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The Antifield BRST-Anti-BRST Quantization of Massive and Non-Massive Abelian Three-Form Gauge Fields

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Abstract The antifield BRST-anti-BRST quantization of abelian three-form gauge fields is achieved in detail. The non-minimal sector of the antifield BRST formalism appears now naturally. It is shown that the quantizations of the massive case implies in fact the BRST or BRST-anti-BRST quantization of abelian three-form gauge fields coupled to abelian two-form gauge fields. This mixing-component system preserves in a certain way the reducibility of the non-massive abelian three-form gauge fields.

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1. Introduction

The most powerful manifestly covariant quantization method for gauge theories is the BRST antibracket-antifield formalism [1-5]. An alternative method for the quantization of the gauge theories is the BRST-anti-BRST formalism. Immediately after its discovery, this symmetry was evidenced in the framework of Yang-Mills theories. The BRST-anti-BRST formalism was implemented from different point of views in the Hamiltonian [6-10], as well as in the Lagrangian context [10-20]. Especially the $sp(2)$ formalism attracted much attention in connection with string field theory [6-7], [11-12], [16], [21-23]. Recently, this formalism was proved to be equivalent at both Hamiltonian and Lagrangian levels with the theory developed in [8-10], [13-14]. Although the BRST-anti-BRST symmetry does not play such an important role as the BRST symmetry, it is a helpful background for the geometrical (superfield) description of the BRST transformation, in the investigation of the perturbative renormalizability of the Yang-Mills models, in the consistent approach of anomalies, as well as for the correct understanding of the non-minimal sector [24-30].

Some models intensively studied lately, especially from the Hamiltonian point of view, are abelian p -form gauge fields free and coupled with vector fields [31-37]. The physical importance of these models consists in the fact that they are profoundly connected on one hand with string theory and on the other with cosmic strings, vortices and black holes [32], [36]. p -form gauge fields appear naturally in the framework of compactification of a higher-dimensional (especially ten-dimensional) theory to four dimensions through the zero modes which behave as massless particles. These mathematical objects allow us to understand zero modes and define some topological invariants associated with vector bundles and called characteristic classes. At the same time, these theories have a mathematical importance as the differential forms are the only fields for which is possible to define a differential operator independent of metric choice.

The BRST structure of abelian p -form gauge fields is well-known [31], but to our knowledge there are no attempts of implementing the antifield BRST-anti-BRST symmetry. This is the aim of our paper in the case $p=3$ for both massive and non-massive models in four dimensions. As it will be seen below, the quantization of the massive model emphasizes the natural coupling between abelian three and two-form gauge fields in the procedure of conversion of the second-class constraints into first-class ones by introducing extrafields. This procedure is of interest in the context of a massive theory which preserves in a certain way (to be further exposed) the reducibility of the correspondent non-massive theory. As far as we know, the problem of conversion has been approached only in the irreducible case [39-40].

This work is organized in four sections. In Sec. 2 we present in brief the BRST quantization of abelian three-form gauge fields. Sec. 3 focuses on the antifield BRST-anti-BRST quantization of this model. In Sec. 4 it is discussed the conversion of the correspondent (second-class) massive theory into a first class one by adding extrafields together with the BRST and BRST-anti-BRST quantization of the latter, showing that its path integral coincides with the original one.

Regarding the antifield BRST quantization we follow the general lines from [4-5], while the antifield BRST-anti-BRST treatment will employ the point of view from [10], [13-14], apart from using the BRST-anti-BRST operators as right derivatives.

2. The antifield BRST quantization for abelian three-form gauge fields

Our starting point is the Lagrangian action

$$S_0^L[A^{\alpha\beta\gamma}] = \int d^4x \left(-\frac{1}{2 \cdot 4!} F_{\alpha\beta\gamma\rho} F^{\alpha\beta\gamma\rho} \right), \quad (1)$$

where

$$F^{\alpha\beta\gamma\rho} = \partial[\alpha_A \beta\gamma\rho] \equiv \partial^\alpha_A \beta\gamma\rho - \partial^\beta_A \alpha\gamma\rho + \partial^\gamma_A \rho\alpha\beta - \partial^\rho_A \gamma\alpha\beta, \quad (2)$$

and the bosonic fields $A^{\alpha\beta\gamma}$ are completely antisymmetric in all indices. Action (1) is invariant under the next gauge transformations

$$\delta_\varepsilon A^{\alpha\beta\gamma} = \partial[\alpha_\varepsilon \beta\gamma] \equiv \partial^\alpha_\varepsilon \beta\gamma + \partial^\gamma_\varepsilon \alpha\beta + \partial^\beta_\varepsilon \gamma\alpha \equiv Z_{\mu\nu}^{\alpha\beta\gamma} \varepsilon^{\mu\nu}, \quad (3)$$

with $\varepsilon^{\mu\nu}$ arbitrary antisymmetric functions and

$$Z_{\mu\nu}^{\alpha\beta\gamma} = \frac{1}{2} \left[\left(\delta_\mu^\beta \delta_\nu^\gamma - \delta_\nu^\beta \delta_\mu^\gamma \right) \partial^\alpha + \left(\delta_\mu^\gamma \delta_\nu^\alpha - \delta_\nu^\gamma \delta_\mu^\alpha \right) \partial^\beta + \left(\delta_\mu^\alpha \delta_\nu^\beta - \delta_\nu^\alpha \delta_\mu^\beta \right) \partial^\gamma \right]. \quad (4)$$

The gauge transformations (3) are reducible since

$$Z_\lambda^{\mu\nu} Z_{\mu\nu}^{\alpha\beta\gamma} = 0, \quad (5)$$

and

$$Z^\sigma Z_\sigma^{\mu\nu} = 0, \quad (6)$$

where the reducibility functions have the form

$$Z_\lambda^{\mu\nu} = \delta_\lambda^\nu \partial^\mu - \delta_\lambda^\mu \partial^\nu, \quad (7)$$

$$Z^\sigma = \partial^\sigma. \quad (8)$$

There are no further reducibility relations, the above ones holding off-shell.

Next, we pass to the antifield BRST quantization of this model. Due to the above reducibility conditions, it is clear [4-5] that the minimal ghost and respectively antifield spectra are given by

$$\left(\eta^{\alpha\beta}, \eta^\alpha, \eta \right) \quad ; \quad \left(A_{\alpha\beta\gamma}^*, \eta_{\alpha\beta}^*, \eta_\alpha^*, \eta^* \right), \quad (9)$$

their ghost numbers (gh) and Grassmann parities (ε) being

field/ antifield	$\eta^{\alpha\beta}$	η^α	η	$A_{\alpha\beta\gamma}^*$	$\eta_{\alpha\beta}^*$	η_α^*	η^*
gh	1	2	3	-1	-2	-3	-4
ε	1	0	1	1	0	1	0

The minimal solution of the master equation reads

$$S_{\min} = S_0^L + \int d^4x \left(A_{\alpha\beta\gamma}^* \partial[\alpha\eta^{\beta\gamma}] + \eta_{\alpha\beta}^* \partial[\alpha\eta^\beta] + \eta_\alpha^* \partial^\alpha \eta \right), \quad (10)$$

where $\partial[\alpha\eta^\beta] = \partial^\alpha \eta^\beta - \partial^\beta \eta^\alpha$. As (10) possesses the supplementary gauge invariances $\delta\eta^{\alpha\beta} = \partial[\alpha\theta^\beta]$, $\delta\eta^\beta = \partial^\beta \omega$, with θ^β all fermionic and ω bosonic arbitrary functions, it follows [5] the next non-minimal sector (denoted by bar variables and respectively by b's).

$$\begin{array}{ccc}
 A^{\alpha\beta\gamma} & -\eta^{\alpha\beta} & -\eta^\alpha & -\eta \\
 \downarrow & \downarrow & \downarrow & \\
 \bar{\eta}^{\alpha\beta} & \bar{\eta}^\sigma & \bar{\eta} & \\
 \downarrow & \downarrow & & \\
 \bar{\eta}'^\alpha & \bar{\eta}' & & \\
 \downarrow & & & \\
 \bar{\eta}'' & & &
 \end{array}
 \quad
 \begin{array}{ccc}
 b^{\alpha\beta} & \dots & b^\alpha \dots b \\
 \vdots & & \vdots \\
 b'^\alpha & & b' \\
 \vdots & & \vdots \\
 b'' & &
 \end{array}
 \quad (11)$$

The ghost numbers and Grassmann parities of the non-minimal sector fields are as follows

field	$\bar{\eta}^{\alpha\beta}$	$\bar{\eta}^\sigma$	$\bar{\eta}$	$\bar{\eta}'^\alpha$	$\bar{\eta}'$	$\bar{\eta}''$	$b^{\alpha\beta}$	b^α	b	b'^α	b'	b''
gh	-1	-2	-3	0	1	-1	0	-1	-2	1	2	0
ε	1	0	1	0	1	1	0	1	0	1	0	0

In fact, the necessity of introducing the non-minimal sector as in the two pyramids from (11) is implied by the fact that now the ghost fields allow some gauge invariances, as was shown above. The role of the non-minimal fields in the first pyramid is to freeze this gauge freedom. More precisely, for every two fields connected by an arrow, the lower field is freezing the gauge freedom of the upper one.

