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Radiative Cooling of Ion Beams in Storage Rings By Broad Band Lasers

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A scheme for cooling non-fully stripped ions in storage rings is described which is based on the spontaneous emission of bound electrons interacting resonantly with a counter propagating laser beam. The laser frequency is near one of the transition frequencies of the ion, with a sufficiently broad bandwidth to cover the full Doppler spectrum of the ions. Except for the resonance, the cooling discussed here is similar to the well-known radiative cooling in electron storage rings. In both cases, the cooling takes place in all three dimensions. However, the present scheme is distinct from the usual laser cooling which employs narrow band lasers to resolve the line shape of individual ions. With a broad band laser, cooling occurs only as a combined effect of the radiation reaction due to spontaneous emission and the acceleration in the cavities; in particular, the radiation reaction alone leads to anti-damping for non-relativistic ions. We set up the rate equation for the laser-ion interaction for a moving ion, and derive the spontaneous emission rate. Formulae for the cooling time and the equilibrium beam dimensions in the relativistic limit are obtained following the similar procedure for electron storage rings. Explicit examples for relativistic ion storage rings under construction show that, with currently achievable laser parameters, the scheme provides an efficient cooling of the ion beams, while serving at the same time as a source of intense gamma rays.

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It is well-known that the relativistic electron beams are cooled in a storage ring as a combined effect of the synchrotron radiation loss and the energy gain in accelerating cavities [1]. For protons or fully stripped ions, the synchrotron radiation, and hence the cooling, is negligible due to their heavy mass. However, non-fully stripped ions can be made to radiate copiously in a counter-propagating laser beam when the laser frequency is near the transition frequency of the ion [2]. Therefore, by choosing the laser bandwidth to cover the full Doppler broadening of all ions, cooling can be achieved in all three directions. In this Letter, we study this process, which will be referred to as the radiative ion cooling, and present some explicit examples.

The radiative ion cooling in the extreme relativistic case is similar to the cooling in electron storage rings, and can be understood from the fact that faster particles radiate more energy. It is somewhat subtle in non-relativistic cases, where it is found that faster ions lose less energy. The analysis of cooling in this case reveals the essential role played by the accelerating cavities.

The radiative ion cooling is distinct from the well-known laser cooling [3], which is based on the damping force arising from the line shape of individual ion. The laser bandwidth must be, therefore, narrower than the linewidth. The laser cooling is highly efficient in the longitudinal direction, but difficult in the transverse directions unless a special coupling mechanism is introduced [4].

To explain the basic principle of the radiative ion cooling, consider the process of absorption of a laser photon by a moving ion and the subsequent spontaneous emission. The laser photon travels in the negative z -direction with the four momenta $k = \omega(1, -\mathbf{e}_z)$ and the ion in the near z -direction with the four momenta $p = M\gamma(1, \boldsymbol{\beta})$, where ω is the laser frequency, $\mathbf{e}_z$ the unit vector in the z -direction, M the ion mass, $\boldsymbol{\beta}$ the velocity vector of the ion, and $\gamma = 1/\sqrt{1 - \beta^2}$. We will for the moment use the units where $\hbar = c = 1$. Let p^* be the four momentum of the excited ion after absorbing a photon; $p^* = k + p$. Denoting the mass of the excited ion by M^* , one obtains the resonance condition

$$\omega = \frac{\omega^*}{\gamma(1 + \beta_z)}, \quad (1)$$

where $\omega^* = (M^{*2} - M^2)/2M$ is the resonant transition energy in the rest frame of the initial ion, and $\beta_z = \boldsymbol{\beta} \cdot \mathbf{e}_z = \beta \cos \psi$, ψ being the angle between $\boldsymbol{\beta}$ and the z -direction. The right hand side of Eq.(1) is the Doppler shifted resonance frequency. The excited ion decays spontaneously to the ground state by emitting a photon with four momentum k^s . Using the fact that the angular distribution of the spontaneous emission is symmetric in the rest frame of p^* , the average change in three momentum of the ion in the laboratory frame can be calculated with the following result:

$$(\Delta \mathbf{p})_L = -\frac{\omega^*}{M^{*2}} \left(M \mathbf{p} + \frac{M^2 + M^{*2}}{2\gamma(1 + \beta_z)} \mathbf{e}_z \right). \quad (2)$$

Here, the subscript L implies that it is the laser induced momentum transfer.

