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The BRST Quantization of Massive Abelian Two-Form Gauge Fields

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Abstract Massive abelian two-form gauge fields are quantized in the framework of the antifield BRST formalism based on path integrals. In this letter it is shown that transforming the original system into i) a first-class one in the initial phase-space, or ii) a reducible family of first-class systems in a larger phase-space, and further quantizing in the BRST manner the resulting first-class systems, their correspondent path integrals coincide with the original one. The procedure is then generalized to the case of massive abelian p -form abelian gauge fields in D dimensions.

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1. Introduction

The most powerful quantization method for theories with gauge invariances is well-known to be the BRST formalism. For systems with both first and second-class constraints or only first-class ones there are implemented many BRST quantization alternatives [1-6]. For theories possessing only second-class constraints this formalism cannot be directly applied because they are not gauge invariant. However, the BRST quantization of such theories can be achieved in principle by changing the original second-class system into a first-class one: i) in the initial phase-space [7] or ii) in a larger phase-space obtained from the original one by adding some extra canonical pairs [8-9] and then quantizing the resulting first-class system(s) accordingly the BRST manner. Many authors [10-19] have applied the methods presented in [8-9] to various models. Nevertheless, the method from [8-9] hasn't been extended yet to the case of second-class systems preserving somehow the reducibility relics of a certain first-class system, and peculiarly in the case of massive abelian two-form gauge fields. This is the aim of the present paper. More precisely, in this work we quantize in the framework of the BRST formalism the massive four-dimensional abelian two-form gauge fields transforming the original system in a first-class one in the light of the procedures i) and ii). Procedure ii) implies in fact the BRST quantization of two-form gauge fields interacting with vector fields through a current-current type term. This interaction is definitely not a topological coupling. More on abelian p-form gauge fields may be found in [20]. There are many reasons for studying these models. The significance of free abelian two-form gauge fields resides in their natural appearance in the study of dual resonance models, relativistic strings, black holes, vortices, quantum gravity and extended supergravity [21-29]. While the free abelian two-form gauge fields describe spin-0 particles of zero rest mass, the massive ones reveal massive spin-1 particles [30].

Our starting point is the Lagrangian action

$$S_0^L[A^{\alpha\beta}] = \int d^4x \left(-\frac{1}{12} F^{\alpha\beta\gamma} F_{\alpha\beta\gamma} - \frac{M^2}{4} A^{\alpha\beta} A_{\alpha\beta} \right), \quad (1)$$

where $F^{\alpha\beta\gamma} = \partial^\alpha A^{\beta\gamma} + \partial^\gamma A^{\alpha\beta} + \partial^\beta A^{\gamma\alpha} \equiv \partial[\alpha A^{\beta\gamma}]$ and $A^{\alpha\beta}$ are antisymmetric in the Lorentz indices. The canonical analysis furnishes the canonical Hamiltonian

$$H = \int d^3x \left(-\pi_{ij} \pi^{ij} + \frac{1}{12} F_{ijk} F^{ijk} + \frac{M^2}{4} A^{\alpha\beta} A_{\alpha\beta} - 2A_{0j} \partial_i \pi^{ij} \right), \quad (2)$$

and the second-class constraints

$$G^i \equiv \pi^{0i} = 0 \text{ (primary)}, \quad (3 \text{ a})$$

$$C^j \equiv 2\partial_i \pi^{ij} - M^2 A^{0j} = 0 \text{ (secondary)}, \quad (3 \text{ b})$$

where $\pi_{\mu\nu}$ are the canonical momenta conjugated with the $A^{\mu\nu}$'s. We observe simply that $[C^j, G^k] = -M^2 g^{jk} \equiv \Delta^{jk}$, with $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$. We denote for later convenience by $(\Delta^{-1})^{jk}$ the inverse of Δ^{jk} and $C_j^{(0)} \equiv 2\partial_i \pi^{ij}$, $C_j^{(1)} \equiv -M^2 A^{0j}$.