Under these considerations, the non-minimal solution of the master equation takes the form

$$S_{\text{non min}} = S_{\min} + \int d^4x \left(\bar{\eta}_{\alpha\beta}^* b^{\alpha\beta} + \bar{\eta}_\sigma^* b^\sigma + \bar{\eta}^* b + \bar{\eta}'_\sigma^* b'^\sigma + \bar{\eta}'^* b' + \bar{\eta}''^* b'' \right), \quad (12)$$

where the ghost numbers and Grassmann parities of the antifields associated to the non-minimal sector are given by the generic formulas

$$\text{gh}(\text{antifield}) = -\text{gh}(\text{corresponding field}) - 1; \varepsilon(\text{antifield}) = \varepsilon(\text{corresponding field}) + 1 \pmod{2}.$$

The correct gauge-fixing fermion is

$$\Psi = \int d^4x \left(\bar{\eta}^{\alpha\beta} \partial^\gamma A_{\gamma\alpha\beta} + \bar{\eta}^\sigma \partial^\alpha \eta_{\alpha\sigma} + \bar{\eta} \partial^\sigma \eta_\sigma + \bar{\eta}'^\sigma \partial^\alpha \bar{\eta}_{\alpha\beta} + \bar{\eta}''^\sigma \partial^\alpha \bar{\eta}'_\sigma + \bar{\eta}' \partial^\sigma \bar{\eta}_\sigma \right). \quad (13)$$

This gauge-fixing fermion implements the Lorentz-type gauge conditions $\partial^\alpha A_{\alpha\beta\gamma} = 0$. The remaining terms are introduced in order to fix the gauge freedom of the ghost fields noticed earlier. Eliminating as usually the antifields in (12), it results the next gauge-fixed action

$$\begin{aligned} S_\Psi = S_0^L + \int d^4x & \left(-\frac{1}{3} \partial[\alpha \bar{\eta} \beta \gamma] \partial_{[\alpha} \eta_{\beta\gamma]} - \frac{1}{2} \partial[\alpha \bar{\eta} \beta] \partial_{[\alpha} \eta_{\beta]} - \partial^\alpha \bar{\eta} \partial_\alpha \eta + \right. \\ & + \left(\partial_\gamma A^{\gamma\alpha\beta} - \frac{1}{2} \partial[\alpha \bar{\eta}' \beta] \right) b_{\alpha\beta} + \left(\partial_\alpha \eta^{\alpha\beta} - \partial^\beta \bar{\eta}' \right) b_\beta + \left(\partial_\alpha \bar{\eta}^{\alpha\beta} - \partial^\beta \bar{\eta}'' \right) b'_{\beta} + \\ & \left. + (\partial_\alpha \bar{\eta}^\alpha) b' + (\partial_\alpha \bar{\eta}'^\alpha) b'' + (\partial_\alpha \eta^\alpha) b \right). \end{aligned} \quad (14)$$

One can check that indeed, there are no residual gauge invariances in (14). As one can see, the non-minimal spectrum for reducible theories is complicated in the antifield BRST formalism, while in the BRST-anti-BRST one it appears naturally, as will be further noticed.

3. The antifield BRST-anti-BRST quantization for abelian three-form gauge fields

In the opening of this section we shall briefly present the antifield BRST-anti-BRST formalism for a general second order reducible gauge theory and subsequently apply this formalism to the case of abelian three-form gauge fields. In the sequel we use the notations from [10], [14]. We recall the basic ideas of the BRST-anti-BRST construction: (i) we double the original gauge symmetries and (ii) we introduce a bigrading named new ghost bigrading and denoted by bingh. These ideas are further developed accordingly the standard antifield BRST method.

3. 1. The ghost spectrum

The starting point is given by an original second order reducible gauge theory, described by the quadruple $(S_0^L[q^{\alpha_0}], Z_{\alpha_1}^{\alpha_0}, Z_{\alpha_2}^{\alpha_1}, Z_{\alpha_3}^{\alpha_2})$, where the above indices are valued as $\alpha_i = 1, \dots, M_i, i = 0, 1, 2, 3$. The original variables q^{α_0} are assumed bosonic, $Z_{\alpha_1}^{\alpha_0}$ stand for the gauge generators, while $Z_{\alpha_2}^{\alpha_1}$ and $Z_{\alpha_3}^{\alpha_2}$ give the first, respectively second order reducibility functions.

We double the gauge generators as

$$Z_{A_1}^{\alpha_0} = \begin{pmatrix} Z_{\alpha_1}^{\alpha_0} \\ Z_{\alpha_1}^{\alpha_0} \end{pmatrix}, \quad A_1 = 1, \dots, 2M_1. \quad (15)$$

The new first order reducibility function matrix will be

$$Z_{A_2}^{A_1} = \begin{pmatrix} Z_{\alpha_2}^{\alpha_1} & 0 \\ -\frac{1}{2}Z_{\alpha_2}^{\alpha_1} & -\frac{1}{2}Z_{\alpha_2}^{\alpha_1} \\ 0 & Z_{\alpha_2}^{\alpha_1} \\ -\frac{1}{2}\delta_{\beta_1}^{\alpha_1} & \frac{1}{2}\delta_{\beta_1}^{\alpha_1} \end{pmatrix}, \quad A_2 = 1, \dots, 3M_2 + M_1. \quad (16)$$

The corresponding second and third order reducibility function matrices read

$$Z_{A_3}^{A_2} = \begin{pmatrix} Z_{\alpha_3}^{\alpha_2} & 0 & 0 & 0 \\ -\frac{1}{2}Z_{\alpha_3}^{\alpha_2} & -\frac{1}{2}Z_{\alpha_3}^{\alpha_2} & 0 & 0 \\ 0 & -\frac{1}{2}Z_{\alpha_3}^{\alpha_2} & -\frac{1}{2}Z_{\alpha_3}^{\alpha_2} & 0 \\ 0 & 0 & Z_{\alpha_3}^{\alpha_2} & 0 \\ \delta_{\beta_2}^{\alpha_2} & \delta_{\beta_2}^{\alpha_2} & 0 & \frac{1}{2}Z_{\alpha_2}^{\alpha_1} \\ 0 & \delta_{\beta_2}^{\alpha_2} & \delta_{\beta_2}^{\alpha_2} & \frac{1}{2}Z_{\alpha_2}^{\alpha_1} \end{pmatrix}, \quad A_3 = 1, \dots, 4M_3 + 2M_2, \quad (17)$$

$$Z_{A_4}^{A_3} = \begin{pmatrix} \delta_{\beta_3}^{\alpha_3} & \delta_{\beta_3}^{\alpha_3} & 0 & 0 & \frac{1}{2}Z_{\alpha_3}^{\alpha_2} & 0 \\ 0 & \delta_{\beta_3}^{\alpha_3} & \delta_{\beta_3}^{\alpha_3} & 0 & \frac{1}{2}Z_{\alpha_3}^{\alpha_2} & \frac{1}{2}Z_{\alpha_3}^{\alpha_2} \\ 0 & 0 & \delta_{\beta_3}^{\alpha_3} & \delta_{\beta_3}^{\alpha_3} & 0 & \frac{1}{2}Z_{\alpha_3}^{\alpha_2} \end{pmatrix}, \quad A_4 = 1, \dots, 3M_3. \quad (18)$$

Correspondingly to the matrices (15-18), we introduce the following ghost spectrum

$$\Phi^{A_1} = \left(\begin{pmatrix} (1,0)^{\alpha_1} \\ \eta_1 \end{pmatrix}, \begin{pmatrix} (0,1)^{\alpha_1} \\ \eta_2 \end{pmatrix} \right), \quad (19 \text{ a})$$

$$\Phi^{A_2} = \left(\begin{pmatrix} (2,0)^{\alpha_2} \\ \eta_1 \end{pmatrix}, \begin{pmatrix} (1,1)^{\alpha_2} \\ \eta_2 \end{pmatrix}, \begin{pmatrix} (0,2)^{\alpha_2} \\ \eta_3 \end{pmatrix}, \begin{pmatrix} (1,1)^{\beta_1} \\ \pi \end{pmatrix} \right), \quad (19 \text{ b})$$

$$\Phi^{A_3} = \left(\begin{pmatrix} (3,0)^{\alpha_3} \\ \eta_1 \end{pmatrix}, \begin{pmatrix} (2,1)^{\alpha_3} \\ \eta_2 \end{pmatrix}, \begin{pmatrix} (1,2)^{\alpha_3} \\ \eta_3 \end{pmatrix}, \begin{pmatrix} (0,3)^{\alpha_3} \\ \eta_4 \end{pmatrix}, \begin{pmatrix} (2,1)^{\beta_2} \\ \pi_1 \end{pmatrix}, \begin{pmatrix} (1,2)^{\beta_2} \\ \pi_2 \end{pmatrix} \right), \quad (19 \text{ c})$$

$$\Phi^{A_4} = \left(\begin{pmatrix} (3,1)^{\beta_3} \\ \pi_1 \end{pmatrix}, \begin{pmatrix} (2,2)^{\beta_3} \\ \pi_2 \end{pmatrix}, \begin{pmatrix} (1,3)^{\beta_3} \\ \pi_3 \end{pmatrix} \right), \quad (19 \text{ d})$$

where the superscript (a,b) signifies $\text{bingh} = (\text{ng}h_1, \text{ng}h_2)$. We use the generic notation $\Phi^A \equiv (\Phi^{A_1}, \dots, \Phi^{A_4})$ and call these variables "fields". As can be noticed from (15-18), the original second order theory becomes third order in the antifield BRST-anti-BRST formalism and is described by the 5-uple $\left(S_0^L \begin{bmatrix} (0,0)^{\alpha_0} \\ q \end{bmatrix}, Z_{A_1}^{\alpha_0}, Z_{A_2}^{A_1}, Z_{A_3}^{A_2}, Z_{A_4}^{A_3} \right)$.

3.2. The antifield spectrum

In the antifield BRST-anti-BRST formalism there exist two antibrackets [10], [14], respectively denoted by $(,)_1$ and $(,)_2$, with $\text{bingh}(,)_1 = (1, 0)$ and $\text{bingh}(,)_2 = (0, 1)$. This implies that to every "field" Φ^A one associates a doublet of antifields $(\Phi_A^{*(1)}, \Phi_A^{*(2)})$, respectively conjugated with the "field" in the two antibrackets. The resolution bidegree $\text{bires}=(\text{res}_1, \text{res}_2)$ is given by $(-\text{ngh}_1, -\text{ngh}_2)$ and is non-zero only for antifields. Thus, one has two Koszul-Tate operators, δ_1 and δ_2 , with $\text{bires}(\delta_1)=(-1, 0)$ and $\text{bires}(\delta_2)=(0, -1)$. The new ghost bidegrees and Grassmann parities of these antifields are defined by

$$\text{bingh}\Phi_A^{*(1)} = (-\text{ngh}_1\Phi^A - 1, -\text{ngh}_2\Phi^A), \text{bingh}\Phi_A^{*(2)} = (-\text{ngh}_1\Phi^A, -\text{ngh}_2\Phi^A - 1), \quad (20 \text{ a})$$

$$\varepsilon(\Phi_A^{*(1)}) = \varepsilon(\Phi_A^{*(2)}) = \varepsilon(\Phi^A) + 1 \pmod{2}. \quad (20 \text{ b})$$