It follows from Eq.(2) that the transverse momentum will be damped to zero. To see that the longitudinal momentum p_z is also damped, first consider the relativistic case $\gamma \gg 1$ and thus $\beta \approx 1$. The second term in Eq.(2) is then negligible, and $(\Delta p_z)_L \approx -(\omega^* M/M^{*2})p_z$. Now consider the effect of the accelerating cavity. On the average, the momentum transfer per photon absorption due to the accelerating cavity can be written as $(\Delta p_z)_A = qV/\mathcal{N} \equiv (\omega^* M/M^{*2})\bar{p}$. Here q is the ion charge, V the accelerating voltage, $\mathcal{N}$ the average number of the photon absorption events during the interval between the cavity crossing, and $\bar{p}$ a constant defined by the above equation. The total longitudinal momentum transfer obtained by adding $(\Delta p_z)_L$ and $(\Delta p_z)_A$ is $\Delta p_z = -(\omega^* M/M^{*2})(p_z - \bar{p})$. This shows clearly that the longitudinal momentum is damped toward the value $\bar{p}$.

The circumstance is somewhat different in the non-relativistic case, $\beta_z \ll 1$. In this case, Eq.(2) becomes, to the first order in β_z ,

$$(\Delta p_z)_L = -\frac{\omega^*}{2M^{*2}}(M^2 + M^{*2}) + \frac{\omega^{*2}}{2M^{*2}}p_z. \quad (3)$$

The first term is a constant which will be cancelled by a corresponding term in acceleration. However, note that the second term is proportional to p_z with a positive coefficient. Thus the radiative process in the non-relativistic case appears to be anti-damping!

The paradox is resolved by the velocity dependence of the acceleration in the non-relativistic case. Indeed, the momentum kick in an accelerating cavity in this case is given by

$$(\Delta p_z)_A = \frac{1}{\mathcal{N}}(\sqrt{p_z^2 + 2qMV} - p_z) \simeq \frac{qMV}{\bar{p}\mathcal{N}}(1 - \frac{p_z - \bar{p}}{\bar{p}}). \quad (4)$$

Note that this expression contains a velocity dependent term with a negative coefficient. This is due to the fact that a faster ion gains less momentum as it spends less time in the cavity. Furthermore, the magnitude of the velocity dependent term relative to the constant term is larger in Eq.(4) than in Eq.(3). Here, and in the following, we assume $\omega^* \ll M$, which is surely true for all cases of interest. Therefore, the total momentum transfer obtained by adding the two contributions and adjusting the voltage V to cancel the constant terms becomes proportional to $(p_z - \bar{p})$ with a negative coefficient, implying that the damping occurs also in the non-relativistic case. This is as it should be since otherwise we would have found an absurd result that the damping phenomena could depend on the reference frame.

We will now examine the relativistic case more closely, to determine the damping rate and the equilibrium beam parameters. Thus we consider a cooling configuration in which a laser beam is colliding head-on with the ion beam in a straight section of a ion storage ring. Both the laser beam and the ion beam are focussed to a waist at the center of the interaction region.

When the ion beam has an angular spread $\Delta\psi$ around $\psi = 0$, and energy spread $\Delta\gamma$ around an average value $\bar{\gamma}$, the full bandwidth required for the incoming laser to interact with all ions is, in view of Eq.(1),

$$\Delta\omega/\omega_L = (\Delta\psi)^2/4 + \Delta\gamma/\bar{\gamma}. \quad (5)$$

Here $\omega_L = \omega^*/\bar{\gamma}(1 + \beta_z) \approx \omega^*/2\bar{\gamma}$ is the central laser frequency. We assume that the incident laser beam has a uniform spectral intensity $dI/d\omega = I/\Delta\omega$ in the frequency interval $\Delta\omega$ centered around ω_L , where I is the total intensity (power per unit area). We consider the transition between two electronic states, state 1 (the ground state) and state 2 (the excited state). Since the laser bandwidth given by Eq.(5) will be much larger than the transition width of individual ion, the laser-ion interaction is accurately represented by the rate equation [5], consisting of $n_1 + n_2 = 1$ and