2. The BRST quantization in the original phase-space

Within this section we quantize the system described by action (1) as follows. As the second-class constraints (3 a-b) split in two equally numbered subsets, primary and secondary, such that the G^i 's are first-class among themselves, we can regard the original system as being provided by a first-class one possessing only the constraints (3 a) in the canonical gauge (3 b). For our model, the Hamiltonian of this first-class system reads [7]

$$H' = H - [H, G_j] (\Delta^{-1})^{jk} C_k + \frac{1}{2} [[H, G_j], G_k] (\Delta^{-1})^{ji} (\Delta^{-1})^{kl} C_l C_l, \quad (4)$$

such that $[H', G_j] = 0$ strongly. The concrete form of (4) is given by

$$H' = \int d^3x \left(-\pi_{ij} \pi^{ij} + \frac{1}{12} F_{ijk} F^{ijk} + \frac{M^2}{4} A^{ij} A_{ij} - \frac{2}{M^2} \partial_i \pi^{ij} \partial^k \pi_{kj} \right) \equiv \int d^3x h' \quad (5)$$

Thus, we associated to the original system a first-class one in the same phase-space, described by the Hamiltonian action

$$S_0^T[A, \pi, u] = \int d^4x \left(\dot{A}^{ij} \pi_{ij} + \dot{A}^{0i} \pi_{0i} - h' - u_i G^i \right), \quad (6)$$

which is invariant under the gauge transformations: $\delta_\varepsilon A^{0j} = \varepsilon^j$, $\delta_\varepsilon A^{ij} = 0$, $\delta_\varepsilon \pi^{\mu\nu} = 0$, $\delta_\varepsilon u_i = \dot{\varepsilon}_i$.

Next, we quantize in the BRST antifield formalism the above first-class system following the general line in [5]. Then, the non-minimal solution of the master equation is

$$S^T = S_0^T + \int d^4x \left(A_{0j}^* \eta^j + u_j^* \eta^j + \bar{\eta}_j^* B^j \right), \quad (7)$$

where the star fields are the antifields of the corresponding fields, η^j are the minimal ghost fields, and the $\bar{\eta}^i$'s together with the B^i 's are non-minimal sector fields. We impose the canonical gauge conditions (3 b) through the gauge-fixing fermion

$$K = \int d^4x \bar{\eta}^i C_i. \quad (8)$$

Eliminating as usually the antifields in (7), we get the gauge-fixed action

$$S_K^T = S_0^T + \int d^4x \left(-M^2 \bar{\eta}_i \eta^i + B_i C^i \right). \quad (9)$$

The non-vanishing BRST invariances of S_K^T are the following: $s A^{0j} = \eta^j$, $s \bar{\eta}_i = -B^i$. Integrating in the path integral correspondent to (9) over all the fields excepting the A^{ij} 's and π_{ij} 's, we get

$$Z_K = \int D A^{ij} D \pi_{ij} \exp \left(i \int d^4x \left(\dot{A}^{ij} \pi_{ij} - h' \right) \right). \quad (10)$$

Formula (10) gives precisely the Hamiltonian path integral over independent variables for our model. The validity of the prior path integral can be checked by means of canonical methods [31], confirming that our method is indeed correct. On the other hand, we are able to infer directly the associated Lagrangian path integral of our model. This can be done by making in (9) the replacement $C_i \rightarrow C_i - \frac{M^2}{2} B_i$ (which doesn't affect its BRST invariances) and integrating then in the novel path integral over all the variables excepting the $A^{\alpha\beta}$'s. Then, we deduce

$$Z_K^L = \int DA^{\alpha\beta} \det(M) \exp(iS_0^L), \quad (11)$$

with S_0^L given in (1). Relations (10-11) express the basic results of our first quantization method in the case of the investigated model.