One observes that at every resolution degree $\text{res}=\text{res}_1+\text{res}_2$, there appear some non-trivial co-cycles of the form

$$\Phi_A^{*(2)} - \Phi_A^{*(1)}. \quad (21)$$

In order to kill them in the homology of the Koszul-Tate operator, $\delta = \delta_1 + \delta_2$ ($\text{res}(\delta)=-1$), one has to introduce some bar variables equal in number with the Φ^A 's, regarded as antifields and denoted by $\bar{\Phi}_A$. Their characteristics are

$$\text{bingh}\bar{\Phi}_A = (-\text{ngh}_1\Phi^A - 1, -\text{ngh}_2\Phi^A - 1), \varepsilon(\bar{\Phi}_A) = \varepsilon(\Phi^A). \quad (22)$$

To accomplish this killing, it is necessary to extend the action of the Koszul-Tate operator, δ , on the bar variables through an operator V acting only on them by

$$V\bar{\Phi}_A = (-)^{\varepsilon(\Phi^A)} (\Phi_A^{*(2)} - \Phi_A^{*(1)}). \quad (23 \text{ a})$$

The resolution degree of V is minus one and V decomposes into two pieces as

$$V = V_1 + V_2, \quad \text{bires}(V_1) = (-1, 0), \quad \text{bires}(V_2) = (0, -1), \quad (23 \text{ b})$$

respectively contained in δ_1 and δ_2 , such that

$$V_1\Phi_A^{*(1)} = (-)^{\varepsilon(\Phi^A)} \Phi_A^{*(2)}, \quad V_2\Phi_A^{*(2)} = -(-)^{\varepsilon(\Phi^A)} \Phi_A^{*(1)}. \quad (24)$$

Under these circumstances, the antifield spectrum, ordered accordingly the bingh number, for an arbitrary second order reducible theory, is the following

$$\Phi_{a_0}^* = \left(\begin{matrix} (-1,0)^*(1) & (0,-1)^*(2) \\ \eta & \alpha_0 \end{matrix} \right), a_0 = 1, \dots, 2M_0, \quad (25 \text{ a})$$

$$\Phi_{a_1}^* = \left(\begin{matrix} (-2,0)^*(1) & (-1,-1)^*(1) & (-1,-1)^*(2) & (0,-2)^*(2) & (-1,-1) \\ \eta & 1\alpha_1 & \eta & 2\alpha_1 & \bar{\eta} & \beta_0 \end{matrix} \right), \\ a_1 = 1, \dots, 4M_1 + M_0, \quad (25 \text{ b})$$

$$\Phi_{a_2}^* = \left(\begin{matrix} (-3,0)^*(1) & (-2,-1)^*(1) & (-1,-2)^*(1) & (-2,-1)^*(2) & (-1,-2)^*(2) \\ \eta & 1\alpha_2 & \eta & 2\alpha_2 & \eta & 3\alpha_2 & \eta & 1\alpha_2 & \eta & 2\alpha_2, \\ (0,-3)^*(2) & (-2,-1)^*(1) & (-1,-2)^*(2) & (-2,-1) & (-1,-2) \\ \eta & 3\alpha_2 & \pi & \alpha_1 & \pi & \alpha_1 & \bar{\eta} & 1\beta_1 & \bar{\eta} & 2\beta_1 \end{matrix} \right), \\ a_2 = 1, \dots, 6M_2 + 4M_1, \quad (25 \text{ c})$$

$$\Phi_{a_3}^* = \left(\begin{matrix} (-4,0)^*(1) & (-3,-1)^*(1) & (-2,-2)^*(1) & (-1,-3)^*(1) & (-3,-1)^*(2) \\ \eta & 1\alpha_3 & \eta & 2\alpha_3 & \eta & 3\alpha_3 & \eta & 4\alpha_3 & \eta & 1\alpha_3, \\ (-2,-2)^*(2) & (-1,-3)^*(2) & (0,-4)^*(2) & (-3,-1)^*(1) & (-2,-2)^*(1) & (-2,-2)^*(2) \\ \eta & 2\alpha_3 & \eta & 3\alpha_3 & \eta & 4\alpha_3 & \pi & 1\alpha_2 & \pi & 2\alpha_2 & \pi & 1\alpha_2, \\ (-1,-3)^*(2) & (-3,-1) & (-2,-2) & (-1,-3) & (-2,-2) \\ \pi & 2\alpha_2 & \bar{\eta} & 1\beta_2 & \bar{\eta} & 2\beta_2 & \bar{\eta} & 3\beta_2 & \pi & \beta_1 \end{matrix} \right), \\ a_3 = 1, \dots, 8M_3 + 7M_2 + M_1, \quad (25 \text{ d})$$

$$\Phi_{a_4}^* = \left(\begin{matrix} (-4,-1)^*(1) & (-3,-2)^*(1) & (-2,-3)^*(1) & (-3,-2)^*(2) & (-2,-3)^*(2) \\ \pi & 1\alpha_3 & \pi & 2\alpha_3 & \pi & 3\alpha_3 & \pi & 1\alpha_3 & \pi & 2\alpha_3, \\ (-1,-4)^*(2) & (-4,-1) & (-3,-2) & (-2,-3) & (-1,-4) \\ \pi & 3\alpha_3 & \bar{\eta} & 1\beta_3 & \bar{\eta} & 2\beta_3 & \bar{\eta} & 3\beta_3 & \bar{\eta} & 4\beta_3, \\ (-3,-2) & (-2,-3) \\ \bar{\pi} & 1\beta_2 & \bar{\pi} & 2\beta_2 \end{matrix} \right), a_4 = 1, \dots, 10M_3 + 2M_2, \quad (25 \text{ e})$$

$$\Phi_{a_5}^* = \left(\begin{matrix} (-4,-2) & (-3,-3) & (-2,-4) \\ \bar{\pi} & 1\beta_3 & \bar{\pi} & 2\beta_3 & \bar{\pi} & 3\beta_3 \end{matrix} \right), a_5 = 1, \dots, 3M_3. \quad (25 \text{ f})$$

By using purely homological arguments, one can determine the correct action of δ on antifields, and thus of δ_1 and δ_2 on them. However, it is more economical to construct a complete description of the stationary surface $\delta S_0^L / \delta q^{\alpha 0} = 0$, denoted by Σ , to which we shall associate a Koszul-Tate resolution, δ . This is why we are interested in giving the complete

descriptions of Σ to which we associate the Koszul-Tate resolutions δ_1 and δ_2 . These descriptions are respectively given by the new equivalent and redundant equation systems

$$\Phi_{a_0}^{(1)} = \begin{pmatrix} -\frac{\delta S_0^L}{\delta q^{\alpha_0}} \\ 0 \end{pmatrix}, \quad \Phi_{a_0}^{(2)} = \begin{pmatrix} 0 \\ -\frac{\delta S_0^L}{\delta q^{\alpha_0}} \end{pmatrix}. \quad (26 \text{ a-b})$$

The structure of the reducibility (of the equations defining the stationary surface) function matrices $\begin{pmatrix} (b)_{a_{i-1}} \\ Z_{a_i} \end{pmatrix}_{b=1,2;i=1,\dots,5}$ respectively corresponding to the descriptions (26 a-b) is given in Appendix A (formulas (A1-A5)). The concrete form of these matrices was established also taking into account the ordering of the antifields from (25 a-f). Using their form, one computes the first pieces in the action of δ_1 and δ_2 on the antifield sector as

$$\delta_b \Phi_{a_0}^* = \Phi_{a_0}^{(b)}, \quad (27 \text{ a})$$

$$\delta_b \Phi_{a_i}^* = (-)^{i-1} \Phi_{a_{i-1}}^* \begin{pmatrix} (b)_{a_{i-1}} \\ Z_{a_i} \end{pmatrix}. \quad (27 \text{ b})$$

Using (27a-b), (A1-A5) and taking into account as usually that

$$\delta_b \Phi_A^{*(c)} = \left(\Phi_A^{*(c)}, S \right)_b \Big|_{\Phi A=0}, \quad (28)$$

where S is the generator of the BRST-anti-BRST symmetry, one determines completely

$\begin{matrix} (0) & (1) & (2) & (3) & (4) \\ S, & S & \text{and some boundary terms from} & S, & S \text{ and } S \end{matrix}$ of the form

$$\begin{matrix} (0) \\ S = S_0^L, \end{matrix} \quad (29 \text{ a})$$

$$\begin{matrix} (1) \\ S = \begin{pmatrix} (-1,0) \\ q \end{pmatrix} \alpha_0 \begin{pmatrix} (1,0) \\ \eta \end{pmatrix} \beta_1 + \begin{pmatrix} (0,-1) \\ q \end{pmatrix} \alpha_0 \begin{pmatrix} (0,1) \\ \eta \end{pmatrix} \beta_1 \end{matrix}, \quad (29 \text{ b})$$

$$\begin{matrix} (2) \\ S = \begin{pmatrix} (-2,0) \\ \eta \end{pmatrix} \alpha_1 \begin{pmatrix} (2,0) \\ \eta \end{pmatrix} \beta_2 + \frac{1}{2} \left(\begin{pmatrix} (-1,-1) \\ \eta \end{pmatrix} 2\alpha_1 + \begin{pmatrix} (-1,-1) \\ \eta \end{pmatrix} 2\alpha_1 \right) \begin{pmatrix} (1,1) \\ \eta \end{pmatrix} \beta_2 + \\ + \left(\begin{pmatrix} (-1,-1) \\ \bar{q} \end{pmatrix} \alpha_0 \begin{pmatrix} (-1,-1) \\ \eta \end{pmatrix} \beta_1 - \begin{pmatrix} (-1,-1) \\ \eta \end{pmatrix} 2\beta_1 \right) \begin{pmatrix} (1,1) \\ \pi \end{pmatrix} \beta_1 + \begin{pmatrix} (0,-2) \\ \eta \end{pmatrix} \alpha_1 \begin{pmatrix} (0,2) \\ \eta \end{pmatrix} \beta_2 \end{matrix}, \quad (29 \text{ c})$$

$$\begin{matrix} (3) \\ S = \begin{pmatrix} (-3,0) \\ \eta \end{pmatrix} \alpha_2 \begin{pmatrix} (3,0) \\ \eta \end{pmatrix} \beta_3 + \frac{1}{2} \left(\begin{pmatrix} (-2,-1) \\ \eta \end{pmatrix} 2\alpha_2 + \begin{pmatrix} (-2,-1) \\ \eta \end{pmatrix} 1\alpha_2 \right) \begin{pmatrix} (2,1) \\ \eta \end{pmatrix} \beta_3 + \end{matrix}$$

