Measurement of the $B_s^0$ lifetime and production rate with $D_s^-\ell^+$ combinations in Z decays

The ALEPH Collaboration

Abstract

The lifetime of the $B_s^0$ meson is measured in approximately 3 million hadronic Z decays accumulated using the ALEPH detector at LEP from 1991 to 1994. Seven different $D_s^-$ decay modes were reconstructed and combined with an opposite sign lepton as evidence of semileptonic $B_s^0$ decays. Two hundred and eight $D_s^-\ell^+$ candidates satisfy selection criteria designed to ensure precise proper time reconstruction and yield a measured $B_s^0$ lifetime of $\tau(B_s^0) = 1.59^{+0.17}_{-0.15} \text{ (stat)} \pm 0.03 \text{ (syst)} \text{ ps}$. Using a larger, less constrained sample of events, the product branching ratio is measured to be $\text{Br}(\bar{b} \to B_s^0) \cdot \text{Br}(B_s^0 \to D_s^-\ell^+\nu X) = 0.82 \pm 0.09 \text{ (stat)}^{+0.13}_{-0.14} \text{ (syst)} \%$. 

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1 Introduction

Measurements of the individual B meson lifetimes provide a well-defined test of the theory of $b$ hadron decays. The lifetimes of the $B^0_d$ and $B^0_s$ are expected to be equal to within a few percent and the $B^-$ lifetime is expected to be approximately 5% greater than the neutral B meson[1].

This letter presents the lifetime of the $B^0_s$ meson measured via reconstructed semileptonic decays, $B^0_s \rightarrow D_s^- \ell^+ \nu X$ \(^1\). The current measurement improves upon the previous ALEPH result[2] with more than four times the number of observed $D_s^- \ell^+$ combinations by increasing the number of reconstructed $D_s^-$ decay channels from two to seven and making use of approximately 3 million hadronic $Z$ decays recorded from 1991 to 1994. The increased $D_s^- \ell^+$ yield is also used to improve ALEPH’s previous measurement[3] of the product branching ratio, $Br(B \rightarrow B_s^-) \cdot Br(B_s^- \rightarrow D_s^- \ell^+ \nu X)$, which is essential to $B^0_s$-$\bar{B}^0_s$ oscillation measurements[4, 5, 6], and for determining the various $b$ hadron fractions[7]. The following sections describe the ALEPH detector, the $D_s^-$ and $B^0_s$ candidate selection, the proper time reconstruction, the lifetime fit and the extraction of the product branching ratio.

2 The ALEPH detector

The ALEPH detector and its performances are described in detail elsewhere[8, 9] and only a brief overview of the apparatus is given here. A high resolution vertex detector (VDET) consisting of two layers of double-sided silicon microstrip detectors surrounds the beam pipe. The inner layer is 6.5 cm from the beam axis and covers 85% of the solid angle and the outer layer is at an average radius of 11.3 cm and covers 69%. The spatial resolution for the $r\phi$ and $z$ projections (transverse to and along the beam axis respectively) is 12 $\mu$m at normal incidence. The vertex detector is surrounded by a drift chamber (ITC) with eight coaxial wire layers with an outer radius of 26 cm and by a time projection chamber (TPC) that measures up to 21 three-dimensional points per track at radii between 30 cm and 180 cm. These detectors are immersed in an axial magnetic field of 1.5 T and together measure the momentum of charged particles with a resolution $\sigma(p)/p = 6 \times 10^{-4}p_{T} \pm 0.005$ ($p$ in GeV/$c$). The resolution of the three-dimensional impact parameter in the transverse and longitudinal view for tracks having information from all tracking detectors and two VDET hits can be parameterized as $\sigma = 25\mu m + 95\mu m/p$ ($p$ in GeV/$c$). The TPC also provides up to 338 measurements of the specific ionization of a charged track ($dE/dx$). The TPC is surrounded by an electromagnetic calorimeter of lead/proportional-chamber construction segmented into $0.9^\circ \times 0.9^\circ$ projective towers and read out in three sections in depth, with energy resolution $\sigma(E)/E = 0.18/\sqrt{E} + 0.009$ ($E$ in GeV). The iron return yoke of the magnet is instrumented with streamer tubes to form a hadron calorimeter, with a thickness of over 7 interaction lengths and is surrounded by two additional double-layers of streamer tubes to aid in muon identification. An algorithm combines all these measurements to provide a determination of the energy flow[9] with a precision on the measurable total energy of $\sigma(E) = 0.6\sqrt{E/GeV} + 0.6$ GeV.

The selection of hadronic events is based on charged tracks and is described elsewhere[10]. The interaction point is reconstructed on an event-by-event basis using the constraint of the average beam spot position[11]. The resolution is 85 $\mu$m for $Z \rightarrow b\bar{b}$ events, projected along the sphericity axis of the event.

\(^1\)In this paper charge conjugate modes are always implied and “lepton” refers to electrons and muons.
3 Sources of \( D_s \ell \) correlations

The five sources of \( D_s \ell \) correlations considered are

1. \( B_s^0 \to D_s^- \ell^+ \nu X \)
2. \( B \to D_s^{(*)-}D^{(*)}X \) followed by \( D^{(*)} \to \ell^+ \nu X \)
3. \( B \to D_s^- X, \ell^+ \nu \)
4. kinematic reflections in the \( D_s^- \to K^{*0}K^- \) (\( K_0^0K^- \)) channels due to \( D^- \to K^{*0}\pi^- \) (\( K_0^0\pi^- \)), and
5. combinatorial background.

These different sources of signal and background have been extensively discussed previously [2, 3, 12] and will simply be reviewed here. Only two decays, \( B_s^0 \to D_s^- \ell^+ \nu X \) and \( B_s^0 \to D_s^- \ell^+ \nu \), are considered to contribute to \( B_s^0 \to D_s^- \ell^+ \nu X \) as the higher order \( \bar{c}s \) resonances decay to \( D^{(*)}K \) final states while the \( D_s^- \to D_s^- \gamma \) decay dominates [13]. The kinematics of the \( B \to D_s^{(*)-}D^{(*)}X \) background are simulated by \( B_d^0 \to D_s^{(*)-}D^{(*)+}(\pi) \) decays but the contributions from \( B^- \), \( B_s^0 \) and \( b \) baryon decays are included in the rate estimate. The branching ratio of the as-yet unobserved \( B \to D_s^- X, \ell^+ \nu \) process is estimated to be less than 0.36% at 90% C.L. The assumptions and rates used to derive these estimates are discussed in section 8. The reflection background is due to misinterpretation of a charged pion as a charged kaon and is discussed in section 4.1.