3. The BRST quantization in a larger phase-space

The purpose of this section is to quantize our model converting the original system into a first-class one by adding some extra canonical pairs, and subsequently applying the BRST formalism. A conversion method is outlined in [8-9]. Here it will be not employed this procedure, but a different way which we extend to the "reducible" case. The above invoked "reducibility" will be made meaningful in the sequel.

For the subsequent presentation it is useful to modify (5) slightly such that the new first-class Hamiltonian to be in the same gauge invariant function equivalence class as H' . Thus, we take as new first-class Hamiltonian in the original phase-space the function

$$\bar{H} = H - \frac{1}{M^2} \left(\frac{1}{2} C^{(1)}_j C^{(1)}_j + C^{(0)}_j C^{(1)}_j \right). \quad (12)$$

Our method consists of two steps. Firstly, we associate in the manner of Sec. 2 to the original system a first-class one in the original phase-space, described by the Hamiltonian (12) and the first-class constraints (3 a). Secondly, we associate to the above system a one-parameter family of first-class systems in a larger phase-space by adding some extra canonical pairs as follows.

i) For every pair $(G^j, C^{(0)}_j)$ we introduce a bosonic canonical pair (q^j, p_j) , such that the new secondary constraints to be precisely

$$\gamma_j \equiv \lambda p_j - C^{(0)}_j = 0, \quad (13)$$

with λ the parameter of the one-parameter family. It is easy to see that G_j and γ_j become now first-class and

$$\partial^j \gamma_j = \lambda \partial^j p_j = 0 \quad (14)$$

ii) Because of relation (14) we introduce an other canonical pair (q^0, p_0) and the supplementary constraint $\gamma \equiv p_0 = 0$, such that its consistency to imply as new constraints

exactly relations (14). Thus, we joined to the original system a one-parameter family of first-class systems with the first-class constraints (3 a), (13) and

$$\bar{\gamma} \equiv -\partial^i p_i = 0, \gamma \equiv p_0 = 0. \quad (15)$$

An interesting feature of the one-parameter family is that now the constraints are reducible, namely

$$\lambda \bar{\gamma} - \partial^j \gamma_j = 0. \quad (16)$$

Under these circumstances, we construct the first-class Hamiltonian of the first-class family as

$$H^* = \bar{H} + \int d^3x \left(\frac{\lambda^2}{2} (\Delta^{-1})^{jk} p_j p_k + \gamma_j (\Delta^{-1})^{jk} C_k^{(1)} + q^0 \bar{\gamma} + g \right), \quad (17)$$

where g is a function satisfying

$$[G_i, g] = 0, [\gamma_i, g] = 0, [\gamma, g] = 0, [\bar{\gamma}, g] = 0. \quad (18 \text{ a-d})$$

The solution of system (18) results from developing g as a series in q^i 's with coefficients depending only on the A^{ij} 's

$$g = g_i^{(1)} q^i + g_{ij}^{(2)} q^i q^j + \dots \quad (19)$$

From (19), it is clear that (18 a) and (18 d) are automatically fulfilled. Introducing (19) in (18 b), we get

$$g_i^{(1)} = \frac{M^2}{\lambda} \partial^k A_{ki}, \quad g_{ik}^{(2)} = -\frac{1}{\lambda} \left[\partial^j \pi_{ji}, g_k^{(1)} \right], \quad (20 \text{ a-b})$$

all the coefficients $g_{i_1 \dots i_n}^{(n)}$, with $n > 2$ vanishing. Replacing (19) and (20 a-b) in (18 c), the last one automatically holds. Then, the complete form of (17) reads

$$H^* = \bar{H} + \int d^3x \left(\frac{\lambda^2}{2} p_j p_j + \gamma_j A^{0j} + q^0 \bar{\gamma} - \frac{M^2}{2\lambda} A^{ij} (\partial_i q_j - \partial_j q_i) + \right. \\ \left. + \frac{M^2}{4\lambda^2} (\partial_i q_j - \partial_j q_i) (\partial^i q^j - \partial^j q^i) \right). \quad (21)$$

