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The performance of a dual recycled interferometer with squeezed light

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We consider a dual recycled interferometer for detecting gravitational waves. Squeezed light is injected into the interferometer through the output port. We analyse the sensitivity of the detector for the two cases of interest, namely the broad band and the narrow band modes of operation. The improvement in sensitivity due to the squeezed light is investigated in such a set up.

I. INTRODUCTION

The detection of gravitational waves promises to be a very challenging problem. Gravitational waves would provide a great deal of information about some astrophysical sources, an example being binary systems. Since the amplitude of these waves is extremely weak, considerable effort has gone into the design of very sensitive detectors.

A gravitational wave of suitable polarisation would bring about an effective change in the pathlength of light in the arms of a Michelson interferometer which can subsequently be detected by monitoring the output intensity. The weak signal has to be picked out from noise, one among which is the photon shot noise [1]. It would therefore be very useful to find ways of reducing this noise. Recycling techniques serve to reduce this noise. The interferometer would be made to work close to a dark fringe for maximum sensitivity. Most of the light from the laser is reflected back and hence wasted. Power recycling works by suitably placing a partially transmitting mirror in front of the laser beam so that this light can be reflected back into the interferometer to increase the effective power of the laser [2]. The sidebands generated by the gravitational wave propagate towards the photodetector (pd). These can also be recycled by a partially transmitting mirror appropriately placed in front of the pd. This would serve to increase the strength of the signal. When both methods are used, we have a dual recycled interferometer [3]. For a Michelson interferometer, the photon shot noise can be understood to be due to the mixing in of vacuum modes which enter through the output port and the laser light. It was suggested that by sending squeezed light into this port, the output photon number fluctuations could be reduced [4]. There are however losses in the system which allow vacuum modes to mix in with the squeezed light, thus decreasing its effectiveness. Detailed calculations show however that for interferometers working close to unit fringe visibility, the squeezed state technique is certainly useful in increasing the sensitivity [5].

In this paper we analyse the sensitivity of a dual recycled interferometer into which we send in squeezed light. Previous work showed that power recycling and the squeezed state technique are compatible [6]. Here we extend the analysis to dual recycling. We consider two cases of interest namely, the broad band and the narrow band modes of operation in dual recycling. Broad band(BB) and narrow band(NB) refer to the nature of the frequency response of the entire interferometer [7]. In each of the above cases we analyse the usefulness of the squeezed state technique and its effect on the bandwidth of the interferometer by calculating the sensitivity. In both cases, squeezed light is potentially useful in increasing the sensitivity. Nevertheless we find that the squeezed state technique is more useful in the NB regime than in the BB. In the NB case, squeezed light enhances the sensitivity without changing the bandwidth of the interferometer. We also show that for small asymmetry in the losses in the two arms of the interferometer, the sensitivity maximum shifts by a small amount with respect to the frequency. The paper is organised as follows. In section II we first calculate the response of a Fabry-Perot (F-P) cavity to a gravitational wave. Having established a few notations we then calculate the electric fields in the interferometer. In section III the noise and signal are defined. The noise power spectral density is calculated for the two cases of interest namely the BB and NB modes of operation of the interferometer in section IV. We present our conclusions in section V.

II. CALCULATION OF THE OUTPUT FIELDS OF THE INTERFEROMETER IN THE PRESENCE OF A GRAVITATIONAL WAVE

In this section we will calculate the electric fields at the output port when a gravitational wave is incident on the detector. The large scale detectors will be using F-P cavities in their arms so as to increase the storage time of light. We will therefore be considering two identical F-P cavities aligned along the arms of the interferometer. We perform the computation in two steps; first, we evaluate the response of a single F-P cavity and then apply this result to the entire configuration which consists of the two F-P cavities and the recycling mirrors (see fig.2).