4 The \( B_s^0 \to D_s^- \ell^+ \nu X \) candidate selection

Candidates for \( B_s^0 \to D_s^- \ell^+ \nu X \) are selected from hadronic \( Z \) decays where a \( D_s^- \) and a lepton are found in the same hemisphere as defined by the thrust axis. The lepton identification in ALEPH is described in detail in reference [14]. In this paper the requirement that the \( dE/dx \) information be available for electron candidates is dropped.

The \( D_s^- \) is reconstructed in five hadronic decay modes, \( \phi \pi^- \), \( K^{*0}K^- \), \( \phi \pi^+\pi^-\pi^- \), \( K^{*0}K^- \) and \( K_0^0K^- \), and two semileptonic decay modes, \( \phi e^- \nu \) and \( \phi \mu^- \nu \). The selection criteria for the first two modes are similar to those used in the previous \( B_s^0 \) lifetime measurement [2]. The next two modes suffer from larger combinatorial background and require more involved selection criteria. The final hadronic mode, \( D_s^- \to K_0^0K^- \), is contaminated by the kinematic reflections from \( D^- \to K_0^0\pi^- \) requiring more stringent cuts be applied to the charged kaon candidate. The following sections describe the details of the \( D_s^- \) selection.

4.1 Kinematic criteria for hadronic \( D_s^- \) decays

Each \( D_s^- \) candidate must carry at least 15% of the beam energy. For all the channels the momentum of the charged kaon candidate must be at least 1.5 GeV/c to ensure good kaon and pion \( dE/dx \) identification. The charged kaon candidates specific ionization, when available, should fulfill \( \chi_\pi + \chi_K < 1 \) [2], where \( \chi_\pi \) is the difference between the measured and expected ionization for the particle hypothesis \( H \) in terms of the expected resolution. Pion candidates, except those from the \( K_0^0 \), should have \( |\chi_\pi| < 3 \) if the \( dE/dx \) information is available, and a momentum greater than 0.5 GeV/c. The \( \phi \), \( K^{*+}(0) \) and \( K_0^0 \) are reconstructed in the \( \phi \to K^+K^- \), \( K^{*+}(0) \to K^{0(*)}\pi^+ \) and \( K_0^0 \to \pi^+\pi^- \) modes and are required to be within 9, 50 and 12 MeV/c^2 of the nominal \( \phi \), \( K^{*+}(0) \) and \( K_0^0 \) masses [13], respectively, according to the resolutions and natural widths. The details of the momentum requirements for each hadronic decay mode are summarized in Table 1.

For the \( \phi \pi^- \) and \( K^{*0}K^- \) modes, \( |\cos \theta^*| > 0.4 \) is required, where \( \theta^* \) is the center-of-mass decay angle of the \( \phi \) or \( K^{*0} \) relative to its flight direction. This cut exploits the fact that the distribution
of $\theta^*$ is proportional to $\cos^2 \theta^*$ for the decay of the pseudoscalar $D_s^-$ into a vector ($\phi$ or $K^*$) and a pseudoscalar. To reduce the larger combinatorial background for $K^{*0}K^-$, $|\cos \lambda'| < 0.8$ is also required where $\lambda'$ is the angle of the $K^{*0}$ in the $D_s^-$ center-of-mass relative to the $D_s^-$ flight direction. For the $\phi\pi^+\pi^-\pi^-$ mode, the momentum of at least one pion is required to exceed 1.5 GeV/c and the $D_s^-$ must carry at least 20% of the beam energy.

The $\frac{dE}{dx}$ information is required to be available for the kaon from the $D^-\to K^{*0}K^-$ and $K^0_sK^-$ modes to reduce the kinematic reflection background from $D^-\to K^{*0}\pi^-$ and $D^-\to K^0_s\pi^-$, respectively. In addition the kinematic reflection background to $D_s^-\to K^0_s\pi^-$ from $D^-\to K^{*0}\pi^-$ is rendered negligible by requiring $p_K < 5$ GeV/c or $\chi_K < 0$. The kinematic reflection background to $D_s^-\to K^0_s\pi^-$ from $\Lambda^-\to K^0_s\pi^-$ is also rendered negligible by requiring the invariant mass to be incompatible with the $\Lambda^-$ when the kaon candidate is given the proton mass.

\[
\begin{array}{|c|c|c|c|c|}
\hline
\phi\pi^- & K^{*0}K^- & K^0_sK^- & \phi\pi^+\pi^-\pi^- & K^{*0}K^- \\
\hline
p_{\phi} > 4.0 & p_{K^*} > 4.0 & p_{K^0_s} > 2.0 & p_\phi > 3.5 & p_{K^0_s} > 2.0 \\
p_\pi > 1.0 & p_{K^0_s} > 2.0 & p_\pi > 1.0 & p_{K^0_s} > 3.0 & \\
\hline
\end{array}
\]

Table 1: The momentum cuts in GeV/c for each hadronic $D_s^-$ decay mode. Additional details are described in the text.

4.2 Kinematic criteria for the semileptonic $D_s^-$ decays

The $\phi$ is reconstructed as in the $\phi\pi^-$ mode. The momentum of the lepton from the $D_s^-$ must be $p_{\mu(e)} > 3(2)$ GeV/c and satisfy the lepton identification criteria cited earlier[14]. Additional electron candidates are identified as tracks satisfying the standard electron criteria with momenta between 1 and 2 GeV/c where the $\frac{dE}{dx}$ information must be available and inconsistent with the proton hypothesis ($|\chi_p| > 2.0$) to eliminate hadron contamination. At 1 GeV/c the electron and proton ionization is indistinguishable.

To unambiguously assign the two leptons (one from the $B_s^0$, the other from the $D_s^-$), $M(\phi, \ell_{D_s^-})$ ($M(\phi, \ell_{B_s^0})$) must be at least 15 MeV/c$^2$ less (more) than the nominal $D_s^-$ mass. If the two leptons are of the same flavour, their invariant mass must be inconsistent\(^2\) with the $J/\psi$ and $\psi'$; in addition, the mass of electron–positron pairs must be greater than 100 MeV to exclude photon conversions. The $B \to D_s^{(*)-}D^{(*)}X$ background is further reduced by requiring the missing energy in the hemisphere, estimated using an energy flow technique[9], to be greater than 10 GeV or the momentum of the lepton from the $B_s^0$ decay to be greater than 5 GeV/c.