Thus, we substitute the original second-class system with a one-parameter family of first-class systems described by the extended action

$$S_0^E[A, \pi, q, p, u, v] = \int d^4x \left(\dot{A}^{0j} \pi_{0j} + \dot{A}^{ij} \pi_{ij} + \dot{q}^0 p_0 + \dot{q}^i p_i - h^* - \right.$$

$$-u\gamma - u^j G_j - v\bar{\gamma} - v^j \gamma_j \Big) , \quad (22)$$

where the u 's and v 's denote the multipliers correspondent to the first-class constraints. Action (22) is invariant under the following gauge transformations: $\delta_\epsilon q^0 = \epsilon_1^0$, $\delta_\epsilon A^{0j} = \epsilon_1^j$, $\delta_\epsilon q^j = \partial^j \epsilon_2^0 + \lambda \epsilon_2^j$, $\delta_\epsilon A^{ij} = \partial^i \epsilon_2^j - \partial^j \epsilon_2^i \equiv \partial^{[i} \epsilon_2^{j]}$, $\delta_\epsilon \pi_{0j} = \delta_\epsilon \pi_{ij} = \delta_\epsilon p_0 = \delta_\epsilon p_i = 0$, $\delta_\epsilon u = \epsilon_1^0$, $\delta_\epsilon u^j = \epsilon_1^j$, $\delta_\epsilon v = \epsilon_2^0 - \epsilon_1^0 + \lambda \epsilon_2$, $\delta_\epsilon v^j = \epsilon_2^j - \epsilon_1^j - \partial^j \epsilon_2$. The additional arbitrary function ϵ_2 appears within the gauge transformations of the v 's because of the reducibility relation (16) [5], [32]. In this case the minimal solution of the master equation takes the form

$$\begin{aligned} S_{\min}^E = & S_0^E + \int d^4x \left(q_0^* \eta_1^0 + q_i^* \left(\partial^i \eta_2^0 + \lambda \eta_2^i \right) + A_{0j}^* \eta_1^j + A_{ij}^* \partial^{[i} \eta_2^{j]} + u^* \dot{\eta}_1^0 + \right. \\ & \left. + u_i^* \dot{\eta}_1^i + v^* \left(\dot{\eta}_2^0 - \eta_1^0 + \lambda \eta_2 \right) + v_j^* \left(\dot{\eta}_2^j - \eta_1^j - \partial^j \eta_2 \right) + \lambda \eta_2^{*0} C - \eta_2^{*i} \partial_i C \right) . \end{aligned} \quad (23)$$

We see that (23) still has the residual gauge invariances $\delta_\theta \eta_2^i = \frac{1}{\lambda} \partial^i \theta$, $\delta_\theta \eta_2^0 = -\theta$, $\delta_\theta \eta_2 = \frac{1}{\lambda} \dot{\theta}$, $\delta_\omega v = \omega$, and $\delta_\omega v^i = -\frac{1}{\lambda} \partial^i \omega$, with θ and ω respectively fermionic and bosonic arbitrary functions, so we need a larger than usual non-minimal sector. This implies the non-minimal solution

$$S_{\text{non min}}^E = S_{\min}^E + \int d^4x \left(\bar{\eta}_2^* B_2^i + \bar{\eta}_2^* B_2 + \bar{\eta}_{1i}^* B_1^i + \bar{\eta}_1^* B_1 + \bar{\eta}^* b \right) . \quad (24)$$