A. Response of a F-P cavity

Assume that a gravitational wave of plus polarisation, propogating along the z-axis, of dimensionless amplitude h_o and frequency ω_g impinges on an interferometer which is located in the x-y plane. The two arms of the interferometer are along the x and y axis. Consider the F-P cavity aligned along the x-axis with a lossless input mirror of amplitude reflectivity r_o ($r_o^2 + t_o^2 = 1$) and end mirror amplitude reflectivity unity. The total round trip travel time for light is $\tau = 2l/c$, where l is the length of the cavity and c is the speed of light. If laser light (of frequency ω_l) propogates for time τ then the phase it acquires due to the gravitational wave is [8],

$$\Delta\phi(t) = \int_{t-\tau}^t \delta\phi(t')dt' = \epsilon_g \sin \omega_g(t - \frac{\tau}{2}), \quad (2.1)$$

where,

$$\epsilon_g = \frac{h_o \omega_l}{\omega_g} \sin \frac{\omega_g \tau}{2}. \quad (2.2)$$

Consider fig.1, where the symbols e_1, e_2, e_o and e_R stand for the electric fields as indicated. The equations for the electric fields are,

$$e_1(t) = t_o e_o(t) + r_o e_2(t), \quad (2.3)$$

$$e_2(t) = e_1(t - \tau) e^{i\Delta\phi(t)}, \quad (2.4)$$

$$e_R(t) = -r_o e_o(t) + t_o e_2(t). \quad (2.5)$$

The sign convention for reflection is chosen such that a reflection from the inner surfaces have a positive sign whereas for the outer surfaces, a negative sign. We require $e_R(t)$ in terms of the cavity parameters and e_o . Substituting for e_2 in Eq.(2.3) one obtains,

$$e_1(t) = t_o e_o(t) + r_o e_1(t - \tau) e^{i\epsilon_g \sin \omega_g(t - \frac{\tau}{2})}. \quad (2.6)$$

We now take the Fourier transforms of the electric fields and expand the exponential to first order in ϵ_g to give us,

$$\begin{aligned} \int E_1(\omega) e^{-i\omega t} d\omega &= t_o \int E_o(\omega) e^{-i\omega t} d\omega + r_o \int E_1(\omega) e^{-i\omega t} e^{i\omega \tau} d\omega \\ &+ r_o(\epsilon_g/2) \int E_1(\omega) e^{-i\omega t} e^{i\omega \tau} e^{i\omega_g(t-\tau/2)} d\omega - r_o(\epsilon_g/2) \int E_1(\omega) e^{-i\omega t} e^{i\omega \tau} e^{-i\omega_g(t-\tau/2)} d\omega. \end{aligned} \quad (2.7)$$

Using the above equation we match the Fourier components of the electric fields on both sides for the frequencies we are interested in, namely $\omega_l, \omega_l \pm \omega_g$. We then arrange them in matrix form as

$$\begin{pmatrix} E_o(\omega_l) \\ E_o(\omega_-) \\ E_o(\omega_+) \end{pmatrix} = \chi \begin{pmatrix} E_1(\omega_l) \\ E_1(\omega_-) \\ E_1(\omega_+) \end{pmatrix}, \quad (2.8)$$

where, χ is a 3×3 matrix. We need to evaluate the amplitude of the reflected light $e_R(t)$. Using Eq.(2.5) and the previous equation we obtain, $E_R = S E_o$, where S is given by,

$$S = \begin{pmatrix} \frac{-r_o + e^{i\omega_l \tau}}{1 - r_o e^{i\omega_l \tau}} & \frac{-t_o^2(\epsilon_g/2)e^{i(\omega_l - \omega_g/2)\tau}}{(1 - r_o e^{i\omega_l \tau})(1 - r_o e^{i(\omega_l - \omega_g)\tau})} & \frac{t_o^2(\epsilon_g/2)e^{i(\omega_l + \omega_g/2)\tau}}{(1 - r_o e^{i\omega_l \tau})(1 - r_o e^{i(\omega_l + \omega_g)\tau})} \\ \frac{t_o^2(\epsilon_g/2)e^{i(\omega_l - \omega_g/2)\tau}}{(1 - r_o e^{i\omega_l \tau})(1 - r_o e^{i(\omega_l - \omega_g)\tau})} & \frac{-r_o + e^{i(\omega_l - \omega_g)\tau}}{1 - r_o e^{i(\omega_l - \omega_g)\tau}} & 0 \\ \frac{-t_o^2(\epsilon_g/2)e^{i(\omega_l + \omega_g/2)\tau}}{(1 - r_o e^{i\omega_l \tau})(1 - r_o e^{i(\omega_l + \omega_g)\tau})} & 0 & \frac{-r_o + e^{i(\omega_l + \omega_g)\tau}}{1 - r_o e^{i(\omega_l + \omega_g)\tau}} \end{pmatrix}. \quad (2.9)$$