$$\begin{aligned}
& + \frac{1}{2} \left(\begin{pmatrix} (-1, -2)^{(1)} \\ \bar{\eta} \end{pmatrix} {}_3\alpha_2 + \begin{pmatrix} (-1, -2)^{(2)} \\ \bar{\eta} \end{pmatrix} {}_2\alpha_2 \right) Z_{\beta_3}^{\alpha_2} \begin{pmatrix} (1, 2)^{\beta_3} \\ \bar{\eta} \end{pmatrix} {}_3 + \begin{pmatrix} (0, -3)^{(2)} \\ \bar{\eta} \end{pmatrix} {}_3\alpha_2 Z_{\beta_3}^{\alpha_2} \begin{pmatrix} (0, 3)^{\beta_3} \\ \bar{\eta} \end{pmatrix} {}_4 + \\
& + \left[\left(\begin{pmatrix} (-2, -1) \\ \bar{\eta} \end{pmatrix} {}_1\alpha_1 - \frac{1}{2} \begin{pmatrix} (-2, -1)^{(1)} \\ \pi \end{pmatrix} {}_1\alpha_1 \right) Z_{\beta_2}^{\alpha_1} + \begin{pmatrix} (-2, -1)^{(2)} \\ \bar{\eta} \end{pmatrix} {}_1\beta_2 - \begin{pmatrix} (-2, -1)^{(1)} \\ \bar{\eta} \end{pmatrix} {}_2\beta_2 \right] \begin{pmatrix} (2, 1)^{\beta_2} \\ \pi \end{pmatrix} {}_1 + \\
& + \left[\left(\begin{pmatrix} (-1, -2) \\ \bar{\eta} \end{pmatrix} {}_2\alpha_1 - \frac{1}{2} \begin{pmatrix} (-1, -2)^{(2)} \\ \pi \end{pmatrix} {}_1\alpha_1 \right) Z_{\beta_2}^{\alpha_1} + \begin{pmatrix} (-1, -2)^{(2)} \\ \bar{\eta} \end{pmatrix} {}_2\beta_2 - \begin{pmatrix} (-1, -2)^{(1)} \\ \bar{\eta} \end{pmatrix} {}_3\beta_2 \right] \begin{pmatrix} (1, 2)^{\beta_2} \\ \pi \end{pmatrix} {}_2, \quad (29 \text{ d})
\end{aligned}$$

$$\begin{aligned}
S^{(4)} &= \left[\left(\begin{pmatrix} (-3, -1) \\ \bar{\eta} \end{pmatrix} {}_1\alpha_2 - \frac{1}{2} \begin{pmatrix} (-3, -1)^{(1)} \\ \pi \end{pmatrix} {}_1\alpha_2 \right) Z_{\beta_3}^{\alpha_2} + \begin{pmatrix} (-3, -1)^{(2)} \\ \bar{\eta} \end{pmatrix} {}_1\beta_3 - \begin{pmatrix} (-3, -1)^{(1)} \\ \bar{\eta} \end{pmatrix} {}_2\beta_3 \right] \begin{pmatrix} (3, 1)^{\beta_3} \\ \pi \end{pmatrix} {}_1 + \\
& + \left[\left(\begin{pmatrix} (-1, -3) \\ \bar{\eta} \end{pmatrix} {}_3\alpha_2 - \frac{1}{2} \begin{pmatrix} (-1, -3)^{(2)} \\ \pi \end{pmatrix} {}_2\alpha_2 \right) Z_{\beta_3}^{\alpha_2} + \begin{pmatrix} (-1, -3)^{(2)} \\ \bar{\eta} \end{pmatrix} {}_3\beta_3 - \begin{pmatrix} (-1, -3)^{(1)} \\ \bar{\eta} \end{pmatrix} {}_4\beta_3 \right] \begin{pmatrix} (1, 3)^{\beta_3} \\ \pi \end{pmatrix} {}_3 + \\
& + \left[\left(\begin{pmatrix} (-2, -2) \\ \bar{\eta} \end{pmatrix} {}_2\alpha_2 - \frac{1}{2} \begin{pmatrix} (-2, -2)^{(2)} \\ \pi \end{pmatrix} {}_1\alpha_2 - \frac{1}{2} \begin{pmatrix} (-2, -2)^{(1)} \\ \pi \end{pmatrix} {}_2\alpha_2 \right) Z_{\beta_3}^{\alpha_2} + \right. \\
& \left. + \begin{pmatrix} (-2, -2)^{(2)} \\ \bar{\eta} \end{pmatrix} {}_2\beta_3 - \begin{pmatrix} (-2, -2)^{(1)} \\ \bar{\eta} \end{pmatrix} {}_3\beta_3 \right] \begin{pmatrix} (2, 2)^{\beta_3} \\ \pi \end{pmatrix} {}_2. \quad (29 \text{ e})
\end{aligned}$$

In (29 a-e), $\left(\begin{pmatrix} (i) \\ S \end{pmatrix} \right)_{i=1, \dots, 4}$ denotes the first pieces from the decomposition of S via the resolution degree

$$S = \sum_{i \geq 0} \begin{pmatrix} (i) \\ S \end{pmatrix}, \text{ res} \begin{pmatrix} (i) \\ S \end{pmatrix} = i, \text{ bingh} \begin{pmatrix} (i) \\ S \end{pmatrix} = (0, 0), \varepsilon \begin{pmatrix} (i) \\ S \end{pmatrix} = 0. \quad (30)$$

In order to determine the remaining terms in $\left(\begin{pmatrix} (j) \\ S \end{pmatrix} \right)_{j=2, \dots, 4}$, as well as in (30), one has to write down the master equation in the BRST-anti-BRST formalism

$$\frac{1}{2}(S, S) + \bar{V}S = 0, \quad (31)$$

where

$$(\cdot, \cdot) = (\cdot, \cdot)_1 + (\cdot, \cdot)_2, \quad (32)$$

and V is given by (23 a), and resolute it for various resolution degrees.

It is remarkable that in the case of an original second order reducible abelian theory, with the reducibility equations holding off-shell and the reducibility functions not depending of the original variables, the complete solution of the master equation (31) is precisely given by

$$S = \sum_{i=0}^4 \frac{(i)}{S}, \quad (33)$$

where the terms appearing in (33) are respectively given by (29 a-e).

3. 3 Abelian three-form gauge fields

The equivalencies for our model with the theory presented earlier are as follows

$$q^{\alpha 0} \leftrightarrow A^{\alpha\beta\gamma}; Z_{\alpha 1}^{\alpha 0} \leftrightarrow Z_{\mu\nu}^{\alpha\beta\gamma}; Z_{\alpha 2}^{\alpha 1} \leftrightarrow Z_{\sigma}^{\mu\nu}; Z_{\alpha 3}^{\alpha 2} \leftrightarrow Z^{\sigma}, \quad (34)$$

where $Z_{\mu\nu}^{\alpha\beta\gamma}, Z_{\sigma}^{\mu\nu}, Z^{\sigma}$ are respectively given by (4), (7), (8). We are exactly under the conditions mentioned in the end of Sec. 3. 2., so the complete solution of the master equation (31) in the case of our model reads

$$\begin{aligned} S = \int d^4x & \left[\frac{1}{24!} F_{\alpha\beta\gamma\rho} F^{\alpha\beta\gamma\rho} + \frac{(-1,0)^*(1)}{A} \frac{[\alpha \quad (1,0)\beta\gamma]}{\eta_1} \frac{(0,-1)^*(2)}{\partial} \frac{[\alpha \quad (0,1)\beta\gamma]}{\eta_2} + \right. \\ & + \frac{(-2,0)^*(1)}{\eta_1} \frac{[\alpha \quad (2,0)\beta]}{1\alpha\beta} \frac{(0,-2)^*(2)}{\partial} \frac{[\alpha \quad (0,2)\beta]}{\eta_3} + \frac{(-1,-1)}{A} \frac{[\alpha \quad (1,1)\beta\gamma]}{\alpha\beta\gamma} \partial + \\ & + \frac{1}{2} \left(\frac{(-1,-1)^*(1)}{\eta_2} \frac{[\alpha \quad (1,1)\beta]}{2\alpha\beta} + \frac{(-1,-1)^*(2)}{\eta_1} \frac{[\alpha \quad (1,1)\beta]}{1\alpha\beta} \right) \partial \frac{[\alpha \quad (1,1)\beta]}{\eta_2} + \left(\frac{(-1,-1)^*(2)}{\eta_1} \frac{[\alpha \quad (1,1)\beta]}{1\alpha\beta} - \frac{(-1,-1)^*(1)}{\eta_2} \frac{[\alpha \quad (1,1)\beta]}{2\alpha\beta} \right) (1,1)^{\alpha\beta} \pi + \\ & + \frac{1}{2} \left(\frac{(-2,-1)^*(1)}{\eta_2} \frac{[\alpha \quad (2,1)\beta]}{2\alpha} + \frac{(-2,-1)^*(2)}{\eta_1} \frac{[\alpha \quad (2,1)\beta]}{1\alpha} \right) \partial \frac{[\alpha \quad (2,1)\beta]}{\eta_2} + \frac{1}{2} \left(\frac{(-1,-2)^*(1)}{\eta_3} \frac{[\alpha \quad (1,2)\beta]}{3\alpha} + \frac{(-1,-2)^*(2)}{\eta_2} \frac{[\alpha \quad (1,2)\beta]}{2\alpha} \right) \partial \frac{[\alpha \quad (1,2)\beta]}{\eta_3} + \\ & + \frac{(-3,0)^*(1)}{\eta_1} \frac{[\alpha \quad (3,0)\beta]}{1\alpha} \partial \frac{[\alpha \quad (3,0)\beta]}{\eta_1} + \frac{(0,-3)^*(2)}{\eta_3} \frac{[\alpha \quad (0,3)\beta]}{3\alpha} \partial \frac{[\alpha \quad (0,3)\beta]}{\eta_4} + \left(\frac{(-2,-1)^*(2)}{\eta_1} \frac{[\alpha \quad (2,1)\beta]}{1\alpha} - \frac{(-2,-1)^*(1)}{\eta_2} \frac{[\alpha \quad (2,1)\beta]}{2\alpha} \right) (2,1)^{\alpha} \pi_1 + \\ & + \left(\frac{(-2,-1)}{\eta_1} \frac{[\alpha \quad (2,1)\beta]}{1\alpha\beta} - \frac{1}{2} \frac{(-2,-1)^*(1)}{\pi} \frac{[\alpha \quad (2,1)\beta]}{\alpha\beta} \right) \partial \frac{[\alpha \quad (2,1)\beta]}{\pi_1} + \left(\frac{(-1,-2)^*(2)}{\eta_2} \frac{[\alpha \quad (1,2)\beta]}{2\alpha} - \frac{(-1,-2)^*(1)}{\eta_3} \frac{[\alpha \quad (1,2)\beta]}{3\alpha} \right) (1,2)^{\alpha} \pi_2 + \\ & + \left(\frac{(-1,-2)}{\eta_2} \frac{[\alpha \quad (1,2)\beta]}{2\alpha\beta} - \frac{1}{2} \frac{(-1,-2)^*(2)}{\pi} \frac{[\alpha \quad (1,2)\beta]}{\alpha\beta} \right) \partial \frac{[\alpha \quad (1,2)\beta]}{\pi_2} + \left(\frac{(-3,-1)^*(2)}{\eta_1} \frac{[\alpha \quad (3,1)\beta]}{1} - \frac{(-3,-1)^*(1)}{\eta_2} \frac{[\alpha \quad (3,1)\beta]}{2} \right) (3,1)^{\alpha} \pi_1 + \\ & + \left(\frac{(-3,-1)}{\eta_1} \frac{[\alpha \quad (3,1)\beta]}{1\alpha} - \frac{1}{2} \frac{(-3,-1)^*(1)}{\pi} \frac{[\alpha \quad (3,1)\beta]}{1\alpha} \right) \partial \frac{[\alpha \quad (3,1)\beta]}{\pi_1} + \left(\frac{(-2,-2)^*(2)}{\eta_2} \frac{[\alpha \quad (2,2)\beta]}{2} - \frac{(-2,-2)^*(1)}{\eta_3} \frac{[\alpha \quad (2,2)\beta]}{3} \right) (2,2)^{\alpha} \pi_2 + \\ & + \left(\frac{(-1,-3)}{\eta_3} \frac{[\alpha \quad (1,3)\beta]}{3\alpha} - \frac{1}{2} \frac{(-1,-3)^*(2)}{\pi} \frac{[\alpha \quad (1,3)\beta]}{2\alpha} \right) \partial \frac{[\alpha \quad (1,3)\beta]}{\pi_3} + \left(\frac{(-1,-3)^*(2)}{\eta_3} \frac{[\alpha \quad (1,3)\beta]}{3} - \frac{(-1,-3)^*(1)}{\eta_4} \frac{[\alpha \quad (1,3)\beta]}{4} \right) (1,3)^{\alpha} \pi_3 + \end{aligned}$$