$$dn_2/dt = -n_2 A_{21} - (n_2 B_{21} - n_1 B_{12})I/c\Delta\omega. \quad (6)$$

Here, n_1 (n_2) is the occupation probability for state 1(2), A_{21} , B_{21} , and B_{12} are respectively the transition rates for spontaneous emission, the stimulated emission, and the absorption in the laboratory frame where the ion is moving. These are given by

$$A_{21} = A_{21}^*/\gamma, \quad B_{12} = (1 + \beta_z)B_{12}^* \quad (7)$$

where $A_{21}^* = 2g_1 f_{12} r_e \omega^{*2}/g_2 c$ and $B_{12}^* = \pi^2 r_e c^2 f_{12}/\hbar \omega^*$ are the Einstein coefficients defined in the ion's rest frame, $g_1(g_2)$ the degeneracy factor of the state 1(2), f_{12} the oscillator strength, r_e the classical electron radius. The first

relation in Eq.(7) is simply the statement of the time dilation effect and the second can be obtained by recognizing that the absorption rate is proportional to the radiation flux seen by the moving ion. Finally, the absorption rate is obtained by the symmetry relation $B_{21} = (g_1/g_2)B_{12}$. From here on we are explicitly indicating the quantities $\hbar$ and c .

The solution of the rate equation reaches a steady state for $t \gg \tau_{\text{spont}}$, where $\tau_{\text{spont}} = 1/A_{21}$ is the spontaneous decay time. The steady state value is found to be $n_2 = (g_2/(g_1 + g_2))D/(1 + D)$, where $D = (g_1 + g_2)B_{12}I/g_2A_{12}c\Delta\omega$ is the saturation parameter. It is customary to write $D = I/I_{\text{sat}}$, where $I_{\text{sat}} = (g_1/4(g_1 + g_2))(\hbar\omega^4/\pi^2c^2\gamma\bar{\gamma})(\Delta\omega/\omega_L)$ is known as the saturation intensity. In writing these expressions, we are distinguishing γ , which is the property of individual ion, from $\bar{\gamma}$, which is the property of the ion beam as a whole. The distinction is relevant in the calculation of the damping time.

The number of spontaneous photons generated per ion per unit time is n_2/τ_{spont} . Taking the interaction length to be twice of the rms laser pulse length σ_{Lz} , we find that the number of the scattered photons generated per ion can be written in the following form:

$$\Delta N^s = 2(1 + \beta_z) \frac{\bar{\sigma}}{1 + D} \frac{I}{\hbar\omega_L} \frac{\sigma_{Lz}}{c}, \quad (8)$$

where $\bar{\sigma} = \pi f_{12} r_e \lambda^* \omega_L / \Delta\omega$, λ^* being the wavelength corresponding to ω^* . Therefore, except for the factor $1/(1 + D)$ representing the saturation effect, the generation of the spontaneous photons by an ion interacting with a broadband laser can be described by a scattering process with an average cross section $\bar{\sigma}$ over the laser bandwidth. Note that $\bar{\sigma}$ is larger than the Thomson cross section by about a factor $(\lambda^*/r_e)(\omega_L/\Delta\omega)$, which is large, about $10^7 - 10^8$ for the examples considered later in this Letter. The radiative ion cooling is feasible due to this fact. It should be remarked that the enhancement factor at exact resonance is even larger, about $(\lambda^*/r_e)^2$ [5].

The average frequency of the spontaneous radiation is $\langle \omega^s \rangle = \gamma\omega^* = 2\gamma\bar{\gamma}\omega_L$, which is one half of the maximum frequency ω_{max}^s occurring in the backscattering direction. The total energy radiated per ion is therefore $\Delta W^s = \gamma\hbar\omega^* \Delta N^s$. Since ΔW^s comes entirely from the ion's kinetic energy in the relativistic case under consideration, it must be equal to qV , the acceleration in the cavity. Also, the requirement that the ion remain in resonance with the entire laser pulse is

$$\Delta W^s \leq Mc^2 \gamma \Delta\omega / \omega_L. \quad (9)$$

We have been assuming that the depletion of the laser intensity during the interaction is small so that I may be regarded as a constant. This is true if the total number of scattered photons per interaction, $N^s = N_i \Delta N^s$, where N_i is the number of ions per pulse, is a small fraction of the number of laser photons. This leads to the condition

$$(1 + \beta_z) \frac{\bar{\sigma}}{1 + D} \frac{N_i}{\Sigma} \ll 1, \quad (10)$$

where Σ is the cross section of the ion beam.