It is useful to write down the S matrix in a much more suggestive form, namely,

$$S = \begin{pmatrix} R(\omega_l) & -\epsilon_1 & \epsilon_2 \\ \epsilon_1 & R(\omega_l - \omega_g) & 0 \\ -\epsilon_2 & 0 & R(\omega_l + \omega_g) \end{pmatrix}. \quad (2.10)$$

The diagonal terms represent the reflectivity of the F-P cavity at the three different frequencies. Since the F-P cavities are lossless, the reflectivities $R(\omega_l, \omega_l \pm \omega_g)$ are just phase factors of modulus unity. The off diagonal terms $-\epsilon_1$ and ϵ_2 represent the amplitude of the 'signal', which depends on the round trip travel time of light in the arm as well as the gravitational wave amplitude h_o [6]. As can be seen, the S matrix is unitary. The matrix S, can be interpreted as a scattering matrix which relates the input field to the output field at the laser and sideband frequencies. In the quantum theory we need to know the initial states of the input light at the three frequencies to be able to evaluate the noise and signal.

B. F-P cavities and the recycling configuration

We now go on to study our set up (see fig.2) which has in addition to two F-P cavities in the arms of the interferometer, the recycling mirrors (M1 and M2). The figure indicated is schematic and the beams are shown as spatially separated although they are superimposed on each other in principle. This has been done mainly for mathematical convenience. Denoting various electric fields as shown in the figure we will ultimately express $E_{a'}$ and $E_{b'}$ in terms of all the interferometer parameters and the input fields which are E_a and E_b . We describe the propagation of the field between the recycling mirrors and the beam splitter(BS) by diagonal matrices S_{p1} (M1 to BS) and S_{p2} (M2 to BS). The lengths between M1, M2 and the beam splitter are given by L_1 and L_2 respectively. Here we choose a convenient notation,

$$e^{i\omega L_1/c} = e^{i\phi_1(\omega)}, \quad e^{i\omega L_2/c} = e^{i\phi_2(\omega)}.$$

In which case, our propagation matrices are,

$$S_{p1} = \begin{pmatrix} e^{i\phi_1(\omega_l)} & 0 & 0 \\ 0 & e^{i\phi_1(\omega_l - \omega_g)} & 0 \\ 0 & 0 & e^{i\phi_1(\omega_l + \omega_g)} \end{pmatrix}, \quad S_{p2} = \begin{pmatrix} e^{i\phi_2(\omega_l)} & 0 & 0 \\ 0 & e^{i\phi_2(\omega_l - \omega_g)} & 0 \\ 0 & 0 & e^{i\phi_2(\omega_l + \omega_g)} \end{pmatrix}. \quad (2.11)$$

Losses in the arms of the interferometer are modeled by the introduction of partially transmitting mirrors [9]. For this purpose we introduce two beam splitters (BS1 and BS2) into the path of each of the beams which enter the two F-P cavities. These beamsplitters carry away some of the mean intensity incident on the F-P cavies as well as introduce noise by allowing vacuum fluctuations to leak in. If the transmission coefficient of one of the beamsplitters is t then the power loss due to losses will be $(1 - t^2)P_{inc}$, where, P_{inc} is the incident power. The power recycling cavity consists of the mirror M1 and the two arms of the interferometer. Hence, for equal losses, the beamsplitters(BS1 and BS2) together, mimic an effective mirror of reflectivity t . The signal recycling cavity likewise consists of M2 and an effective mirror of reflectivity t . We now set up the equations to enable us to calculate the intracavity fields. The equations for the electric fields, which after starting from the two ports and enter the BS are,

$$E_1 = \frac{S_{p1}(t_1 E_a + r_1 E_{a'}) + S_{p2}(t_2 E_b + r_2 E_{b'})}{\sqrt{2}}, \quad E_2 = \frac{S_{p1}(t_1 E_a + r_1 E_{a'}) - S_{p2}(t_2 E_b + r_2 E_{b'})}{\sqrt{2}}. \quad (2.12)$$