Next, we show that imposing the canonical gauge conditions $C_i = 0$, $q^\mu \equiv (q^0, q^i) = 0$ through the gauge-fixing fermion

$$\Psi = \int d^4x \left(\bar{\eta}_1^i C_i + \lambda M^2 \bar{\eta}_2 q^0 - M^2 \bar{\eta}_2^i q_i + \bar{\eta}_1 \eta_2^0 + v \bar{\eta} \right) , \quad (25)$$

we obtain the same path integral as in Sec. 2 for the massive abelian two-form gauge fields. Indeed, eliminating the antifields from (24) in the usual manner and integrating in the correspondent path integral over all the fields excepting the A^{ij} 's and π_{ij} 's, we get

$$Z_\Psi = \int DA^{ij} D\pi_{ij} \left(\exp i \int d^4x \left(A^{ij} \pi_{ij} - \bar{h} + \frac{2}{M^2} \partial^i \pi_{ij} \partial_k \pi^{kj} \right) \right) , \quad (26)$$

which is nothing but (10). The Lagrangian path integral results in this case through the replacement $C_i \rightarrow C_i - \frac{M^2}{2} B_{1i}$ in (24) (which obvious doesn't affect its BRST invariances) and integration over all the fields excepting $A^{\alpha\beta}$'s in its correspondent path integral. Thus we obtain precisely

$$Z_\Psi^L = Z_K^L . \quad (27)$$

Relations (26-27) represent the basic results of this section and the achievement of the purposed aims.

4. The origin of the first-class family

In this section we clarify the provenance of the one-parameter family described by action (22). If one passes to the total action of (22) (putting $v = v^i = 0$) and eliminates the remaining multipliers together with all the momenta on their equations of motion [33], one obtains the Lagrangian action of the first-class family of the form

$$S'_0 L[A^{\alpha\beta}, q^\alpha] = \int d^4x \left(-\frac{1}{2 \cdot 3!} F^{\alpha\beta\gamma} F_{\alpha\beta\gamma} - \frac{M^2}{2 \cdot 2!} \left(A^{\alpha\beta} - \frac{1}{\lambda} F^{\alpha\beta} \right) \left(A_{\alpha\beta} - \frac{1}{\lambda} F_{\alpha\beta} \right) \right), \quad (28)$$

where $F_{\alpha\rho} = \partial_\alpha q_\rho - \partial_\rho q_\alpha$. Action (28) is invariant under the first-order reducible gauge transformations:

$$\delta_\varepsilon A_{\alpha\gamma} = \partial_\alpha \varepsilon_\gamma - \partial_\gamma \varepsilon_\alpha, \quad \delta_\varepsilon q_\alpha = \partial_\alpha \varepsilon + \lambda \varepsilon_\alpha. \quad (29 \text{ a-b})$$

System (28) can be regarded as coming from the gauging of the rigid (Noetherian) symmetries $\delta_\varepsilon q_\alpha = \lambda \varepsilon_\alpha$ (here ε_α are constant) of the system

$$S_0^L[q^\alpha] = \int d^4x \left(-\frac{M^2}{4\lambda^2} F_{\alpha\beta} F^{\alpha\beta} \right). \quad (30)$$

Indeed, making ε_α functions of x in the above Noetherian symmetries it is clear that (30) is not gauge invariant as $\delta_\varepsilon F_{\alpha\rho} = \lambda (\partial_\alpha \varepsilon_\rho - \partial_\rho \varepsilon_\alpha)$. This implies that $F_{\alpha\beta}$ must be coupled to a tensor field transforming as (29 a) such that $A_{\alpha\beta} - \frac{1}{\lambda} F_{\alpha\beta}$ to be gauge invariant. Constructing with this object a Lorentz scalar as in (28), we can naturally add to this one another gauge-invariant scalar built from $F_{\alpha\beta\gamma}$. In this way, action (28) depicts abelian two-form gauge fields coupled with vector fields (abelian one-form gauge fields) through a current-current type term, where the gauge-invariant current is

$$j^\rho_\alpha = \frac{M}{2} \left(A^\rho_\alpha - \frac{1}{\lambda} F^\rho_\alpha \right). \quad (31)$$

The last current corresponds to the above mentioned Noetherian invariances, which also hold for action (28).

