{ref}
lin. and har.	1153(1698)	33 (93)	0.029	1.	1.
lin. and anhar.	1267 (1835)	55 (139)	0.044	1.1	1.67
non-lin and anhar	1352 (1938)	67 (166)	0.05	1.17	2.04

We have applied our model to study the excitation of the projectile in the reaction $^{136}\text{Xe} + ^{208}\text{Pb}$ at $E/A = 700$ MeV. Therefore we will take $E_1 = 15.2$ MeV, which is the energy of the GDR in ^{136}Xe . When the anharmonic terms are considered we obtain $E_2 = 28.3$ MeV, compatible with the observed position of the double GDR peak [2,3].

We have performed a reference calculation which is that of a harmonic oscillator ($\alpha = 0, \beta = 0$) excited by a linear field. In this case the results, shown in the first line of table 1, are very similar to the ones obtained within first and second order perturbation theory and closely correspond to the prediction of the sophisticated perturbative calculation of ref [4].

A second calculation corresponds to the response of an anharmonic oscillator ($\alpha = -38.6\text{MeV/fm}^3, \beta = -188.18\text{MeV/fm}^4$), still assuming the external field as linear (see table 1). We see that the cross section of the two-phonon state has been increased by 1.7 while the one of the first phonon state remains constant. The origin of this variation is two-fold: i) the energies are changed by the anharmonic terms in the hamiltonian; ii) the wave functions are modified due to the mixing between multiphonon states. In order to study the respective contributions of these two effects we have performed two calculations, one keeping the energies of the harmonic oscillator but changing the wave functions and the other changing the energies and keeping the wave functions of the harmonic oscillator. The latter leads to a value of σ_2 equal to 48 mb while the former gives 40 mb. Therefore both these effects equally contribute to the increase of the two-phonon cross section.

The next step is the inclusion, in the external field, of the non-linear terms in the creation and annihilation operators of phonons. Within the present model, with only one mode of excitation, the action of the term in $O^\dagger O$ is essentially to dynamically lower the energy of the multiphonon components of the states $|\phi_\alpha \rangle$ during the collision time. Therefore one may expect an increase of the cross section. On table 1 (third row) we observe a strong increase of the two-phonon cross section and of the ratio σ_2/σ_1 which are

now respectively a factor 2 larger than that in the harmonic limit. With such an increase the disagreement between experiment and theory starts to be strongly reduced. However, in order to compare the theoretical results with the experimental data, one should include more internal degrees of freedom (GDR, GQR, low-lying collective states, etc.).

It should be noticed that the calculation reported in [4] does not predict an increase of the two-phonon cross section. In that paper, the mixing between one- and two-phonon states has been considered in order to describe the width of the GDR. However, in the calculation of the cross section, some important effects have been neglected. For example, it is clear that the $J = 2$ two-GDR state is mixed to the ISGQR and IVGQR. Therefore, through the latter components, one can get a one-phonon $L = 2$ contribution to the cross section for the double excitation of the GDR. Also, the one-GDR state is mixed to the $J = 1$ GDR*ISGQR state and therefore one can get some $L = 1$ one-phonon contribution to the cross section at an energy very close to that of the two-GDR state. In addition to the above ones, other effects are neglected in [4]: i) direct two-phonon interaction (which would for example introduce a splitting of the various spin components of a two phonon multiplet); ii) the coupling with many-phonon states. At this point, a complete calculation including all the mentioned effects is called for and is now under investigation.

3. Discussion of the results

In order to understand at a qualitative level the changes in the excitation probabilities due to anharmonicities, let us calculate P_1 and P_2 within perturbation theory. We have, at the first order of perturbation theory,

$$P_1 = \left| \int_{-\infty}^{+\infty} dt \langle \phi_1 | W(t) | \phi_0 \rangle e^{iE_1 t} \right|^2 = \left| \langle \phi_1 | O^\dagger + O | \phi_0 \rangle \right|^2 w_T^2 \hat{g}_1(E_1) \quad (3)$$

where we have introduced the quantity

$$\hat{g}_1(E) = \int_{-\infty}^{+\infty} dt \exp(-iE_1 t) g_1(t) \quad (4)$$

in which

$$g_1(t) = \int_{-\infty}^{+\infty} d\tau E_\perp(t + \tau) E_\perp(\tau) \quad (5)$$

is the first-order correlation function of the electric field. In fact, in this correlation function we should introduce the expectation value $\langle E_\perp(t + \tau) E_\perp(\tau) \rangle$ to consider a possible quantum or statistical average on the electromagnetic field which is not taken into account in present calculations. The semiclassical approach usually used for the Coulomb excitation of the two-phonon states does not contain either possible fluctuations in the excitation function nor quantum effects which could result from a quantum description of the interaction.