We now allow for some fraction of the light to be lost by the beamsplitters BS1 and BS2 of transmission coefficients t_{11}, t_{12} . The equation for the electric fields that reach the F-P cavities after passing through these beamsplitters are,

$$E_3 = t_{11} E_1 + r_{11} E_{v1}, \quad E_4 = t_{12} E_2 + r_{12} E_{v2}, \quad (2.13)$$

where E_{v1} and E_{v2} are vacuum modes at the three frequencies that enter as noise. The action of the F-P cavities are described by the scattering matrices calculated earlier. For arm 1 we use the S matrix in Eq.(2.9), whereas for arm 2 we use the same matrix but with a negative sign for h_o . A phase offset δ is also introduced between the two arms. The light reaching the BS (after passing through the F-P cavities) is,

$$E_5 = e^{i\delta/2} S_1 E_3, \quad E_6 = e^{-i\delta/2} S_2 E_4. \quad (2.14)$$

The last set of equations we need are for $E_{a'}$ and $E_{b'}$. They are obtained as,

$$E_{a'} = \frac{S_{p1}(E_5 + E_6)}{\sqrt{2}}, \quad E_{b'} = \frac{S_{p2}(E_5 - E_6)}{\sqrt{2}}. \quad (2.15)$$

The above set of equations have to be solved selfconsistently for the intracavity fields. Once $E_{b'}$ is obtained, the equation for the light which is finally detected at the photodetector E_c is,

$$E_c = t_2 E_{b'} - r_2 E_b. \quad (2.16)$$

III. DEFINING THE NOISE AND SIGNAL

Having obtained the classical Fourier amplitudes of the electric fields at the three frequencies of interest, one can make the following correspondence to the annihilation operators. For example,

$$E_a(\omega) = \sqrt{\frac{\hbar\omega}{\epsilon V}} \hat{a}(\omega), \quad (3.1)$$

where $\hbar$ is plancks constant, ϵ is the permittivity and V is the quatisation volume. For the frequencies at $\omega_l \pm \omega_g$, the normalisation constant is taken to be the same as that at frequency ω_l (since $\omega_g \ll \omega_l$). We now have to specify the quantum state of the modes at the various frequencies. $\hat{a}(\omega_l)$ and $\hat{b}(\omega_l)$ are the operators for coherent light entering from the laser through the input port and squeezed light entering through the output port respectively, both at the laser frequency. The other operators at the sideband frequencies are in vacuum states. The coherent state is assumed to have $\bar{n}$ mean number of photons and the phase of the laser is set to zero. For the squeezed state we have the mean number of photons as $\sinh^2(s)$ where the squeeze factor is $\xi = e^{i\theta}s$, θ being the squeeze angle. At the laser frequency ω_l we get,

$$c(\omega_l) = Aa(\omega_l) + Bb(\omega_l) + \dots, \quad (3.2)$$

where,

$$A = \frac{(t_1 t_2 R(\omega_l) T_- e^{i\phi_2(\omega_l)} e^{i\phi_1(\omega_l)})}{2[1 - T_+/2R(\omega_l)(r_1 e^{i2\phi_1(\omega_l)} + r_2 e^{i2\phi_2(\omega_l)}) + r_1 r_2 t_{l1} t_{l2} R^2(\omega_l) e^{i2\phi_1(\omega_l)} e^{i2\phi_2(\omega_l)}]}, \quad (3.3)$$

$$B = \frac{t_2^2 R(\omega_l) e^{i2\phi_1(\omega_l)} e^{i2\phi_2(\omega_l)} (-r_1 R(\omega_l) e^{i2\phi_1(\omega_l)} + T_+/2)}{[1 - T_+/2R(\omega_l)(r_1 e^{i2\phi_1(\omega_l)} + r_2 e^{i2\phi_2(\omega_l)}) + r_1 r_2 t_{l1} t_{l2} R^2(\omega_l) e^{i2\phi_1(\omega_l)} e^{i2\phi_2(\omega_l)}]} - r_2. \quad (3.4)$$