Next, we shall quantize action (28) in the antifield BRST formalism. The non-minimal solution of the master equation reads

$$\begin{aligned} S'_0 L_{\text{non min}} = S_0^L + \int d^4x & \left(A_{\alpha\beta}^* \partial^\alpha [\eta^\beta] + q_\alpha^* \partial^\alpha C - \lambda q_\alpha^* \eta^\alpha + \eta_\alpha^* \partial^\alpha \eta + \right. \\ & \left. + \lambda C^* \eta + \bar{\eta}_\alpha^* b^\alpha + \bar{\eta}^* b \right). \end{aligned} \quad (32)$$

If one takes $q_\alpha = 0$ in (28), one reobtains the original massive theory. The gauge-fixing fermion implementing these canonical gauge conditions is

$$\Psi' = \int d^4x \left(\bar{\eta}_\alpha q^\alpha + (\lambda)^{-1/2} \bar{\eta} C \right) . \quad (33)$$

Integrating now in the path integral of (32) over all the fields excepting $A_{\alpha\beta}$'s, we get

$$Z_{\Psi'}^L = \int DA_{\alpha\beta} \det(\lambda) \exp(iS_0^L) . \quad (34)$$

Comparing (34) with (11) and (27), it results $\lambda = M$. This is the value of the parameter for which the path integral derived in the extended formalism coincides with the one deduced in the Lagrangian formalism [32].

At this point we are able to make clear the meaning of the original system preserving the relic of the reducibility of a certain first-class system. It is obvious that the original action comes from the reducible system (28) taking $q_\alpha = 0$. This conclusion remains also valid at the path integral level. More precisely, the path integral of the original massive theory is derived through the quantization of a reducible theory with a gauge-fixing fermion implementing the canonical gauge conditions $q_\alpha = 0$ (see (26) and (34), and also (25) and (33)).

5. Massive abelian p-form gauge fields

The approach presented in the last section can be generalized to the case of massive abelian p-form gauge fields in D dimensions. Starting with the Lagrangian action

$$S_{0p}^L[A^{\alpha_1 \dots \alpha_p}] = \int d^Dx \left(-\frac{1}{2(p+1)!} F_{\alpha_1 \dots \alpha_{p+1}} F^{\alpha_1 \dots \alpha_{p+1}} - \frac{1}{2 \cdot p!} M^2 A_{\alpha_1 \dots \alpha_p} A^{\alpha_1 \dots \alpha_p} \right) , \quad (35)$$

we arrive following the same procedure as in Sec. 3 to the Lagrangian action

$$S_{0p,p-1}^L[A^{\alpha_1 \dots \alpha_p}, q^{\alpha_1 \dots \alpha_{p-1}}] = \int d^Dx \left(-\frac{1}{2(p+1)!} F_{\alpha_1 \dots \alpha_{p+1}} F^{\alpha_1 \dots \alpha_{p+1}} - \frac{1}{2 \cdot p!} (M A_{\alpha_1 \dots \alpha_p} - F_{\alpha_1 \dots \alpha_p}) (M A^{\alpha_1 \dots \alpha_p} - F^{\alpha_1 \dots \alpha_p}) \right) . \quad (36)$$

The notations in (35-36) are as usually

$$F_{\alpha_1 \dots \alpha_{p+1}} = \frac{1}{p!} \partial_{[\alpha_1} A_{\alpha_2 \dots \alpha_{p+1}]} , \quad (37 a)$$

$$F_{\alpha_1 \dots \alpha_p} = \frac{1}{p-1!} \partial_{[\alpha_1} q_{\alpha_2 \dots \alpha_p]} , \quad (37 b)$$

the A's and q's being completely antisymmetric in all indices. Action (36) allows the gauge invariances

$$\delta_\varepsilon A_{\alpha_1 \dots \alpha_p} = \partial_{[\alpha_1} \varepsilon_{\alpha_2 \dots \alpha_p]} , \quad (38 a)$$

$$\delta \varepsilon q \alpha_1 \dots \alpha_{p-1} = \partial [\alpha_1 \varepsilon_{\alpha_2 \dots \alpha_{p-1}}] + M \varepsilon_{\alpha_1 \dots \alpha_{p-1}}, \quad (38 \text{ b})$$

where the ε 's are completely antisymmetric arbitrary functions.