$$+ \left(\begin{pmatrix} -2, -2 \\ \bar{\eta} \end{pmatrix}_{2\alpha} - \frac{1}{2} \begin{pmatrix} -2, -2 \\ \pi \end{pmatrix}_{2\alpha}^{*(1)} - \frac{1}{2} \begin{pmatrix} -2, -2 \\ \pi \end{pmatrix}_{1\alpha}^{*(2)} \right) \partial^\alpha \begin{pmatrix} 2, 2 \\ \pi \end{pmatrix}_2 \Bigg]. \quad (35)$$

Accordingly [10], [13-14] regarding the gauge-fixing procedure, we introduce the fields $\mu^{(1)A}$ and the antifields $\rho_{(1)}^B$ respectively conjugated in the first antibracket with the Φ_A 's and $\Phi_A^{*(2)}$'s, such that

$$\left(\mu(x)^{(1)A}, \Phi(y)_B \right)_1 = \delta_B^A \delta^4(x-y), \quad (36 a)$$

$$\left(\Phi(x)_A^{*(2)}, \rho(y)_{(1)}^B \right)_1 = \delta_A^B \delta^4(x-y), \quad (36 b)$$

and construct the functional

$$S_1 = S + \int d^4x \left(\mu^{(1)A} \Phi_A^{*(2)} \right), \quad (37)$$

which is the solution of the master equation $(S_1, S_1)_1 = 0$ [10], [14]. Then, the gauge-fixed action will be

$$S_\Psi = S_1 \left[\Phi^A, \Phi_A^{*(2)}, \mu^{(1)A}, \Phi^{*(1)A} = \frac{\overleftarrow{\delta} \Psi}{\delta \Phi^A}, \Phi_A = \frac{\overleftarrow{\delta} \Psi}{\delta \mu^{(1)A}}, \rho_{(1)}^A = \frac{\overleftarrow{\delta} \Psi}{\delta \Phi_A^{*(2)}} \right], \quad (38)$$

where Ψ is the gauge-fixing fermion, which we take of the form

$$\Psi = \int d^4x \left(\mu_{(1)}^A \frac{\overleftarrow{\delta} F}{\delta \Phi^A} \right), \quad (39)$$

with $F[\Phi^A]$ a bosonic functional depending only on the "fields".

In the case of abelian three-form gauge fields we choose $F[\Phi^A]$ of the form

$$F = \int d^4x \left(-\frac{1}{6} A_{\alpha\beta\gamma} A^{\alpha\beta\gamma} + \frac{1}{2} \eta_1 \alpha_\beta \eta_1^{\alpha\beta} + \frac{1}{2} \eta_2 \alpha_\beta \eta_2^{\alpha\beta} + \frac{1}{2} \eta_2 \alpha \eta_2^\alpha + \eta_1 \alpha \eta_2^\alpha \right). \quad (40)$$

The corresponding gauge-fixed action, after eliminating all the variables on their equations of motion [40] excepting the Φ^A 's, reads

$$\begin{aligned} S_\Psi = S_0^L[A^{\alpha\beta\gamma}] + \int d^4x & \left[-\frac{1}{3} \partial^{[\alpha} \eta_1^{\beta\gamma]} \hat{c}_{[\alpha} \eta_2 \beta \gamma] + \left(\partial^\alpha A_{\alpha\beta\gamma} + \partial[\beta(\eta_3 - \eta_1)_{\gamma]} \right) \pi^{\alpha\beta} - \right. \\ & \left. - \frac{1}{2} \partial^{[\alpha} \eta_2^{\beta]} \partial_{[\alpha} (\eta_3 + \eta_1)_{\beta]} - \partial^\alpha \eta_4 \partial_\alpha \eta_1 + (\partial_\beta \eta_3 - 2\partial^\alpha \eta_1 \alpha_\beta) \pi_1^\beta - \partial^\alpha \eta_2 \alpha \pi_2 + \right. \end{aligned}$$

$$+\left(\partial_\beta \eta_2 - 2\partial^\alpha \eta_{2\alpha\beta}\right)\pi_2^\beta + \frac{1}{2}\partial^\alpha(\eta_1 - \eta_3)_\alpha(\pi_1 - \pi_3) - \frac{1}{2}\partial^\alpha(\eta_1 + \eta_3)_\alpha(\pi_1 + \pi_3)\Bigg] . \quad (41)$$

It is clear that action (41) is identical to action (14) modulo the identifications

$$\begin{array}{lllll} \eta_1^{\alpha\beta} = \eta^{\alpha\beta} & -\eta_2^{\alpha\beta} = \bar{\eta}^{\alpha\beta} & \eta_2^\alpha = \eta^\alpha & \eta_1 = \eta & \eta_4 = \bar{\eta} \\ (\eta_1 + \eta_3)^\alpha = \bar{\eta}^\alpha & 2(\eta_1 - \eta_3)^\alpha = \bar{\eta}'^\alpha & \frac{1}{2}\eta_3 = \bar{\eta}' & -\frac{1}{2}\eta_2 = \bar{\eta}'' & -2\pi_1^\alpha = b^\alpha \\ -2(\pi_1 + \pi_3) = b' & \pi_1 - \pi_3 = b'' & -\pi_2 = b & \pi^{\alpha\beta} = b^{\alpha\beta} & 2\pi_2^\alpha = b'^\alpha \end{array}$$

From (41) and the above table one can see that indeed the non-minimal sector from the BRST method is automatically incorporated within the BRST-anti-BRST one.

4. Massive abelian three-form gauge fields

The starting point of this section is the Lagrangian action [38]

$$S_{0M}^L[A^{\alpha\beta\gamma}] = \int d^4x \left(-\frac{1}{2 \cdot 4!} F_{\alpha\beta\gamma\rho} F^{\alpha\beta\gamma\rho} - \frac{M^2}{2 \cdot 3!} A_{\alpha\beta\gamma} A^{\alpha\beta\gamma} \right) . \quad (42)$$

The canonical analysis of this system furnishes the canonical Hamiltonian

$$H = \int d^3x \left(3\pi_{ijk}\pi^{ijk} - 3A_{0jk}\partial_i\pi^{ijk} + \frac{M^2}{2 \cdot 3!} A_{\alpha\beta\gamma} A^{\alpha\beta\gamma} \right) \equiv \int d^3x h , \quad (43)$$

together with the second-class constraints

$$G^{ij} \equiv \pi^{0ij} = 0 \quad (\text{primary}) , \quad (44 \text{ a})$$

$$C^{ij} \equiv 3\hat{c}_k\pi^{kij} - \frac{M^2}{2} A^{0ij} \quad (\text{secondary}) . \quad (44 \text{ b})$$

We observe that

$$[C^{ij}, G_{kl}] \equiv \Delta^{ij}_{kl} = -\frac{M^2}{4} (\delta_k^i \delta_l^j - \delta_l^i \delta_k^j) . \quad (45 \text{ a})$$

Denoting by $(\Delta^{-1})^{ij}_{kl}$ the inverse of Δ^{ij}_{kl} , we get

$$(\Delta^{-1})^{ij}_{kl} = -\frac{2}{M^2} \Delta^{ij}_{kl} . \quad (45 \text{ b})$$

4. 1. Conversion to an equivalent first-class system

The above constraints being purely second-class, we cannot quantize our massive model directly in the BRST or BRST-anti-BRST formalisms. In order to quantize it in such ways, it is necessary to implement some gauge invariances transforming the original constraints into some first-class ones.

In this section we present a method different from [39-40] of converting the prior second-class constraints into some first-class ones. Our procedure consists of two steps. Firstly, we associate to the original system a first-class one in the same original phase-space, described by the Hamiltonian [41]

$$\bar{H} = \int d^3x \left(3\pi_{ijk}\pi_{ijk} + \frac{M^2}{2 \cdot 3!} A_{ijk}A^{ijk} \right) = \int d^3x \bar{h}, \quad (46)$$

and possessing the sole and primary constraints (44 a), such that $[G^{ij}, \bar{H}] = 0$ strongly.