The dotted points indicate the amplitudes of the vacuum states due to the losses. They are not required for the calculation as we will be evaluating normal ordered expressions. In the above expressions,

$$R(\omega_l) = \frac{e^{i\omega_l \tau} - r_o}{1 - e^{i\omega_l \tau} r_o}. \quad (3.5)$$

represents the reflection coefficient of the identical F-P cavities to laser light. We will take the cavities to be resonant at ω_l , hence $R(\omega_l) = 1$. The expressions for T_+ and T_- are,

$$T_+ = t_{l1} e^{i\delta} + t_{l2} e^{-i\delta}, \quad T_- = t_{l1} e^{i\delta} - t_{l2} e^{-i\delta}. \quad (3.6)$$

The power recycling cavity which is comprised of the recycling mirror M1 and the two F-P cavities is also resonant at the laser frequency. This implies that $e^{i2\phi_1(\omega_l)} = 1$. The choice of the phase factor $e^{i2\phi_2(\omega_l)}$ (which depends on the distance between BS and M2) is decided by whether we want to make our detector broad band or narrow band.

The mean number of photons in the output field in the carrier mode (ω_l) is,

$$\bar{N} = \langle \hat{N} \rangle = \langle \hat{c}(\omega_l)^\dagger \hat{c}(\omega_l) \rangle. \quad (3.7)$$

The noise is just the fluctuation ΔN in the photon number and is given through the relation,

$$\Delta N^2 = \langle \hat{N}^2 \rangle - \langle \hat{N} \rangle^2. \quad (3.8)$$

The expectation values for the various terms such as $\langle \hat{a}^\dagger \hat{b}^2 \rangle$ etc can be obtained by just using the definition of the coherent and squeezed states as defined by the displacement operators[4]. ΔN when computed for $s > 0$ and under the approximation $\sinh^2(s) \ll \bar{n}$ gives,

$$\Delta N^2 = |A|^2[1 - \rho|B|^2], \quad (3.9)$$

where $\rho = 1 - e^{-2s}$ and A, B are given by Eqs.(3.3) and (3.4). In the above equation the phase of squeezing is chosen to minimise the fluctuations. We keep the above expression in its most general form so that we can come back to it for special cases. The signal is obtained by beating the sidebands at frequency $\omega_l \pm \omega_g$ with laser light. In calculating the sidebands, the only term which is important is the one contributed by the laser light. The expressions for the sidebands are,

$$c(\omega_l + \omega_g) = Y_+ a(\omega_l), \quad (3.10)$$

$$c(\omega_l - \omega_g) = Y_- a(\omega_l), \quad (3.11)$$

where,

$$Y_{\pm} = \frac{N_{\pm}}{M_{\pm}P}, \quad (3.12)$$

and,

$$N_+ = -t_1 t_2 \epsilon_2 e^{i2\phi_2(\omega_l + \omega_g)} (-r_1 t_{l1} t_{l2} R(\omega_l + \omega_g) e^{i2\phi_1(\omega_l + \omega_g)} + T_+/2), \quad (3.13)$$

$$N_- = t_1 t_2 \epsilon_1 e^{i2\phi_2(\omega_l - \omega_g)} (-r_1 t_{l1} t_{l2} R(\omega_l - \omega_g) e^{i2\phi_1(\omega_l - \omega_g)} + T_+/2), \quad (3.14)$$

$$M_{\pm} = 1 - T_+/2R(\omega_l \pm \omega_g) (r_1 e^{i2\phi_1(\omega_l \pm \omega_g)} + r_2 e^{i2\phi_2(\omega_l \pm \omega_g)} + r_1 r_2 t_{l1} t_{l2} R(\omega_l \pm \omega_g)^2 e^{i2\phi_1(\omega_l \pm \omega_g)} e^{i2\phi_2(\omega_l \pm \omega_g)}), \quad (3.15)$$