The damping times of betatron and the phase oscillations can be calculated following the well known steps [1]; Assuming that the laser-ion interaction takes places in a dispersion-free straight section, the damping time of the horizontal betatron oscillation τ_x is the same as that of the vertical oscillation τ_y because the variation in the radiated energy due to a variation in the orbit vanishes. The damping times are

$$\tau_x = \tau_y = 2Mc\bar{\gamma}C/\Delta W^s, \quad (11)$$

where C is the circumference of the storage ring. The damping time of the energy oscillation, τ_ϵ , involves the derivative of the radiated energy ΔW^s with respect to the ion energy γ at a fixed orbit. Noting that the energy dependence of ΔW^s is contained in the factor $D/(1 + D)$, where D is proportional to $\gamma\bar{\gamma}$, and remembering that $\bar{\gamma}$ is regarded as a constant, one obtains

$$\tau_\epsilon = \tau_x(1 + D). \quad (12)$$

Equation(12) is specific to the assumption that the spectral intensity $dI/d\omega$ is constant within the bandwidth. If $dI/d\omega$ were, for example, proportional to ω , we would have obtained that D is proportional to $\bar{\gamma}^2$, and thus ΔW^s is independent of the ion energy. There would then be no damping of the energy oscillation. On the other hand, if $dI/d\omega$ were proportional to $1/\omega$, then the damping of the energy oscillation would be faster.

The oscillation amplitudes reach an equilibrium when the damping balances the excitation arising from the quantum nature of the emission process. The equilibrium amplitudes can again be calculated following the similar calculation

for the case of electron storage rings [1]. However, the spectral-angular probability distribution of the spontaneous emission required in the calculation is given by that of the undulator radiation in the present case.

We find that the rms relative energy spread at equilibrium is given by

$$\sigma_\delta = \sqrt{1.4\omega^*\hbar/Mc^2}. \quad (13)$$

The excitation of the horizontal betatron motion in the case of electron storage rings is due mainly to the emission in the bending magnets with non-vanishing dispersion [1]. In the present case, where we assume that the interaction takes place in a dispersion-free straight section, the excitation is due only to the small effect that the emitted photons are not exactly parallel to the ion momentum. The situation is similar to the case of the vertical betatron excitation in electron storage rings with horizontal orbit. The equilibrium rms horizontal emittance in this case is found to be

$$\epsilon_x = \frac{3}{20} \frac{\hbar\omega^*}{\bar{\gamma}^2 Mc^2} < \beta_x >, \quad (14)$$

where $< \beta_x >$ is the average horizontal beta-function in the interaction region. The equilibrium vertical emittance is obtained by replacing β_x by β_y .

For the transition between the $1S$ ground state and the $2P$ excited state of a hydrogen-like ion with atomic number Z , we have $\hbar\omega^* = 3\alpha^2 m_e c^2 Z^2/8 \simeq 10.19Z^2[\text{eV}]$ and $f_{12} = 0.416$. The scattering of laser beams in the optical wavelength region by relativistic heavy ions could therefore produce short wavelength radiation extending to the gamma-ray region. The $2S$ state, although degenerate with $2P$ state, is not coupled by the dipole transition. The degeneracy parameters are therefore $g_1 = 1$, $g_2 = 3$.

Since the spontaneous emission rate is proportional to $D/(1+D)$, the optimum value of the laser intensity is $I \simeq I_{sat}$. However, limitations in the laser or the storage ring RF power may force I to a smaller value. The Rayleigh length z_R and the rms pulse length σ_{Lz} of the laser beam need to be suitably chosen taking proper account of the ion beam parameters, rms transverse size at the waist σ_x , the length σ_z , and the beta function β_x . In particular, the relative magnitudes of the ion beam emittance $\epsilon_x = \sigma_x^2/\beta_x$ and the laser beam emittance $\epsilon_L = \lambda_L/4\pi$ will be important in this consideration. The energy in the laser pulse is related to the intensity by $W_L = \sigma_{Lz} z_R \lambda_L I/c$, where λ_L is the laser wavelength corresponding to the frequency ω_L . The required laser power is then $P_L = W_L/t_b$, where t_b is the time separation between the bunches.