Secondly, we associate to this first-class system a one-parameter family of first-class systems in a larger phase-space obtained from the original one by adding some extradegrees of freedom. The extension of the original phase-space goes as follows.

(i) For every pair (G^{ij}, C_{ij}) we introduce a bosonic canonical pair (q^{ij}, p_{ij}) , with the new fields antisymmetric in their indices, such that the new secondary constraints to be

$$\gamma_{ij} \equiv \lambda p_{ij} - 3\partial^k \pi_{kij} = 0, \quad (47)$$

where λ is the parameter of the first-class family. It is simply to check that G^{ij} and γ_{ij} are first-class and

$$\partial^i \gamma_{ij} = \lambda \partial^i p_{ij} = 0, \quad (48)$$

(ii) For every relation (48) we introduce the new canonical pairs (q^{0i}, p_{0i}) and the constraints $\gamma_j \equiv p_{0j} = 0$, such that their consistency to imply the constraints (48). In this way we associated to the original system a one-parameter family of first-class systems with the first class constraints (44 a), (47) and

$$\bar{\gamma}_j \equiv -\partial^i p_{ij} = 0, \quad \gamma_j \equiv p_{0j} = 0. \quad (49 \text{ a-b})$$

Now, the first-class constraints become reducible

$$\lambda \bar{\gamma}_j + \partial^i \gamma_{ij} = 0. \quad (50)$$

The first-class Hamiltonian of the first-class family is built as follows

$$H^* = \bar{H} + \int d^3x \left(\frac{\lambda^2}{2} (\Delta^{-1})^{ij} p_{ij} p^{kl} - \frac{M^2}{2} (\Delta^{-1})^{ij} A^{0kl} \gamma_{ij} - 2q^{0j} \partial^i p_{ij} + g \right), \quad (51)$$

where g is taken such that

$$[\gamma_i, g] = 0, [\bar{\gamma}_i, g] = 0, [\gamma_{ij}, g] = 0, [G^{ij}, g] = 0. \quad (52 \text{ a-d})$$

The solution of the system (52) is obtained in a simple manner. We represent g as a series in q^{ij} 's, with some coefficients depending only on A^{ijk} 's. With this representation, (52 a) and (52 d) are automatically fulfilled. Developing g as

$$g = g_{ij}^{(1)} q^{ij} + g_{ij\,kl}^{(2)} q^{ij} q^{kl} + \dots, \quad (53)$$

and introducing (53) in (52 c), we obtain

$$g_{ij}^{(1)} = \frac{M^2}{2\lambda} \partial^k A_{kij}, \quad g_{ij\,kl}^{(2)} = -\frac{3}{2\lambda} \left[\partial^{i1} \pi_{i1\,ij}, g_{kl}^{(1)} \right], \quad (54)$$

all the coefficients $g_{i_1 j_1 \dots i_n j_n}^{(n)}$, with $n > 2$ vanishing. Replacing (53-54) in (52 b), the last equation is indeed verified. Using (53-54) and (51), we obtain the first-class Hamiltonian of the first-class family of the form

$$H^* = \bar{H} + \int d^3x \left(-\lambda^2 p_{ij} p^{ij} + A^{0ij} \gamma_{ij} - 2q^{0j} \partial^i p_{ij} + \right. \\ \left. + \frac{M^2}{3! \lambda^2} \left(\frac{1}{2} \partial [i q_{jk}] \partial [i q_{jk}] - A^{ijk} \partial [i q_{jk}] \right) \right) \equiv \int d^3x h^*. \quad (55)$$

The extended action of the first-class family reads

$$S_0^E = \int d^4x \left(\pi_{0ij} \dot{A}^{0ij} + \pi_{kij} \dot{A}^{kij} + p_{0i} \dot{q}^{0i} + p_{ij} \dot{q}^{ij} - h^* - u_{ij} G^{ij} - u^j \gamma_j - v^j \bar{\gamma}_j - v^{ij} \gamma_{ij} \right). \quad (56)$$

Eliminating from the total action corresponding to (56) $(v^j = 0, v^{ij} = 0)$ the remaining multipliers and the momenta on their equations of motion [42], we obtain the Lagrangian action of the first-class family

$$S_0^L[A^{\alpha\beta\gamma}, q^{\alpha\beta}] = S_0^L[A^{\alpha\beta\gamma}] - \int d^4x \frac{M^2}{2 \cdot 3!} \left(A^{\alpha\beta\gamma} - \frac{1}{\lambda} F^{\alpha\beta\gamma} \right) \left(A_{\alpha\beta\gamma} - \frac{1}{\lambda} F_{\alpha\beta\gamma} \right), \quad (57)$$

where $q^{\alpha\beta} = (q^{0j}, q^{ij})$ and $F^{\alpha\beta\gamma} = \partial[\alpha q^{\beta\gamma}]$. This action is invariant under the gauge transformations

$$\delta_\varepsilon A^{\alpha\beta\gamma} = \partial[\alpha_\varepsilon \beta\gamma], \quad \delta_\varepsilon q^{\alpha\beta} = \partial[\alpha_\varepsilon \beta] + \lambda_\varepsilon \alpha^\beta, \quad (58 \text{ a-b})$$

where $\varepsilon^{\alpha\beta}$'s and ε^α 's are arbitrary functions. If we put all the ε 's constant in (58 a-b), we get the Noetherian symmetries $\delta_\varepsilon A_{\alpha\beta\gamma} = 0$ and $\delta_\varepsilon q_{\alpha\beta} = \lambda_\varepsilon \alpha_{\alpha\beta}$, leading via Noether's theorem to the conserved current

$$j_{\alpha\beta}^\gamma = \left(A_{\alpha\beta}^\gamma - \frac{1}{\lambda} F_{\alpha\beta}^\gamma \right). \quad (59)$$

Now, it is clear that action (57) describes abelian three-form gauge fields interacting with abelian two-form gauge fields through a mixing-component term of the type current-current, where the current is given by (59) and is gauge invariant under the transformations (58 a-b).

4. 2. The BRST quantization of the first-class family

Let's quantize in the antifield BRST formalism the system described by action (57). The gauge transformations (58 a-b) are second order reducible, the gauge generator and first and second order reducibility matrices respectively taking the form

$$Z_{\alpha_1}^{\alpha_0} = \begin{pmatrix} Z_{\mu\nu}^{\alpha\beta\gamma} & 0 \\ \frac{\lambda}{2}(\delta_\mu^\alpha \delta_\nu^\beta - \delta_\nu^\alpha \delta_\mu^\beta) & Z_\mu^{\alpha\beta} \end{pmatrix}, \quad (60 \text{ a})$$

$$Z_{\alpha_2}^{\alpha_1} = \begin{pmatrix} Z_\rho^{\mu\nu} & 0 \\ -\lambda \delta_\rho^\mu & Z^\mu \end{pmatrix}, \quad (60 \text{ b})$$

$$Z_{\alpha_3}^{\alpha_2} = \begin{pmatrix} Z^\rho \\ \lambda \end{pmatrix}. \quad (60 \text{ c})$$

We observe that if $\lambda = 0$, then relations (60 a-c) furnish exactly the corresponding gauge generator and reducibility matrices for an action equal with the sum between the free abelian two and three-form gauge field actions. This action may be inferred from (57) in the limit $\lambda \rightarrow 0$ when $\lambda = M$. Thus, we fixed the correct value of the parameter ($\lambda = M$). From (60 a-c) it results the minimal ghost respectively antifield spectra

ghost	$\eta^{\mu\nu}$	C^μ	η^μ	C	η
gh	1	1	2	2	3
ε	1	1	0	0	1

antifield	$A_{\alpha\beta\gamma}^*$	$q_{\alpha\beta}^*$	$\eta_{\mu\nu}^*$	C_μ^*	η_μ^*	C^*	η^*
gh	-1	-1	-2	-2	-3	-3	-4
ε	1	1	0	0	1	1	0

The minimal solution of the master equation reads

$$\begin{aligned} S_{\min} = & \int d^4x \left(-\frac{1}{2 \cdot 4!} F_{\alpha\beta\gamma\rho} F^{\alpha\beta\gamma\rho} - \frac{1}{2 \cdot 3!} (M A^{\alpha\beta\gamma} - F^{\alpha\beta\gamma}) (M A_{\alpha\beta\gamma} - F_{\alpha\beta\gamma}) + \right. \\ & + A_{\alpha\beta\gamma}^* \partial^\alpha [\eta^{\beta\gamma}] + q_{\alpha\beta}^* \partial^\alpha [C^\beta] + M q_{\alpha\beta}^* \eta^{\alpha\beta} + \eta_{\alpha\beta}^* \partial^\alpha [\eta^{\beta\gamma}] + C_\alpha^* \partial^\alpha C - M C_\alpha^* \eta^\alpha + \\ & \left. + \eta_\alpha^* \partial^\alpha \eta + M C^* \eta \right). \end{aligned} \quad (61)$$

We notice that $M \rightarrow 0$ in (61) induces in S the sum between the solutions of the master equations for free abelian three and two-form gauge fields, as expected. Because (61) has the additional gauge invariances $\delta_\theta \eta^{\alpha\beta} = -\frac{1}{M} \partial[\alpha\theta\beta]$, $\delta_\theta C^\alpha = \theta^\alpha$ and $\delta_\omega \eta^\alpha = \frac{1}{M} \partial^\alpha \omega$, $\delta_\omega C = \omega$, with the fermions θ^α and the boson ω arbitrary functions, it results [5] the next non-minimal sector

$$\left(A^{\alpha\beta\gamma, q\alpha\beta} \right)_{\bar{\eta}^{\alpha\beta}} - \left(\eta^{\alpha\beta}, C^\alpha \right)_{\bar{\eta}^\alpha} - \left(\eta^\alpha, C \right)_{\bar{\eta}} - \eta_{b\alpha\beta \dots b^\alpha \dots b} \quad (62)$$

The components of the non-minimal sector (62) coincide with the corresponding ones from Sec. 2. Thus, the non-minimal solution of the master equation takes the form

$$S_{\text{nonmin}} = S_{\text{min}} + \int d^4x \left(\bar{\eta}_{\alpha\beta}^* b^{\alpha\beta} + \bar{\eta}_\alpha^* b^\alpha + \bar{\eta}^* b \right). \quad (63)$$

If one takes $q^{\alpha\beta} = 0$ in (57), one reobtains the original massive theory. These conditions together with the additional gauge invariances of (61) imply the gauge-fixing fermion

$$\Psi = \int d^4x \left(\bar{\eta}^{\alpha\beta} q_{\alpha\beta} + \bar{\eta}^\alpha C_\alpha + \bar{\eta} C \right). \quad (64)$$

The gauge-fixed action will be

$$S_\Psi = S_0^L \left[A^{\alpha\beta\gamma, q\alpha\beta} \right] + \int d^4x \left[\bar{\eta}_{\alpha\beta} \left(\partial[\alpha C\beta] + M \eta^{\alpha\beta} \right) + \bar{\eta}_\alpha (\partial^\alpha C - M \eta^\alpha) + M \bar{\eta} \eta + q^{\alpha\beta} b_{\alpha\beta} + C^{\alpha\beta} b_\alpha + C b \right]. \quad (65)$$

The path integral correspondent to (65), which is actually the Lagrangian path integral of the massive abelian three-form gauge fields after integration over all the variables excepting the $A^{\alpha\beta\gamma}$'s, is expressed by

$$Z_\Psi = \int DA^{\alpha\beta\gamma} \det(M) \exp \left(i S_{0M}^L \left[A^{\alpha\beta\gamma} \right] \right). \quad (66)$$

Formula (66) can be derived starting from the Hamiltonian path integral of the original system deduced with the aid of general canonical methods [5], [43]. This ends the antifield BRST treatment of massive abelian three-form gauge fields.