and

$$P = 1 - r_1 T_+/2. \quad (3.16)$$

The expressions for ϵ_1 and ϵ_2 have been described in [Eqs.(2.9) and (2.10)]. It is instructive at this stage to interpret the sideband amplitude. For equal losses in both arms, $t_{l1} = t_{l2} = t$. If we evaluate the sideband at $\omega_l + \omega_g$ when we are close to the dark fringe ($\delta \simeq 0$), we obtain,

$$c(\omega_l + \omega_g) = -t \left[\frac{t_1}{1 - r_1 t} \right] [\epsilon_2] \left[\frac{t_2}{1 - r_2 t R(\omega_l + \omega_g) e^{i2\phi_2(\omega_l + \omega_g)}} \right] e^{i\phi_2(\omega_l + \omega_g)} a(\omega_l). \quad (3.17)$$

The first term in square brackets is the enhancement factor due to power recycling. The second term expresses the strength of the signal. The last term is the signal recycling factor. As can be seen in each recycling factor the losses behave as a partially transmitting mirror of reflectivity t . The formula for the change in photon number due to the gravitational wave, which is the signal is given by,

$$\delta N(t) = \langle c^\dagger(\omega_l) c(\omega_l + \omega_g) + c^\dagger(\omega_l) c(\omega_l - \omega_g) + c.c. \rangle = \delta N_{max} \sin(\omega_g t + \phi), \quad (3.18)$$

where, in terms of the electric field amplitudes,

$$\delta N_{max} = 2\bar{n} |A^* Y_- + A Y_+^*|. \quad (3.19)$$

In the above equation δN_{max} is the amplitude and ϕ is the phase of the signal. The latter can be chosen to be zero by a time shift. δN_{max} is proportional to h_o through Y_+ and Y_- (which, being the sideband amplitudes, themselves are proportional to h_o).

IV. NOISE POWER SPECTRAL DENSITY

We assume that in the absence of a gravitational wave, the photon number fluctuations mimic a signal. The strength of the signal [Eq. (3.19)] is equal to δN_{max} (which is proportional to h_o). Equating this signal to the r.m.s noise, ΔN [Eq.(3.9)] we obtain,

$$\delta N_{max}(h_o) = \Delta N. \quad (4.1)$$

This gives us an equation for h_o from which we can calculate the power spectral density for noise ($S_h(\omega_g)$) as,

$$S_h(\omega_g) = h_o^2 / \delta\omega, \quad (4.2)$$

where $\delta\omega$ is the inverse of the measurement time of the signal. The sensitivity is defined as the inverse of $S_h(\omega_g)$.

A. Broad band

For the broad band case the distance between the signal recycling mirror and the BS is adjusted such that the whole signal recycling cavity is resonant at the laser frequency. Since the individual F-P cavities are already resonant at the laser frequency, this implies that $e^{i2\phi_2(\omega_l)} = 1$. Close to a dark fringe, the sidebands generated by the gravitational wave emerge towards the photodetector, where they are recycled by mirror M2. The two sidebands are now of almost the same strength, being equally displaced from the central frequency. Carrying out the computations as indicated in Eq.(4.1) and Eq.(4.2), we obtain the expression for $S_h(\omega_g)$:

$$S_h(\omega_g) = \frac{(1-r_1t)^2(1-r_2t)^2(1-r_o)^4(1+\omega_g^2/\omega_p^2)}{\bar{n}\delta\omega t^2 t_2^2 t_1^2 t_o^4 \omega_l^2 \tau^2} \left[1 - \rho \frac{(t-r_2)^2}{(1-r_2t)^2}\right], \quad (4.3)$$

where $\omega_p = (1-r_o)(1-r_2t)/((r_o+r_2t)\tau)$. The broad band behaviour of the sensitivity is apparent from the form of the function $1+\omega_g^2/\omega_p^2$, which is slowly varying. We note that for a squeeze factor $s > 0$, the term in square brackets is always less than 1 (since $\rho > 0$). So evidently squeezing helps to reduce $S_h(\omega_g)$. If we maximise the sensitivity within a frequency range over which it is almost flat, then we find that the maximum sensitivity is at $r_1 = r_2 = t$. This is so if we are constrained only by the losses of the interferometer. The expression for the sensitivity becomes,

$$1/S_h(\omega_g) = \frac{\bar{n}\delta\omega t^2 t_o^4 \omega_l^2 \tau^2}{(1-t^2)(1-r_o)^4 [1+\omega_g^2/\omega_p^2]}. \quad (4.4)$$