4.3 $B_s^0$ selection

The tracks forming $D_s^-$ candidates are required to form a vertex in three dimensions and at least two of the charged $D_s^-$ decay products must have at least one $r\phi$ and $z$ VDET hit and at least one ITC hit. For the $D_s^-\to K^0_sK^-$ mode, only the charged kaon track should fulfil this requirement. The $D_s^-$ vertex probability must be greater than 0.01% and the calculated uncertainty on the $D_s^-$ decay length must be less than 1.0 mm.

These $D_s^-$ candidates are in turn used to form a three-dimensional vertex with a candidate lepton with a momentum of at least 3 GeV/c and three-dimensional VDET information and an ITC hit.\(^2\)Specifically, $|M(\psi^{(1)}) - M(\mu^+\mu^-)| > 100$ MeV/c$^2$ and $M(\psi^{(1)}) - M(e^+e^-) > 150$ MeV/c$^2$ or $M(e^+e^-) - M(\psi^{(1)}) > 100$ MeV/c$^2$ are required.

3
to create the $B_s^0$ candidates. The $D_s^-\ell^+$ vertex probability should be greater than 0.01\% and the uncertainty on the $B_s^0$ decay length must be less than 0.5 mm. The requirements on the tracks used to form the vertices and on the vertex probabilities are somewhat different from the previous analysis[2]. Based on studies of the Monte Carlo simulation, these selection criteria give a 210 $\mu$m resolution on the $B_s^0$ decay length for the hadronic $D_s^- $ decays. This resolution is similar to reference[2] with a slight reduction in the tails of the distributions. The $B_s^0$ decay length resolution is approximately 20\% worse for the semileptonic $D_s^- $ decay modes.

The reconstructed $B_s^0$ momentum for the $D_s^- \to \phi \pi^+ \pi^- \pi^-$ mode must be at least 50\% of the beam energy in order to reduce the greater combinatorial background. The $D_s^-\ell^+$ invariant mass should be in the range 3.0 to 5.5 GeV/c$^2$ (for $D_s^- \to \phi \ell^- \nu$ the lower bound is 2.5 GeV/c$^2$ because of the additional neutrino). This cut considerably reduces the $B \to D_s^{(*)-} D^{(*)} X$ and $B \to D_s^- X_s \ell^+ \nu$ backgrounds[3].

The requirement that $L_{D^-}/\sigma(L_{D^-}) > -0.5$, where $L_{D^-}$ is the distance between the reconstructed $B_s^0$ and $D_s^-$ vertices projected along the $D_s^-$ direction, reduces the combinatorial background by approximately 50\% and retains about 90\% of the $B_s^0$ signal while inducing a very small bias on the $B_s^0$ decay length measurement. From a study of simulated $B_s^0$ semileptonic decays, this bias was determined to be $-19 \pm 3 \mu$m. The bias was taken to be $-20 \mu$m with a 100\% systematic uncertainty. For the average reconstructed $B_s^0$ momentum of 33 GeV/c, this leads to a bias of $-0.01 \pm 0.01$ ps on the $B_s^0$ lifetime.

5 Sample composition

To determine the relative contributions of the signal and background processes in each channel, the $D_s^-$(\phi) mass spectra are fitted with a Gaussian (Breit-Wigner) to represent the signal and a second degree polynomial to represent the combinatorial background for the hadronic (semileptonic) $D_s^-$ decay modes. The masses are fixed to their nominal values[13] and the widths are fixed to their expected values determined from simulated semileptonic $B_s^0$ decays (Table 2). For the $D^- \to K^0 \pi^-$ channel, the fraction of the reflection background from $D^- \to K^0 \pi^-$ decays is fitted using a shape determined from the simulation. An additional Gaussian at the nominal $D^-$ mass represents the contribution of the Cabibbo-suppressed hadronic decays of the $D^-$. The fitted mass spectra for the hadronic and semileptonic $D_s^-$ decay modes are shown in Figure 1. The estimated contribution of the three main components within the mass windows (Table 2) is shown in Table 3 for each mode. A small correction has been applied to the $\phi e^- \nu$ yield to account for an estimated background of 0.9 $\pm$ 0.2 events due to hadrons misidentified as electrons. The small contribution of the $B \to D_s^- X_s \ell^+ \nu$ decay is considered as a systematic uncertainty only. The contribution to the $D_s^-\ell^+$ yield from $B_s^0 \to D_s^- \tau^+ \nu X$ followed by $\tau^+ \to \ell^+ \nu \bar{\nu}$ is estimated to be less than one event.

The reconstruction efficiencies for the signal and the mass resolution for each $D_s^-$ decay mode are listed in Table 2. The reconstruction efficiency for the $B \to D_s^{(*)-} D^{(*)} X$ background relative to the $B_s^0 \to D_s^- \ell^+ \nu$ signal for the hadronic and semileptonic $D_s^-$ decays is determined to be 15.4 $\pm$ 1.4 \% and 8.4 $\pm$ 1.2 \%, respectively, where the uncertainties are due to the Monte Carlo statistics. For the $B \to D_s^- X_s \ell^+ \nu$ background, the relative efficiencies for the hadronic and semileptonic $D_s^-$ decays are 54.2 $\pm$ 1.8 \% and 39.0 $\pm$ 3.6 \%, respectively.

Approximately 134 of the 208 $D_s^-\ell^+$ combinations are attributed to $B_s^0 \to D_s^- \ell^+ \nu X$ decays.
Table 2: The second and third columns contain the $D_s^-$ or $\phi$ mass resolution and mass acceptance window about the nominal $D_s^-$ or $\phi$ mass for each mode. The signal detection efficiency is shown in the last column where the uncertainty is due to Monte Carlo statistics.