At the second order of perturbation theory, A_2 contains the sum over all possible intermediate states. However, since most of the energies of the intermediate states are high and they are weakly coupled to the ground state, except the first phonon state, the latter will provide the main contribution to this sum. Therefore we can write

$$P_2 \approx \frac{1}{4} \left| \langle \phi_2 | (O^\dagger + O) | \phi_1 \rangle \langle \phi_1 | O^\dagger + O | \phi_0 \rangle \right|^2 w_T^4 \hat{g}_2(E_2 - E_1, E_1) \quad (6)$$

where we have introduced the quantity

$$\tilde{g}_2(E, E') = \int_{-\infty}^{+\infty} dt \exp(-iEt) \int_{-\infty}^{\tau+t} dt' \exp(-iE't') g_2(t, t') \quad (7)$$

in which

$$g_2(t, t') = \frac{1}{2} \int_{-\infty}^{+\infty} d\tau \int_{-\infty}^{+\infty} d\tau' T(E_{\perp}(t+\tau) E_{\perp}(t'+\tau') E_{\perp}(\tau) E_{\perp}(\tau')) \quad (8)$$

is the second-order correlation function of the electric field. T stands for the time ordering operator. Again, we should introduce in the second-order correlation function the expectation value $\langle E_{\perp}(t+\tau) E_{\perp}(t'+\tau') E_{\perp}(\tau) E_{\perp}(\tau') \rangle$ to consider a possible quantum or statistical average on the electromagnetic field.

If we now consider the ratio P_2/P_1^2 , this reads at the leading order of the perturbation theory

$$\frac{P_2}{P_1^2} \simeq \frac{1}{4} \frac{|\langle \phi_2 | (O^{\dagger} + O) | \phi_1 \rangle|^2 \tilde{g}_2(E_2 - E_1, E_1)}{|\langle \phi_1 | (O^{\dagger} + O) | \phi_0 \rangle|^2 \tilde{g}_1(E_1)^2} \quad (9)$$

In the harmonic case this ratio is equal to $1/2$, the matrix element appearing in equation (9) being directly proportional. It should be noticed that this is true only if $g_2(E', E) = g_1(E)^2$ which is the case in the simplest classical description of the Coulomb excitation. However, the above definitions of the correlation functions may also take into account non trivial effects in the electromagnetic field properties as discussed in non linear optics theories. Indeed, it is well-known that depending on the properties of the electromagnetic radiation source, such as its coherence or its chaoticity, one may observe additional increase of the $g_2(E', E)/g_1(E)^2$ ratio. This effects are also now under investigation but they will not be considered in the presented results.

When anharmonicities are taken into account, the ratio (9) will depart from its standard value $1/2$ due to the two factors: the shift of the eigenenergies and the change in the eigenfunctions.

Considering the effects due to the modification of the wave functions one may worry about the origin of the extra strength. In fact, it originates from the fact that the wave function of an α -phonon state is now a sum over all the harmonic oscillator n -quantum states $|\phi_{\alpha}\rangle = \sum_n C_{\alpha}^n |n\rangle$ and so $\langle \phi_{\alpha'} | O^{\dagger} | \phi_{\alpha} \rangle = \sum_n C_{\alpha'}^{n+1*} C_{\alpha}^n \sqrt{n+1}$. Therefore the extra strength comes from the increase of the matrix element of $O^{\dagger}$ with n .