Similarly to Sec. 4, action (36) results from gauging the rigid symmetries $\delta \varepsilon q \alpha_1 \dots \alpha_{p-1} = M \varepsilon_{\alpha_1 \dots \alpha_{p-1}}$ (with all ε 's constant) of the action

$$S_0^L[q^{\alpha_1 \dots \alpha_{p-1}}] = \int dDx \left(-\frac{1}{2 \cdot p!} F_{\alpha_1 \dots \alpha_p} F^{\alpha_1 \dots \alpha_p} \right). \quad (39)$$

The conserved gauge invariant current for action (36) associated to the prior rigid symmetries has the form

$$j^{\alpha_1}_{\alpha_2 \dots \alpha_p} = \frac{1}{p!} \left(M A^{\alpha_1}_{\alpha_2 \dots \alpha_p} - F^{\alpha_1}_{\alpha_2 \dots \alpha_p} \right). \quad (40)$$

The gauge transformations (38) are $(p-1)$ -order reducible. The minimal solution of the master equation for action (36) reads

$$S_{\min p, p-1}^L = S_p^L + S_{p-1}^L + \int dDx \left(-\frac{M}{2 \cdot p!} A_{\alpha_1 \dots \alpha_p} \left(M A^{\alpha_1 \dots \alpha_p} - 2 F^{\alpha_1 \dots \alpha_p} \right) + \right. \\ \left. + M \sum_{k=1}^{p-1} (-)^{p-k} C_{\alpha_1 \dots \alpha_k}^* \eta^{\alpha_1 \dots \alpha_k} + M C^* \eta \right), \quad (41)$$

where S_p^L and S_{p-1}^L are the minimal solutions of the master equations respectively for abelian p and $(p-1)$ -form gauge fields. The concrete forms of S_p^L and S_{p-1}^L are not important in the sequel due to the peculiar choices of the gauge-fixing fermions (see (43 a-b)). The only interesting (for us) components in these solutions are the original actions. The notations $C_{\alpha_1 \dots \alpha_k}^*$, C^* and $\eta^{\alpha_1 \dots \alpha_k}$, η respectively denote the minimal antifield spectrum for abelian $(p-1)$ -form gauge fields and minimal ghost spectrum for abelian p -form gauge fields, their ghost numbers and Grassmann parities being: $gh(\eta^{\alpha_1 \dots \alpha_k}) = -gh(C_{\alpha_1 \dots \alpha_k}^*) = k$, $gh\eta = -ghC^* = p$, $\varepsilon(\eta^{\alpha_1 \dots \alpha_k}) = \varepsilon(C_{\alpha_1 \dots \alpha_k}^*) = k \pmod{2}$, $\varepsilon(\eta) = \varepsilon(C^*) = p \pmod{2}$. The non-minimal solution correspondent to (41) reads

$$S_{\text{non min } p, p-1}^L = S_{\min p, p-1}^L + \int dDx \left(\sum_{k=1}^{p-1} \bar{\eta}_{\alpha_1 \dots \alpha_k}^* b^{\alpha_1 \dots \alpha_k} + \bar{\eta}^* b \right), \quad (42)$$

where the variables $\bar{\eta}_{\alpha_1 \dots \alpha_k}^*$, $\bar{\eta}^*$, $b^{\alpha_1 \dots \alpha_k}$, b are cohomological trivial [5]. The gauge-fixing fermions implementing the canonical gauge conditions $q \alpha_1 \dots \alpha_{p-1} = 0$ leading us back to the original action (35) take the forms

$$\Psi_1 = \int dDx \left(\bar{\eta}^{\alpha_1 \dots \alpha_{p-1}} q_{\alpha_1 \dots \alpha_{p-1}} + \sum_{k=1}^{p-2} \bar{\eta}^{\alpha_1 \dots \alpha_k} C_{\alpha_1 \dots \alpha_k} + \bar{\eta} C \right), \quad (43 \text{ a})$$