<table>
<thead>
<tr>
<th>$D_s^-$ decay mode</th>
<th>Mass resolution (MeV/$c^2$)</th>
<th>Mass cut (MeV/$c^2$)</th>
<th>$D_s^- \ell^+\nu X$ efficiency(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi\pi^-$</td>
<td>7.0</td>
<td>± 15</td>
<td>9.7 ± 0.1</td>
</tr>
<tr>
<td>$K^{*0}\pi^-$</td>
<td>7.0</td>
<td>± 15</td>
<td>6.2 ± 0.1</td>
</tr>
<tr>
<td>$K_s^{*0}\pi^-$</td>
<td>10.0</td>
<td>± 20</td>
<td>2.3 ± 0.1</td>
</tr>
<tr>
<td>$\phi\pi^+\pi^-\pi^-$</td>
<td>5.0</td>
<td>± 10</td>
<td>5.7 ± 0.1</td>
</tr>
<tr>
<td>$K^{*0}\pi^+\pi^-$</td>
<td>7.0</td>
<td>± 15</td>
<td>3.4 ± 0.1</td>
</tr>
<tr>
<td>$\phi e^-\nu$</td>
<td>4.7</td>
<td>± 6</td>
<td>7.6 ± 0.2</td>
</tr>
<tr>
<td>$\phi \mu^-\nu$</td>
<td>4.7</td>
<td>± 6</td>
<td>5.5 ± 0.2</td>
</tr>
</tbody>
</table>

Table 3: The estimated contributions to the yield for each decay mode. The estimated contribution of one $D^- \rightarrow K^{*0}\pi^-$ event to the $K^{*0}\pi^-$ mode is not shown in the table but is included in the totals.

<table>
<thead>
<tr>
<th>$D_s^-$ decay mode</th>
<th>$D_s^- \ell^+\nu X$ signal</th>
<th>$D_s^- \ell^+\nu X$ background</th>
<th>Combinatorial background</th>
<th>Total per mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi\pi^-$</td>
<td>36.7</td>
<td>3.6</td>
<td>6.7</td>
<td>47</td>
</tr>
<tr>
<td>$K^{*0}\pi^-$</td>
<td>47.0</td>
<td>4.3</td>
<td>23.6</td>
<td>76</td>
</tr>
<tr>
<td>$K_s^{*0}\pi^-$</td>
<td>9.3</td>
<td>0.8</td>
<td>5.9</td>
<td>16</td>
</tr>
<tr>
<td>$\phi\pi^+\pi^-\pi^-$</td>
<td>9.8</td>
<td>0.9</td>
<td>9.3</td>
<td>20</td>
</tr>
<tr>
<td>$K^{*0}\pi^+\pi^-$</td>
<td>9.8</td>
<td>0.9</td>
<td>4.3</td>
<td>15</td>
</tr>
<tr>
<td>$\phi e^-\nu$</td>
<td>16.6</td>
<td>0.9</td>
<td>7.5</td>
<td>25</td>
</tr>
<tr>
<td>$\phi \mu^-\nu$</td>
<td>4.9</td>
<td>0.2</td>
<td>3.9</td>
<td>9</td>
</tr>
<tr>
<td>TOTAL</td>
<td>134.2</td>
<td>11.6</td>
<td>61.3</td>
<td>208</td>
</tr>
</tbody>
</table>

6 Proper time reconstruction

The measured proper time of the $B_s^0$ is given by

$$t_m = \frac{M_{B_s^0} L_{B_s^0}}{p_{B_s^0}}$$

where $M_{B_s^0} = 5375 \pm 6 \text{ MeV}/c^2$[13] is the mass of the $B_s^0$, $L_{B_s^0}$ is the distance between the primary vertex and the $D_s^- \ell^+$ vertex projected on the $D_s^- \ell^+$ direction and $p_{B_s^0}$ is the $B_s^0$ momentum computed from the reconstructed $D_s^-$ and lepton momenta and the neutrino energy estimated using the missing energy technique of reference [2]. Monte Carlo simulation shows that the $B_s^0$ boost resolution for the majority of the events is 5% as described in reference [2].

The temporal resolution is parametrized by a sum of two Gaussians and analytically convolved with the decay time distribution. The explicit expression for the probability density function for the
The $B_s^0$ signal is

$$
\bar{P}(t_m; \tau_s) = \sum_{i=1}^{3} \rho_i \int_{0}^{\infty} dt \frac{1}{\sqrt{2\pi}S_{\delta_i}} e^{-\frac{1}{2}(\frac{t}{\sigma_i} - \delta_i)^2} \frac{1}{\tau_s} e^{-\frac{t}{\tau_s}}
$$

where $\tau_s$ is the $B_s^0$ lifetime, $\rho_i$ is the fractional contribution of the $i^{th}$ Gaussian, $\delta_i$ is the offset and $\sigma_i$ is the proper time resolution measured with the simulation. A scale factor $S$ corrects for the differences in proper time resolution between the data and the simulation.

A separate parametrization of the proper time resolution for both the $B_s^0 \rightarrow D^-\ell^+ \nu$ and $B_s^0 \rightarrow D^*_-\ell^+ \nu$ decays for each of the seven $D_s^-$ decay modes is determined from Monte Carlo simulation with an input $B_s^0$ lifetime of 1.50 ps.

To determine $S$ the negative part of the $L_{B_s^0}/p_{B_s^0}$ distribution is fitted with two Gaussians for “fake” $D_s\ell$ combinations in events where the $uds$ content is enhanced to 50% by requiring the tracks in the hemisphere opposite to the $B_s^0$ candidate to be consistent with zero lifetime[11]. “Fake” $D_s\ell$ are candidates from either the like-sign $D_s\ell$ combinations, the unlike-sign $D_s\ell$ sideband or a $D_s$ formed from the $\phi$ or $K^*$ sidebands as defined in section 7. The ratio of the data to the simulation for the proper time resolution is measured to be $S = 1.2 \pm 0.1$.

Extensive analysis of simulated data with generated $B_s^0$ lifetimes of 1.0, 1.5 and 2.0 ps has shown that the parametrization of the temporal resolution as a sum of Gaussians yields an average bias of $-1.0 \pm 0.5$% on the fitted lifetime. This bias arises from neglecting the dependence of the proper time resolution on the proper time itself in the formulation of $\bar{P}(t_m; \tau_s)$.