We now consider two examples of radiative ion cooling; first the hydrogen-like Pb ion beam in LHC [6], for which the relevant parameters are; $C = 27$ km, $\gamma = 3000$, $Mc^2\gamma = 575$ TeV, $Z = 82$, $N_i = 9.4 \times 10^7$, $t_b = 135$ ns, $\sigma_z = 7.5$ cm, $\epsilon_x = 3 \times 10^{-9}$ m-rad, $\beta_x = 0.5$ m, $\sigma_x = 1.2 \times 10^{-3}$ cm, $\Delta\gamma/\bar{\gamma} = 2 \times 10^{-4}$. We find $\hbar\omega^* = 68.7$ keV, $< \hbar\omega^s > = 0.2$ GeV, $\lambda_L = 1080$ Å. In this case $\epsilon_x < \epsilon_L$, and the laser cross section is larger than that of the ion beam as long as $z_R \geq 1.68$ cm. A reasonable choice is then $z_R = \sigma_{Lz} = \sigma_z = 7.5$ cm. With $I = I_{sat}$, the required accelerating voltage exceeds significantly the current LHC design. Thus we choose $I = I_{sat}/712$, so that required acceleration per revolution becomes 1 GeV, which can be achieved by an accelerating voltage of 12.3 MV for the hydrogen-like Pb ions. The requirements on laser are $\Delta W_L \simeq 56$ μJ per pulse and an average power $P_L \simeq 420$ W, which are feasible either with the conventional lasers or free electron lasers. The gamma ray production rate is 4.7×10^8 photons per pulse and 3.5×10^{15} per seconds. The conditions Eqs.(9) and (10) are both satisfied. The spontaneous decay length $c\tau_{spont} \simeq 4.2 \times 10^{-3}$ cm is much less than the interaction length, 15 cm. The damping times in all three directions are found to be 52 seconds. The equilibrium beam parameters are; $\sigma_\delta = 7.1 \times 10^{-4}$, which is larger than the initial energy spread, and $\epsilon_x = 3. \times 10^{-15}$ m-rad.

The second example is the hydrogen-like oxygen ion beam in RHIC [7], for which the relevant parameters are; $C = 3.8$ km, $\gamma = 135$, $Mc^2\gamma = 2$ TeV, $Z = 8$, $N_i = 8.3 \times 10^9$, $t_b = 224$ ns, $\sigma_z = 20$ cm, $\epsilon_x = 3 \times 10^{-7}$ m-rad $\beta_x = 1$ m, $\sigma_x = 5.5 \times 10^{-2}$ cm, $\Delta\gamma/\bar{\gamma} = 2.5 \times 10^{-3}$. We find $\hbar\omega^* = 653$ eV, $< \hbar\omega^s > = 177$ keV, $\lambda_L = 5100$ Å. With $z_R = 7.3$ m, the laser cross section is the same as the ion beam cross section at the waist location. However, since $\epsilon_x > \epsilon_L$, the ion beam cross section is larger than the laser cross section away from the waist. To minimize this effect, we choose $\sigma_{Lz} = \beta_x/2 = 0.5$ m. Contrary to the previous example, we are allowed to choose $I = I_{sat}$ here, since the required acceleration voltage is found to be about 0.3 MV per turn, which should be feasible. The requirements on laser are $\Delta W_L \simeq 6.2$ mJ per pulse and an average power $P_L \simeq 28$ kW, which should be feasible with a free electron laser in an intracavity configuration. The gamma ray production rate is 2×10^{11} photons per pulse and 8.8×10^{17} per seconds. The conditions Eqs.(9) and (10) are both satisfied. The spontaneous decay length $c\tau_{spont} \simeq 2.1$ cm is also much less than the interaction length, 100 cm. The transverse damping times are $\tau_x = \tau_y = 26$ seconds while τ_z is twice as long. The equilibrium beam parameters are; $\sigma_\delta = 2.5 \times 10^{-4}$, which is about ten times smaller than the initial value, and $\epsilon_x = 4 \times 10^{-13}$ m-rad.

It should be remarked that the equilibrium beam emittances obtained in the above examples do not take into account the intrabeam scattering effect.

In summary, we have shown that the radiative ion cooling in relativistic ion storage rings under construction with currently achievable lasers is an efficient way to cool the ion beam. The intense short wavelength radiation produced in the process could be useful for nuclear physics experiments or other applications. The radiative ion cooling would also permit storing higher current, low emittance ion beams, leading to the possibility of free ion lasers [8].

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