4. 3. The BRST-anti-BRST quantization of the first-class family

In this section we shall quantize the system described by action (57) in the antifield BRST-anti-BRST formalism. We are under the conditions from the end of Sec. 3. 2., i.e. the model described by action (57) is off-shell second order reducible, abelian and the reducibility function matrices are field independent. The ghost spectrum (19 a-b) becomes in our case

$$\Phi^{A_1} = \left(\begin{matrix} (1,0)^{\alpha\beta} & (1,0)^\alpha & (0,1)^{\alpha\beta} & (0,1)^\alpha \\ \eta_1 & C_1 & \eta_2 & C_2 \end{matrix} \right), \quad (67 \text{ a})$$

$$\Phi^{A_2} = \left(\begin{matrix} (2,0)^\alpha & (2,0) & (1,1)^\alpha & (1,1) & (0,2)^\alpha & (0,2) & (1,1)^{\alpha\beta} & (1,1)^\alpha \\ \eta_1 & C_1 & \eta_2 & C_2 & \eta_3 & C_3 & \pi & \Pi \end{matrix} \right), \quad (67 \text{ b})$$

$$\Phi^{A_3} = \left(\begin{matrix} (3,0) & (2,1) & (1,2) & (0,3) & (2,1)^\alpha & (2,1) & (1,2)^\alpha & (1,2) \\ \eta_1 & \eta_2 & \eta_3 & \eta_4 & \pi_1 & \Pi_1 & \pi_2 & \Pi_2 \end{matrix} \right), \quad (67 \text{ c})$$

$$\Phi^{A_4} = \left(\begin{matrix} (3,1) & (2,2) & (1,3) \\ \pi_1 & \pi_2 & \pi_3 \end{matrix} \right). \quad (67 \text{ d})$$

The antifield spectrum will result from the reducibility function matrices (A1-A5), where $Z_{\alpha_1}^{\alpha_0}, Z_{\alpha_2}^{\alpha_1}, Z_{\alpha_3}^{\alpha_2}$ are respectively given by (60 a-c) and is given by

$$\Phi_{a_0}^* = \left(\begin{matrix} (-1,0)^*(1) & (-1,0)^*(1) & (0,-1)^*(2) & (0,-1)^*(2) \\ A & \alpha\beta\gamma & q & \alpha\beta \end{matrix} \right), \quad (68 \text{ a})$$

$$\Phi_{a_1}^* = \left(\begin{matrix} (-2,0)^*(1) & (-2,0)^*(1) & (-1,-1)^*(1) & (-1,-1)^*(1) & (-1,-1)^*(2) & (-1,-1)^*(2) \\ \eta_1 & \alpha\beta & C_1 & \alpha & \eta_2 & \alpha\beta & C_2 & \alpha \\ (0,-2)^*(2) & (0,-2)^*(2) & (-1,-1) & (-1,-1) \\ \eta_2 & \alpha\beta & C_2 & \alpha & \bar{A} & \alpha\beta\gamma & \bar{q} & \alpha\beta \end{matrix} \right), \quad (68 \text{ b})$$

$$\Phi_{a_2}^* = \left(\begin{matrix} (-3,0)^*(1) & (-3,0)^*(1) & (-2,-1)^*(1) & (-2,-1)^*(1) & (-1,-2)^*(1) & (-1,-2)^*(1) \\ \eta_1 & \alpha & C_1 & & \eta_2 & \alpha & C_2 & & \eta_3 & \alpha & C_3 & \\ (-2,-1)^*(2) & (-2,-1)^*(2) & (-1,-2)^*(2) & (-1,-2)^*(2) & (0,-3)^*(2) & (0,-3)^*(2) \\ \eta_1 & \alpha & C_1 & & \eta_2 & \alpha & C_2 & & \eta_3 & \alpha & C_3 & \\ (-2,-1)^*(1) & (-2,-1)^*(1) & (-1,-2)^*(2) & (-1,-2)^*(2) & (-2,-1) & (-2,-1) \\ \pi & \alpha\beta & \Pi & \alpha & \pi & \alpha\beta & \Pi & \alpha & \eta_1 & \alpha\beta & \bar{C} & \alpha \\ (-1,-2) & (-1,-2) \\ \eta_2 & \alpha\beta & \bar{C} & 2\alpha \end{matrix} \right), \quad (68 \text{ c})$$

$$\Phi_{a_3}^* = \left(\begin{matrix} (-4,0)^*(1) & (-3,-1)^*(1) & (-2,-2)^*(1) & (-1,-3)^*(1) & (-3,-1)^*(2) & (-2,-2)^*(2) \\ \eta_1 & & \eta_2 & & \eta_3 & & \eta_4 & & \eta_1 & & \eta_2 & \\ (-1,-3)^*(2) & (0,-4)^*(2) & (-3,-1)^*(1) & (-3,-1)^*(1) & (-2,-2)^*(1) & (-2,-2)^*(1) \\ \eta_3 & & \eta_4 & & \pi & \alpha & \Pi & \alpha & \pi & \alpha & \Pi & \alpha \end{matrix} \right),$$

$$\begin{aligned}
& \begin{pmatrix} (-2,-2)^*(2) & (-2,-2)^*(2) & (-1,-3)^*(2) & (-1,-3)^*(2) & (-3,-1) & (-3,-1) & (-2,-2) \\ \pi_{1\alpha} & \bar{\Pi}_1 & \pi_{2\alpha} & \bar{\Pi}_2 & \bar{\eta}_{1\alpha} & \bar{C}_1 & \bar{\eta}_{2\alpha} \end{pmatrix}, \\
& \begin{pmatrix} (-2,-2) & (-1,-3) & (-1,-3) & (-2,-2) & (-2,-2) \\ \bar{C}_2 & \bar{\eta}_{3\alpha} & \bar{C}_3 & \pi_{\alpha\beta} & \bar{\Pi}_\alpha \end{pmatrix}, \tag{68 d}
\end{aligned}$$

$$\begin{aligned}
\Phi_{a_4}^* = & \begin{pmatrix} (-4,-1)^*(1) & (-3,-2)^*(1) & (-2,-3)^*(1) & (-3,-2)^*(2) & (-2,-3)^*(2) \\ \pi_1 & \pi_2 & \pi_3 & \pi_1 & \pi_2 \end{pmatrix}, \\
& \begin{pmatrix} (-1,-4)^*(2) & (-4,-1) & (-3,-2) & (-2,-3) & (-1,-4) & (-3,-2) \\ \pi_3 & \bar{\eta}_1 & \bar{\eta}_2 & \bar{\eta}_3 & \bar{\eta}_4 & \pi_{1\alpha} \end{pmatrix}, \\
& \begin{pmatrix} (-3,-2) & (-2,-3) & (-2,-3) \\ \bar{\Pi}_1 & \pi_{2\alpha} & \bar{\Pi}_2 \end{pmatrix}, \tag{68 e}
\end{aligned}$$

$$\Phi_{a_5}^* = \begin{pmatrix} (-4,-2) & (-3,-3) & (-2,-4) \\ \pi_1 & \pi_2 & \pi_3 \end{pmatrix}. \tag{68 f}$$