7 Lifetime fit

The likelihood function to be maximized for the lifetime fit is

$$
\mathcal{L}(\tau_s) = \prod_{i=1}^{N} f_{1i} \bar{P}(t_{m_i}^{i}; \tau_s) + f_2 \bar{P}(t_{m}^{i}; \tau_d) + f_3 \bar{P}(t_{m}^{i}; \tau_b) + f_4 B(t_{m}^{i})
$$

where

- $f_1$ is the relative fraction of $B_s^0 \rightarrow D^-\ell^+ \nu X$ events in the $D_s^-$ or $\phi$ peak,
- $f_2$ is the relative fraction of $B \rightarrow D_s^{(*)} - D^{(*)} X$ events,
- $\tau_d$ is the effective lifetime of the simulated $B \rightarrow D_s^{(*)} - D^{(*)} X$ events,
- $f_3$ is the relative fraction of reflection background for $D_s^- \rightarrow K^{*0} K^-$,
- $\tau_b$ is the average $b$ lifetime,
- $f_4$ is the relative fraction of combinatorial background,
- $B(t_{m})$ is the proper time distribution of the combinatorial background, and
- $N$ is the number of $D_s^-\ell^+$ candidates.

$B(t_{m})$ is estimated from the proper time distributions of the candidates in the $D_s^-$ and $\phi$ mass “sideband”. For $D_s^{\pm}, \ell^+ (\phi \ell^+ \ell^\mp)$ candidates, the sideband extends from 2.05 to 2.30(1.021 to 1.120) GeV/$c^2$. For the like-sign candidates, $D_s^{\pm}, \ell^\mp (\phi \ell^\pm \ell^\pm)$, the lower limit of the sideband is reduced to 1.95(0.997) GeV/$c^2$. The proper time distributions for the hadronic and semileptonic $D_s^-$ decays are separately parametrized by the sum of two Gaussians and two exponentials (one for $t_m < 0$ and one for $t_m > 0$).

Although not explicitly noted in eqn.(1), the $f_i$ are calculated separately for each $D_s^-$ decay mode as described in section 5.
The measured lifetime from the unbinned maximum likelihood fit for the entire data sample is 
\( \tau(B_s) = 1.56^{+0.17}_{-0.15} \) ps (statistical uncertainty only) without correcting for the small biases due the 
parametrization of the resolution and the cut on \( L_{D_s^-}/\sigma(L_{D_s^-}) \). The proper time distribution and fit 
are shown in Figure 2.

7.1 Systematic uncertainties and corrections

The individual contributions to the systematic uncertainty on the \( B_s^0 \) lifetime are summarized in Table 
4 and discussed below.

<table>
<thead>
<tr>
<th>Source</th>
<th>Uncertainty (ps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combinatorial background</td>
<td>+0.013 -0.016</td>
</tr>
<tr>
<td>( B \rightarrow D_s^{(<em>)} \rightarrow D^{(</em>)} ) background</td>
<td>+0.023 -0.016</td>
</tr>
<tr>
<td>( B \rightarrow D_s^- X \ell \nu ) background</td>
<td>+0.001 -0.009</td>
</tr>
<tr>
<td>Proper time resolution parametrization bias</td>
<td>+0.008 -0.008</td>
</tr>
<tr>
<td>Parametrization of proper time resolution</td>
<td>+0.015 -0.015</td>
</tr>
<tr>
<td>( L_{D_s^-}/\sigma(L_{D_s^-}) &gt; -0.5 ) bias</td>
<td>+0.010 -0.010</td>
</tr>
<tr>
<td>( B_s^0 ) boost resolution</td>
<td>+0.010 -0.010</td>
</tr>
<tr>
<td>Other</td>
<td>+0.006 -0.005</td>
</tr>
<tr>
<td>Total in quadrature</td>
<td>+0.035 -0.033</td>
</tr>
</tbody>
</table>

Table 4: The components of the estimated systematic uncertainty on the measured \( B_s^0 \) 
lifetime. Additional details are described in the text.

The systematic uncertainty due to the combinatorial background is estimated by varying the 
calculated signal fraction of all decay modes by one standard deviation and by varying the proper 
time distribution of the combinatorial background, \( B(t_m) \). The systematic uncertainty due to \( B(t_m) \) 
is estimated by altering the parametrization (using a single Gaussian and two exponentials or by 
forcing the two Gaussians to have identical means), by using the unlike-sign or the like-sign sideband 
only, or by halving the width of the sidebands. In addition, a simultaneous fit to the signal and 
sidebands was performed similarly to the previous ALEPH measurement[2]. In this fit, the proper 
time distributions of the hadronic and semileptonic \( D_s^- \) sidebands were fitted separately with two 
non-zero lifetime components and a zero lifetime component.

The uncertainty due to the \( B \rightarrow D_s^{(*)} \rightarrow D^{(*)} \) background is estimated by varying both the calculated 
yield and the \( \tau_d \) lifetime within their estimated uncertainties. For simulated \( B \rightarrow D_s^{(*)} \rightarrow D^{(*)} \) events, \( \tau_d \) is fitted as \( 1.90 \pm 0.20 \) ps, which is longer than the input \( b \) hadron lifetime of \( 1.50 \) ps due to the 
under-estimation of the boost. To account for the unknown multiplicity of "\( X \)" in the \( B \) decay, 
the uncertainty has been increased to \( \pm 0.30 \) ps. The uncertainty due to the \( B \rightarrow D_s^- X \ell \nu \) decays is 
estimated by assuming that the rate is equal to the upper limit quoted earlier and varying the effective 
lifetime. The effective lifetime fitted for simulated \( B \rightarrow D_s^- X \ell \nu \) is \( 1.75 \pm 0.20 \) ps which also is longer 
than the input \( b \) hadron lifetime due to the under-estimated boost.

The lifetime is corrected by \( +0.016 \pm 0.008 \) ps for the bias observed in simulated \( B_s^0 \rightarrow D_s^- \ell \nu X \) 
events when they are fitted with the parametrization of the proper time resolution. An additional 
uncertainty arises due to the statistical precision in the parametrization of the proper time resolution 
and is estimated by varying the parameters by one standard deviation of their fitted values.

The lifetime is also corrected for the bias due to \( L_{D_s^-}/\sigma(L_{D_s^-}) > -0.5 \) by \( +0.010 \pm 0.010 \) ps which 
was described in section 4.3.
As in the previous measurement[2], the relative fractions of the tails of the reconstructed $B^0_s$ boost were varied by $\pm 20\%$ to estimate the uncertainty due to the neutrino energy resolution. This yields an uncertainty of $\pm 0.010$ ps. The $B^0_s$ momentum distribution was modified in the simulation and resulted in a negligible uncertainty in the lifetime.