for p odd, respectively

$$\Psi_2 = \int d^D x \left(\bar{\eta}^{\alpha_1 \dots \alpha_{p-1}} q_{\alpha_1 \dots \alpha_{p-1}} + \sum_{k=1}^{p-2} \bar{\eta}^{\alpha_1 \dots \alpha_k} C_{\alpha_1 \dots \alpha_k} + M^{-1/2} \bar{\eta} C \right), \quad (43 \text{ b})$$

for p even. The variables $\left(\bar{\eta}^{\alpha_1 \dots \alpha_j} \right)_{j=1, \dots, p-1}$, $\bar{\eta}$ are the fields conjugated in the antibracket with the correspondent $\bar{\eta}^*$'s. The characteristics of the non-minimal sector variables are: $gh\left(\bar{\eta}^{\alpha_1 \dots \alpha_j} \right) = -gh\left(\bar{\eta}_{\alpha_1 \dots \alpha_j}^* \right) - 1 = gh\left(b^{\alpha_1 \dots \alpha_j} \right) - 1 = -(p-j)$, $gh\bar{\eta} = -gh\bar{\eta}^* - 1 = ghb - 1 = -p$, $\varepsilon\left(\bar{\eta}^{\alpha_1 \dots \alpha_j} \right) = \varepsilon\left(\bar{\eta}_{\alpha_1 \dots \alpha_j}^* \right) + 1 = \varepsilon\left(b^{\alpha_1 \dots \alpha_j} \right) + 1 = (p-j) \pmod{2}$, $\varepsilon(\bar{\eta}) = \varepsilon(\bar{\eta}^*) + 1 = \varepsilon(b) + 1 = p \pmod{2}$.

Eliminating as usually the antifields in (42) with the aid of the appropriate gauge-fixing fermion (43 a) or (43 b) and integrating in the correspondent path integral over all the variables excepting the $A_{\alpha_1 \dots \alpha_p}$'s, we infer

$$Z_{\Psi}^L = \int DA_{\alpha_1 \dots \alpha_p} \det(M) \exp\left(i S_{0p}^L \right). \quad (44)$$

Expression (44) gives the Lagrangian path integral for massive abelian p -form gauge fields deduced within the framework of the antifield BRST formalism.

We notice that for $p=1$ and $D=4$, action (36), which now reads

$$S_{0,0}^L = \int d^4 x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (MA_{\mu} - \partial_{\mu} q) (MA^{\mu} - \partial^{\mu} q) \right), \quad (45)$$

allows the gauge invariances $\delta_{\varepsilon} A_{\mu} = \partial_{\mu} \varepsilon$, $\delta_{\varepsilon} q = M\varepsilon$, representing the only irreducible case among the models investigated here. Action (45) is identical with the one obtained in [34] by converting the second-class constraints of the massive vector fields into some first-class ones using a conversion method for irreducible theories.

6. Conclusion

Using the BRST formalism, we proved that converting second-class constraints into some first-class ones in two different manners, we can quantize consistently a massive theory which preserves in a certain way the reducibility of the non-massive one. The method of conversion exposed in Sec. 3. is directly linked to the manifestly covariance of the Lagrangian action of the first-class family. This covariance is due to the presence of the functions $C_j^{(0)}$ in the novel secondary constraints (13). Indeed, if in (13) we hadn't had introduced the $C_j^{(0)}$'s then they would have read $\gamma_j \equiv \lambda p_j = 0$ and thus g would have vanished (see (20 a-b)), further leading to a non-covariant Lagrangian action. The BRST quantization of massive abelian p -form gauge fields in D dimensions is accomplished following the same line as in the case $p=2$, $D=4$. Action (36) for $p=1$ and $D=4$ leads to the unique irreducible case among the other models investigated in this work.

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