The solution of the master equation (31) now reads

$$\begin{aligned}
\bar{S} = & S + \int d^4x \left[-\frac{1}{2 \cdot 3!} \left(M_{\alpha\beta\gamma}^{(0,0)} - \bar{F}_{\alpha\beta\gamma}^{(0,0)} \right) \left(M_{\alpha\beta\gamma}^{(0,0)} - \bar{F}_{\alpha\beta\gamma}^{(0,0)} \right) \right] + \\
& + \frac{(-1,0)^*(1)}{q_{\alpha\beta}} \left(M_{\alpha\beta}^{(1,0)} + \partial^\alpha \bar{C}_1^{(1,0)\beta} \right) + \frac{(0,-1)^*(2)}{q_{\alpha\beta}} \left(M_{\alpha\beta}^{(0,1)} + \partial^\alpha \bar{C}_2^{(0,1)\beta} \right) + \\
& + \frac{(-2,0)^*(1)}{C_{1\alpha}} \left(-M_{\alpha}^{(2,0)} + \partial^\alpha \bar{C}_1^{(2,0)} \right) + \frac{(0,-2)^*(2)}{C_{2\alpha}} \left(-M_{\alpha}^{(0,2)} + \partial^\alpha \bar{C}_3^{(0,2)} \right) + \\
& + \frac{1}{2} \left(\frac{(-1,-1)^*(1)}{C_{2\alpha}} + \frac{(-1,-1)^*(2)}{C_{1\alpha}} \right) \left(-M_{\alpha}^{(1,1)} + \partial^\alpha \bar{C}_2^{(1,1)} \right) + \frac{(-1,-1)}{q_{\alpha\beta}} \left(M_{\alpha\beta}^{(1,1)} + \right. \\
& \left. + \partial^\alpha \bar{\Pi}^{(1,1)\beta} \right) + \left(\frac{(-1,-1)^*(2)}{C_{1\alpha}} - \frac{(-1,-1)^*(1)}{C_{2\alpha}} \right) (1,1)^\alpha + M \left(\frac{(-3,0)^*(1)}{C_1} \bar{\eta}_1^{(3,0)} + \right. \\
& \left. + \frac{(0,-3)^*(2)}{C_3} \bar{\eta}_4^{(0,3)} \right) + \frac{M}{2} \left(\left(\frac{(-2,-1)^*(1)}{C_2} + \frac{(-2,-1)^*(2)}{C_1} \right) \bar{\eta}_2^{(2,1)} + \left(\frac{(-1,-2)^*(1)}{C_3} + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& + \left(\begin{smallmatrix} (-1,-2)^*(2) \\ \bar{C} \end{smallmatrix} \begin{smallmatrix} (1,2) \\ \eta \end{smallmatrix} \begin{smallmatrix} 2 \\ 3 \end{smallmatrix} \right) + \left(\begin{smallmatrix} (-2,-1) \\ \bar{C} \end{smallmatrix} \begin{smallmatrix} 1\alpha \\ -\frac{1}{2} \end{smallmatrix} \begin{smallmatrix} (-2,-1)^*(1) \\ \bar{\Pi} \end{smallmatrix} \begin{smallmatrix} \alpha \\ \alpha \end{smallmatrix} \right) \left(-M \begin{smallmatrix} (2,1)\alpha \\ \pi \end{smallmatrix} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} + \partial \begin{smallmatrix} \alpha \\ \bar{\Pi} \end{smallmatrix} \begin{smallmatrix} (2,1) \\ 1 \end{smallmatrix} \right) + \\
& + \left(\begin{smallmatrix} (-1,-2) \\ \bar{C} \end{smallmatrix} \begin{smallmatrix} 2\alpha \\ -\frac{1}{2} \end{smallmatrix} \begin{smallmatrix} (-1,-2)^*(2) \\ \bar{\Pi} \end{smallmatrix} \begin{smallmatrix} \alpha \\ \alpha \end{smallmatrix} \right) \left(-M \begin{smallmatrix} (1,2)\alpha \\ \pi \end{smallmatrix} \begin{smallmatrix} 2 \\ 2 \end{smallmatrix} + \partial \begin{smallmatrix} \alpha \\ \bar{\Pi} \end{smallmatrix} \begin{smallmatrix} (1,2) \\ 2 \end{smallmatrix} \right) + \left(\begin{smallmatrix} (-2,-1)^*(2) \\ \bar{C} \end{smallmatrix} \begin{smallmatrix} 1 \\ - \end{smallmatrix} \right. \\
& \left. - \begin{smallmatrix} (-2,-1)^*(1) \\ \bar{C} \end{smallmatrix} \begin{smallmatrix} (2,1) \\ \bar{\Pi} \end{smallmatrix} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} + \left(\begin{smallmatrix} (-1,-2)^*(2) \\ \bar{C} \end{smallmatrix} \begin{smallmatrix} 2 \\ - \end{smallmatrix} \begin{smallmatrix} (-1,-2)^*(1) \\ \bar{C} \end{smallmatrix} \begin{smallmatrix} 3 \end{smallmatrix} \right) \begin{smallmatrix} (1,2) \\ \bar{\Pi} \end{smallmatrix} \begin{smallmatrix} 2 \\ 2 \end{smallmatrix} + M \left(\begin{smallmatrix} (-3,-1) \\ \bar{C} \end{smallmatrix} \begin{smallmatrix} 1 \\ - \end{smallmatrix} \right. \right. \\
& \left. \left. - \frac{1}{2} \begin{smallmatrix} (-3,-1)^*(1) \\ \bar{\Pi} \end{smallmatrix} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} \right) \begin{smallmatrix} (3,1) \\ \pi \end{smallmatrix} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} + M \left(\begin{smallmatrix} (-1,-3) \\ \bar{C} \end{smallmatrix} \begin{smallmatrix} 3 \\ -\frac{1}{2} \end{smallmatrix} \begin{smallmatrix} (-1,-3)^*(2) \\ \bar{\Pi} \end{smallmatrix} \begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \right) \begin{smallmatrix} (1,3) \\ \pi \end{smallmatrix} \begin{smallmatrix} 3 \\ 3 \end{smallmatrix} + \right. \\
& \left. + M \left(\begin{smallmatrix} (-2,-2) \\ \bar{C} \end{smallmatrix} \begin{smallmatrix} 2 \\ -\frac{1}{2} \end{smallmatrix} \begin{smallmatrix} (-2,-2)^*(1) \\ \bar{\Pi} \end{smallmatrix} \begin{smallmatrix} 2 \\ 2 \end{smallmatrix} - \frac{1}{2} \begin{smallmatrix} (-2,-2)^*(2) \\ \bar{\Pi} \end{smallmatrix} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} \right) \begin{smallmatrix} (2,2) \\ \pi \end{smallmatrix} \begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \right), \tag{69}
\end{aligned}$$

where S is given in (35). We remark that solution (69) preserves the same feature as (57), namely when $M \rightarrow 0$ one recovers the sum between the solutions of the master equations for free abelian two and three-form gauge fields.

In order to obtain the gauge-fixed action for our model in this formalism we act as in Sec. 3.3. (see (37-39)). Taking the gauge-fixing boson

$$F = \frac{1}{2M} \int d^4x \left(q_{\alpha\beta} q^{\alpha\beta} + C_1 \alpha C_1^\alpha + C_2 \alpha C_2^\alpha + C_1 C_1 + C_2 C_2 + C_3 C_3 \right), \tag{70}$$

and integrating in the path integral corresponding to (69) over all the variables except $A^{\alpha\beta\gamma}$, we reobtain precisely (66).

At this point we are able to make clear the meaning of the original system preserving somehow the reducibility of a certain first-class system. It is obvious that the original action of the massive theory (42) results from the second-order reducible system (57) (with $\lambda = M$). This conclusion also holds at the path integral level. More precisely, the path integral of the massive theory is derived through the quantization of a reducible theory with a gauge-fixing fermion implementing the canonical gauge conditions $q_{\alpha\beta} = 0$ (see (64) and (70)).

The approach presented in this section can be generalized to the case of massive abelian p -forms gauge fields in D dimensions. Acting as in Sec. 4.1., the quantization (BRST or BRST-anti-BRST) of massive p -form gauge fields reduces to the quantization of the $(p-1)$ -order reducible system

$$\begin{aligned}
S_{0,p,p-1}^L \left[A^{\alpha_1 \dots \alpha_p}, q^{\alpha_1 \dots \alpha_{p-1}} \right] = \int d^Dx \left(-\frac{1}{2(p+1)!} F_{\alpha_1 \dots \alpha_{p+1}} F^{\alpha_1 \dots \alpha_{p+1}} - \right. \\
\left. - \frac{1}{2 \cdot p!} \left(M A_{\alpha_1 \dots \alpha_p} - F_{\alpha_1 \dots \alpha_p} \right) \left(M A^{\alpha_1 \dots \alpha_p} - F^{\alpha_1 \dots \alpha_p} \right) \right). \tag{71}
\end{aligned}$$

Following the same line as in Secs. 4.2.-4.3., we obtain for the system (71) the Lagrangian path integral

$$Z_{\Psi}^{(p)} = \int DA^{\alpha_1 \dots \alpha_p} \det(M) \exp \left(i S_{0p}^L \right), \quad (72)$$

where S_{0p}^L is the Lagrangian action of massive abelian p -form gauge fields, which can be obtained from (71) taking $q\alpha_1 \dots \alpha_{p-1} = 0$.

We notice that for $p=1$ and $D=4$, action (71), which now reads

$$S_{01,0}^L = \int d^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \left(M A_\mu - \partial_\mu q \right) \left(M A^\mu - \partial^\mu q \right) \right), \quad (73)$$

allows the gauge invariances $\delta_\varepsilon A_\mu = \partial_\mu \varepsilon$, $\delta_\varepsilon q = M\varepsilon$, representing the only irreducible case among the models investigated here. Action (73) is identical with the one obtained in [44] by converting the second-class constraints of the massive vector fields into some first-class ones using a conversion method for irreducible theories and also with the one derived by Stueckelberg [45].

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Appendix A

The reducibility function matrices respectively corresponding to the two descriptions (18 a-b) of the stationary surface Σ read

$$\begin{aligned} \begin{pmatrix} (1)a_0 \\ Z_{a_1} \end{pmatrix} &= \begin{pmatrix} Z_{\alpha_1}^{\alpha_0} & 0 \\ 0 & Z_{\alpha_1}^{\alpha_0} \\ 0 & 0 \\ 0 & 0 \\ 0 & \delta_{\beta_0}^{\alpha_0} \end{pmatrix}, & \begin{pmatrix} (2)a_0 \\ Z_{a_1} \end{pmatrix} &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ Z_{\alpha_1}^{\alpha_0} & 0 \\ 0 & Z_{\alpha_1}^{\alpha_0} \\ -\delta_{\beta_0}^{\alpha_0} & 0 \end{pmatrix}, \end{aligned} \quad (A1 \text{ a-b})$$

$$\begin{pmatrix} (1)a_1 \\ Z_{a_2} \end{pmatrix} = \begin{pmatrix} Z_{\alpha_2}^{\alpha_1} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2}Z_{\alpha_2}^{\alpha_1} & \frac{1}{2}Z_{\alpha_2}^{\alpha_1} & 0 & 0 \\ 0 & 0 & 0 & Z_{\alpha_2}^{\alpha_1} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\delta_{\beta_1}^{\alpha_1} & \delta_{\beta_1}^{\alpha_1} & 0 & Z_{\beta_1}^{\beta_0} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta_{\beta_1}^{\alpha_1} & 0 & 0 \\ 0 & 0 & 0 & \delta_{\beta_1}^{\alpha_1} & 0 \end{pmatrix}, \quad (A2 \text{ a})$$

$$\begin{pmatrix} (2)a_1 \\ Z_{a_2} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ Z_{\alpha_2}^{\alpha_1} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2}Z_{\alpha_2}^{\alpha_1} & \frac{1}{2}Z_{\alpha_2}^{\alpha_1} & 0 & 0 \\ 0 & 0 & 0 & Z_{\alpha_2}^{\alpha_1} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\delta_{\beta_1}^{\alpha_1} & \delta_{\beta_1}^{\alpha_1} & 0 & Z_{\beta_1}^{\beta_0} \\ -\delta_{\beta_1}^{\alpha_1} & 0 & 0 & 0 & 0 \\ 0 & -\delta_{\beta_1}^{\alpha_1} & 0 & 0 & 0 \end{pmatrix}, \quad (A2 \text{ b})$$

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