The uncertainties due to following additional small effects are added in quadrature and are labeled as “Other” in Table 4:

- Monte Carlo studies show that if the lifetime to be measured differs substantially from the lifetime, $\tau_{gen}$, used to create the parametrization of the proper time resolution (section 6), then the measured lifetime will be slightly biased towards $\tau_{gen}$. This bias is treated as an additional systematic uncertainty by fitting the lifetime with parameters derived from different $\tau_{gen}$ of 1.0, 1.5 and 2.0 ps.
- The scale factor for the proper time resolution is varied: $S = 1.2 \pm 0.1$.
- The estimated relative production rate of $B^0_s \rightarrow D^-_s \ell^+ \nu$ to $B^0_s \rightarrow D^-_s \ell^+ \nu$ is approximated by the measured $B \rightarrow D^{(*)} \ell \nu$ rates[13] to be $2.3 \pm 0.6$.
- The mass of the $B^0_s$ meson is varied by 6 MeV/$c^2$[13].
- $\tau_s$ is varied by 0.05 ps.

The final result of the maximum likelihood fit for the $B^0_s$ lifetime, including the measurement biases, is

$$\tau(B^0_s) = 1.59^{+0.17} _{-0.15} \text{ (stat)} \pm 0.03 \text{ (syst)} \text{ ps}.$$  

8 Product branching ratio measurement

From yields of $D^-_s \ell^+$ combinations, it is possible to calculate the product branching ratio $\text{Br}(\bar{b} \rightarrow B^0_s) \cdot \text{Br}(B^0_s \rightarrow D^-_s \ell^+ \nu X)$, which can be used to estimate the production rate of $B^0_s$ mesons in $Z \rightarrow b\bar{b}$ decay.

Due to the sensitivity of such a measurement to the reconstruction efficiency, estimated from simulation, all requirements not essential to a yield measurement were removed: the requirements on the vertex probability, VDET and ITC assignments, the $B^0_s$ and $D^-_s$ decay length resolution and the cut $L_{D^-_s}/\sqrt{\langle L_{D^-_s} \rangle} > -0.5$. The number of events within the mass acceptance windows (Table 2) and background estimates for the 3.1 million hadronic Z decays analysed are detailed in Table 5. The reconstruction efficiency estimates for signal and background processes are listed in Table 6. A likelihood function is constructed, taking into account the expected backgrounds in each channel. All channels are then simultaneously fitted for the product branching ratio.

The background processes are estimated as follows. The combinatorial background in each channel is estimated from the sidebands of the mass spectrum, fitted with a second-order polynomial, as described in section 5. The background due to real $D^-_s$ mesons paired with misidentified hadrons or non-prompt leptons (“Accidental Combinations” in Tables 5 and 8) is expected to be small. In the $\phi \pi^-$ mode, it is estimated from a fit to the wrong-sign mass spectrum, then scaled by the ratio of right-sign to wrong-sign sideband events. As the other channels carry significantly less weight in the fit, this background in the other channels is scaled from the $\phi \pi^-$ channel. This yields a conservative error common to all modes. The reflection background is fitted directly in the mass spectrum of the $K^{*0}K^-$ decay mode, as described in section 5.

The background due to $B \rightarrow D^{(*)}_s \rightarrow D^{(*)} \ell^+ \nu X$, $D^{(*)} \rightarrow \ell^+ \nu X$ is estimated from the product of the inclusive branching ratio, $\text{Br}(B \rightarrow D^-_s X)$, measured at the $\Upsilon(4S)[15, 16]$, and the average rate for secondary leptons in $b$ decays, $\text{Br}(b \rightarrow c \rightarrow \ell)$, measured at LEP[21]. Combining these rates
<table>
<thead>
<tr>
<th>Mode</th>
<th>Yield</th>
<th>Combin.</th>
<th>$D^{(<em>)}-D^{(</em>)}X$</th>
<th>Accid.</th>
<th>$D_s^- \ell^+\nu$</th>
<th>Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi\pi^-$</td>
<td>$145 \pm 12.0$</td>
<td>$48.2 \pm 1.6$</td>
<td>$15.3 \pm 1.8$</td>
<td>$2.5 \pm 8.1$</td>
<td>$8.2 \pm 4.0$</td>
<td>$70.8 \pm 15.2$</td>
</tr>
<tr>
<td>$K^oK^-$</td>
<td>$257 \pm 16.0$</td>
<td>$148.6 \pm 2.9$</td>
<td>$12.0 \pm 1.4$</td>
<td>$2.8 \pm 9.0$</td>
<td>$6.4 \pm 3.1$</td>
<td>$77.0 \pm 19.6$</td>
</tr>
<tr>
<td>$\phi e^-\nu$</td>
<td>$102 \pm 10.1$</td>
<td>$44.4 \pm 1.8$</td>
<td>$4.6 \pm 0.7$</td>
<td>$1.5 \pm 4.8$</td>
<td>$3.4 \pm 1.7$</td>
<td>$48.1 \pm 11.5$</td>
</tr>
<tr>
<td>$\phi\mu^-\nu$</td>
<td>$53 \pm 7.3$</td>
<td>$27.7 \pm 1.5$</td>
<td>$3.0 \pm 0.5$</td>
<td>$0.7 \pm 2.1$</td>
<td>$2.4 \pm 1.2$</td>
<td>$19.2 \pm 7.8$</td>
</tr>
<tr>
<td>$K^oK^-$</td>
<td>$111 \pm 10.5$</td>
<td>$83.8 \pm 2.5$</td>
<td>$6.9 \pm 0.8$</td>
<td>$0.7 \pm 2.3$</td>
<td>$3.7 \pm 1.8$</td>
<td>$15.9 \pm 11.2$</td>
</tr>
<tr>
<td>$\phi\pi^+\pi^-\pi^-$</td>
<td>$180 \pm 13.4$</td>
<td>$184.3 \pm 2.7$</td>
<td>$5.7 \pm 0.7$</td>
<td>$0.0 \pm 0.4$</td>
<td>$3.1 \pm 1.5$</td>
<td>$-13.0 \pm 13.8$</td>
</tr>
<tr>
<td>$K^oK^-$</td>
<td>$55 \pm 7.4$</td>
<td>$42.6 \pm 1.8$</td>
<td>$2.7 \pm 0.3$</td>
<td>$0.3 \pm 1.0$</td>
<td>$1.5 \pm 0.7$</td>
<td>$7.9 \pm 7.7$</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>$903 \pm 30.0$</td>
<td>$579.6 \pm 5.8$</td>
<td>$50.1 \pm 2.6$</td>
<td>$8.5 \pm 27.7$</td>
<td>$28.7 \pm 14.0$</td>
<td>$225.9 \pm 44.0$</td>
</tr>
</tbody>
</table>