This increase of the matrix elements can also be derived using arguments about sum rules. In fact, given an excitation operator A , a sum rule exists for every quantum state

$$\sum_{\alpha'} |\langle \phi_{\alpha'} | A | \phi_{\alpha} \rangle|^2 (E_{\alpha'} - E_{\alpha}) = \frac{1}{2} \langle \phi_{\alpha} | [A, [H, A]] | \phi_{\alpha} \rangle \equiv S_{\alpha}^A \quad (10)$$

Therefore we can first derive a sum rule for the transitions induced by the linear part of the external operator. Since $[X, [H, X]] = \hbar^2/m$, we simply get $S_{\alpha}^X = \hbar^2/2m$. If we now consider the transitions from the ground state, since the sum rule is dominated by the transition from the ground state to the first excited state we get $|\langle \phi_1 | X | \phi_0 \rangle|^2 \approx \hbar^2/2mE_1$; whereas if we consider the transitions from the first excited state we get $S_1^X = \hbar^2/2m = \sum_{\alpha} |\langle \phi_{\alpha} | X | \phi_1 \rangle|^2 (E_{\alpha} - E_1)$. Considering again that mostly two terms

contribute to the latter sum – the transition from the first excited state back to the ground state and the transition to the second excited state – we easily get using the previous equation $|\langle \phi_2 | X | \phi_1 \rangle|^2 \approx \hbar^2/m(E_2 - E_1)$. The ratio of matrix elements reads

$$\frac{|\langle \phi_2 | (O^\dagger + O) | \phi_1 \rangle|^2}{|\langle \phi_1 | (O^\dagger + O) | \phi_0 \rangle|^2} \simeq 2 \frac{E_1}{E_2 - E_1} \quad (11)$$

which is greater than 2 when anharmonicities are present.

Let us now estimate the ratio $\tilde{g}_2(E_2 - E_1, E_1)/\tilde{g}_1(E_1)^2$ of relation (9). In order to get a qualitative discussion we may neglect the principal value integral hidden in $\tilde{g}_2$ and replace $\tilde{g}_2(E', E)$ by $\tilde{g}_1(E) \cdot \tilde{g}_1(E')$. Then we can analytically compute the function $\tilde{g}_1$ in terms of the modified Bessel function of order 1 and we get for large impact parameters b

$$\frac{\tilde{g}_2(E_2 - E_1, E_1)}{\tilde{g}_1(E_1)^2} \longrightarrow \frac{E_2 - E_1}{E_1} e^{2(2E_1 - E_2)\tau} \quad (12)$$

where $\tau \simeq b/\gamma v$ is the associated reaction time.

Considering now the various relations derived before, we can write the ratio P_2/P_1^2 in the large impact parameter limit

$$\frac{P_2}{P_1^2} \longrightarrow \frac{1}{2} e^{2(2E_1 - E_2)\tau} \quad (13)$$

Since $(2E_1 - E_2) > 0$ this corresponds to a strong increase of the two phonon probability over the predicted value of 1/2 in the harmonic case. This increase is stronger at larger impact parameter or at lower incident energies. We expect a departure of this exponential behaviour in the small impact parameter region, where the excitation probabilities are higher and perturbative calculations deviate from the coupled-channel results.

4. Conclusion

We have explored the possibility that the large discrepancy between the theoretical results and experimental data on the two-phonon excitation cross section can be reduced when anharmonicities in the internal hamiltonian and non-linearities in the external field are taken into account. In order to have a first insight on the problem, we have considered the simplified model of an anharmonic oscillator. The anharmonicities were introduced in such a way that the main properties of the first excited state were kept unchanged while those of the second one were slightly modified ($\sim 10\%$ variation on its energy). From the presented results we can conclude that the inclusion of small anharmonicities and non-linearities may strongly enhance the excitation cross section of the two-phonon states by a factor up to 2 without modifying much the population of the one-phonon state.

REFERENCES

1. see the contribution of A. Van der Woude
2. H. Emling, Progr. Part. Nucl. Phys. **33** (1994) 729
3. For a review, see Ph. Chomaz and N. Frascaria; Phys. Rep. 252 (1995) 275.
4. V.Yu.Ponomarev et al., Phys. Rev. Lett. **72** (1994) 1168; see the contribution of V. Yu. Ponomarev et al.
5. C. Volpe et al, Nucl. Phys. **A589** (1995) 521.