Table 5: Event yields and background estimates. The estimated contribution of $10.2 \pm 5.1$ reflection background events to the $K^oK^-$ mode is not shown in the table but is included in the totals.

thus includes the contributions from the $B^0_s$ and from $b$ baryons. The measured relative rate of two-body decays, $\Gamma(B \to D_s^{(*)}-D^{(*)})/\Gamma(B \to D_s^-X)$[15, 16], is used to calculate the efficiencies for this background (Table 7).

The background due to $B \to D_s^-X, \ell^+\nu$ is calculated as follows. The fraction of $B \to D_s^-X$ decays in which the $D_s^-$ is not produced from the $W^{-}$ ("lower vertex production") was recently measured to be $f_{\text{lower}} = 0.172 \pm 0.083$, and a limit set at $f_{\text{lower}} < 0.31$ at 90% C.L.[17]. Combined with the inclusive $D_s^-$ production rate, $Br(B \to D_s^-X)$[15, 16], and the inclusive $b$ semileptonic decay rate, $Br(b \to X\ell\nu)[21]$, the rate and limit for this background are estimated to be $Br(B \to D_s^-X, \ell^+\nu) = 0.20 \pm 0.10\%$ and $< 0.36 \%$ at 90% C.L., respectively. These estimates are consistent with an earlier search[18] and theoretical estimates[19].

The calculated $B^0_s \to D_s^-\ell^+\nu X$ signal for each mode after subtraction of the estimated backgrounds is given in Table 5. Using the number of observed events, the estimated reconstruction efficiencies and background contributions, and the branching ratios in Table 7, the product branching ratio is simultaneously fitted to all seven decay modes. The result is

$$Br(B^0_s \to D_s^-\ell^+\nu X) = 0.82 \pm 0.09 \text{ (stat)} \pm 0.13 \text{ (syst)} \%$$

which is the sum of the electron and muon decay channels divided by two\(^3\). The specific contributions to the systematic uncertainty are listed in Table 8.

### 9 Conclusion

The $B^0_s$ has been reconstructed in the semileptonic decay mode $B^0_s \to D_s^-\ell^+\nu X$ using seven different $D_s^-$ decay modes in approximately 3 million hadronic $Z$ decays. The result of an unbinned, maximum likelihood fit to the $B^0_s$ lifetime for the observed 208 $D_s^-\ell^+$ combinations is

$$\tau(B^0_s) = 1.59 \pm 0.17 \text{ (stat)} \pm 0.03 \text{ (syst)} \text{ ps}$$

\(^3\)The product branching ratio quoted in reference [7] was calculated neglecting $B \to D_s^-X, \ell^+\nu$; including this process, $f(b \to B^0_s) = 11.0 \pm 1.2^{+2.5}_{-2.6}\%$ and $f(b \to B^0_s^\ast)$ is $38.8 \pm 1.3 \pm 2.1\%$. The baryon fraction remains unchanged, $f(b \to \Lambda_b) = 11.5 \pm 2.2 \pm 3.4\%$. 

9
Table 6: Reconstruction efficiencies for signal and physics background processes. The uncertainties are due to Monte Carlo statistics.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$B_0 \to D_s^- t^+ \nu X$</th>
<th>$\bar{B} \to D_s^{(<em>)-} D^{(</em>)} X$</th>
<th>$B \to D^- X_t t^+ \nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi \pi^-$</td>
<td>$16.5 \pm 0.1 %$</td>
<td>$3.7 \pm 0.3 %$</td>
<td>$9.7 \pm 0.2 %$</td>
</tr>
<tr>
<td>$K^*0K^-$</td>
<td>$10.1 \pm 0.2 %$</td>
<td>$2.3 \pm 0.2 %$</td>
<td>$5.9 \pm 0.2 %$</td>
</tr>
<tr>
<td>$\phi e^- \nu$</td>
<td>$17.9 \pm 0.3 %$</td>
<td>$2.1 \pm 0.2 %$</td>
<td>$7.5 \pm 0.5 %$</td>
</tr>
<tr>
<td>$\phi \mu^- \nu$</td>
<td>$13.4 \pm 0.3 %$</td>
<td>$1.4 \pm 0.2 %$</td>
<td>$5.3 \pm 0.5 %$</td>
</tr>
<tr>
<td>$K^0\phi K^-$</td>
<td>$10.6 \pm 0.2 %$</td>
<td>$2.4 \pm 0.2 %$</td>
<td>$6.2 \pm 0.2 %$</td>
</tr>
<tr>
<td>$\phi\pi^+\pi^-\pi^-$</td>
<td>$12.1 \pm 0.2 %$</td>
<td>$2.7 \pm 0.2 %$</td>
<td>$7.1 \pm 0.2 %$</td>
</tr>
<tr>
<td>$K^0\phi K^-$</td>
<td>$5.9 \pm 0.1 %$</td>
<td>$1.3 \pm 0.1 %$</td>
<td>$3.5 \pm 0.1 %$</td>
</tr>
</tbody>
</table>

Table 7: Rates used in the calculation of the product $\text{Br}(\bar{b} \to B_0^0) \cdot \text{Br}(B_0^0 \to D_s^- t^+ \nu X)$ and in estimating background process contributions. Additional details are given in the text.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_b$</td>
<td>$0.2204 \pm 0.0020$</td>
<td>[20]</td>
</tr>
<tr>
<td>$\text{Br}(D_s^- \to \phi \pi^-)$</td>
<td>$3.5 \pm 0.4 %$</td>
<td>[13]</td>
</tr>
<tr>
<td>$\Gamma(D_s^- \to K^*0K^-) / \Gamma(D_s^- \to \phi \pi^-)$</td>
<td>$0.95 \pm 0.10$</td>
<td>[13]</td>
</tr>
<tr>
<td>$\Gamma(D_s^- \to \phi e^- \nu) / \Gamma(D_s^- \to \phi \pi^-)$</td>
<td>$0.54 \pm 0.05$</td>
<td>[13]</td>
</tr>
<tr>
<td>$\Gamma(D_s^- \to K^0\phi K^-) / \Gamma(D_s^- \to \phi \pi^-)$</td>
<td>$1.01 \pm 0.16$</td>
<td>[13]</td>
</tr>
<tr>
<td>$\Gamma(D_s^- \to \phi\pi^+\pi^-\pi^-) / \Gamma(D_s^- \to \phi \pi^-)$</td>
<td>$0.51 \pm 0.12$</td>
<td>[13]</td>
</tr>
<tr>
<td>$\Gamma(D_s^- \to K^0\phi K^-) / \Gamma(D_s^- \to \phi \pi^-)$</td>
<td>$1.6 \pm 0.6$</td>
<td>[13]</td>
</tr>
<tr>
<td>$\text{Br}(\phi \to K^+K^-)$</td>
<td>$49.1 \pm 0.9 %$</td>
<td>[13]</td>
</tr>
<tr>
<td>$\text{Br}(K^0 \to \pi^+\pi^-)$</td>
<td>$68.61 \pm 0.28 %$</td>
<td>[13]</td>
</tr>
<tr>
<td>$\text{Br}(B \to D_s^- X)$</td>
<td>$10.46 \pm 0.73 %$</td>
<td>[15, 16]</td>
</tr>
<tr>
<td>$\Gamma(B \to D_s^{(<em>)-} D^{(</em>)}) / \Gamma(B \to D_s^- X)$</td>
<td>$52.8 \pm 6.1 %$</td>
<td>[15, 16]</td>
</tr>
<tr>
<td>$\text{Br}(b \to t\nu X)$</td>
<td>$11.08 \pm 0.31 %$</td>
<td>[21]</td>
</tr>
<tr>
<td>$\text{Br}(b \to c \to \ell)$</td>
<td>$8.22 \pm 0.46 %$</td>
<td>[21]</td>
</tr>
<tr>
<td>$\Gamma(B \to D_s^- X)_{lower\ vertex} / \Gamma(B \to D_s^- X)$</td>
<td>$0.172 \pm 0.083$</td>
<td>[17]</td>
</tr>
<tr>
<td>Source</td>
<td>Uncertainty (%)</td>
<td></td>
</tr>
<tr>
<td>----------------------------------------------------------------------</td>
<td>-----------------</td>
<td></td>
</tr>
<tr>
<td>( \text{Br}(D_s^- \to \phi\pi^-) )</td>
<td>+0.106 -0.085</td>
<td></td>
</tr>
<tr>
<td>Accidental combination background</td>
<td>+0.030 -0.097</td>
<td></td>
</tr>
<tr>
<td>( B \to D_s^- X_s \ell^+\nu ) background</td>
<td>+0.045 -0.045</td>
<td></td>
</tr>
<tr>
<td>Other branching ratios</td>
<td>+0.036 -0.039</td>
<td></td>
</tr>
<tr>
<td>( B \to D_s^{(<em>)-} D^{(</em>)} X ) background</td>
<td>+0.016 -0.016</td>
<td></td>
</tr>
<tr>
<td>Combinatorial background</td>
<td>+0.016 -0.015</td>
<td></td>
</tr>
<tr>
<td>Reflection background</td>
<td>+0.013 -0.012</td>
<td></td>
</tr>
<tr>
<td>( R_b )</td>
<td>+0.008 -0.007</td>
<td></td>
</tr>
<tr>
<td>Efficiency estimates (simulation)</td>
<td>+0.005 -0.005</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>+0.128 -0.144</td>
<td></td>
</tr>
</tbody>
</table>

Table 8: Sources of systematic uncertainty in percent on the product branching ratio \( \text{Br}(b \to B_s^0) \cdot \text{Br}(B_s^0 \to D_s^- \ell^+\nu X) \).

in agreement with the results from reference [12]. An alternative and independent ALEPH measurement using \( D_{s-} \) -hadron correlations[24] found \( \tau(B_s^0) = 1.61^{+0.20}_{-0.29} \) (stat) \( ^{+0.18}_{-0.16} \) (syst) ps using data from 1991 to 1993. The two ALEPH results can be combined to give

\[
\tau(B_s^0) = 1.59^{+0.15}_{-0.14} \text{ ps },
\]

which is consistent with the \( B_s^0 \) lifetime \( 1.58 \pm 0.06 \) ps[25], as expected from theoretical considerations[1].

The product branching ratio is measured to be

\[
\text{Br}(b \to B_s^0) \cdot \text{Br}(B_s^0 \to D_s^- \ell^+\nu X) = 0.82 \pm 0.09 \text{ (stat)}^{+0.13}_{-0.14} \text{ (syst) \% },
\]
in agreement with previous measurements[3, 22, 23].

Acknowledgements

It is a pleasure to thank our colleagues in the accelerator divisions of CERN for the excellent performance of the LEP accelerator. Thanks are also due to the technical personnel of the collaborating institutions for their support in constructing and maintaining the ALEPH experiment. Those of us not from member states wish to thank CERN for its hospitality.
References


The mass distribution of the $D_s^-$ candidates for $D_s^- \ell^+$ combinations for hadronic $D_s^-$ decays, and (b) the $K^+K^-$ mass distribution for semileptonic $D_s^-$ decays, showing the fits described in the text. The shaded histograms represent the like-sign mass spectra.

Figure 1: (a) The mass distribution of the $D_s^-$ candidates for $D_s^- \ell^+$ combinations for hadronic $D_s^-$ decays, and (b) the $K^+K^-$ mass distribution for semileptonic $D_s^-$ decays, showing the fits described in the text. The shaded histograms represent the like-sign mass spectra.
Figure 2: The proper time distributions of (a) the $D_s^-\ell^+$ events in the $D_s^-$ and $\phi$ mass window showing the fitted contributions of the signal and the combinatorial and $B \rightarrow D_s^{(*)} X$ backgrounds as described in the text. (b) The fit to the proper time distribution of the events in the $D_s^-\ell^+$ sideband and the like-sign events.