We can also see from Eq.(4.3) that for $r_2 = t$ the squeezing factor drops out and therefore does not contribute to the sensitivity. We can understand why this happens in the following way. The signal recycling cavity consists of the signal recycling mirror M2 and an effective mirror formed by the two F-P cavities of reflectivity t . Setting $r_2 = t$ implies that the signal recycling cavity which is resonant at the laser frequency has equal reflectivity mirrors - M2 and the effective mirror formed by the two F-P cavities. The transmission of the resonant frequency is maximised in this case. All of the squeezed light is transmitted through the cavity, to be dissipated in the losses of the interferometer. For squeezing to be useful, some of the squeezed light must be reflected back from the cavity to be able to interfere with the laser light. But this can only happen when $r_2 \neq t$ where the sensitivity is not maximum. We therefore see that squeezing is not so useful in this case if we are limited only by the losses.

B. case (ii) Narrow band

In this case one of the sidebands is made resonant with the signal recycling cavity by choosing appropriately the distance between M2 and the BS. Let us consider the expression Eq.(3.10) and Eq.(3.11) for the sidebands. For equal losses ($t_{l1} = t_{l2} = t$) the equations read as,

$$c(\omega_l + \omega_g) = \frac{-t_1 t_2 t \epsilon_2 e^{i\phi_2(\omega_l + \omega_g)}}{(1-r_1t)\gamma_+}, \quad (4.5)$$

$$c(\omega_l - \omega_g) = \frac{t_1 t_2 t \epsilon_1 e^{i\phi_2(\omega_l - \omega_g)}}{(1-r_1t)\gamma_-}, \quad (4.6)$$

where,

$$\gamma_+ = 1 - r_2 t R(\omega_l + \omega_g) e^{2i\phi_2(\omega_l + \omega_g)}, \quad \gamma_- = 1 - r_2 t R(\omega_l - \omega_g) e^{2i\phi_2(\omega_l - \omega_g)}. \quad (4.7)$$

If we wish to enhance $c(\omega_l + \omega_g)$ then γ_+ must be as small as possible. This could be done by adjusting L_2 such that,

$$R(\omega_l + \omega_g) e^{2i\phi_2(\omega_l + \omega_g)} = 1. \quad (4.8)$$

Now $R(\omega_l + \omega_g)$ is the F-P cavity reflectivity for the sideband at $\omega_l + \omega_g$. For $\omega_g \tau \ll 1$,

$$R(\omega_l + \omega_g) \simeq \frac{-r_o + 1 + i\omega_g \tau}{-r_o + 1 - i\omega_g \tau}. \quad (4.9)$$

In this approximation,

$$R(\omega_l + \omega_g) = e^{i2t \tan^{-1}(\omega_g \tau / (1-r_o))} = e^{i\psi(\omega_g)}. \quad (4.10)$$

For $R(\omega_l - \omega_g)$, which represents the F-P cavity reflectivity for the other sideband we have,

$$R(\omega_l - \omega_g) = e^{-i\psi(\omega_g)}. \quad (4.11)$$

The narrow banding condition now reads as,

$$e^{2i\phi_2(\omega_l + \omega_g)} e^{i\psi(\omega_g^o)} = 1, \quad (4.12)$$

where ω_g^o is the particular gravitational wave frequency for which the signal recycling cavity has been tuned for resonance. Using the resonance condition Eq.(4.8), $\gamma_+ = 1 - r_2 t$. We must also consider the expression for the other sideband to see by how much it is suppressed. We have,

$$\gamma_- = 1 - r_2 t e^{2i\phi_2(\omega_l - \omega_g)} e^{-i\psi(\omega_g^o)}. \quad (4.13)$$

Since $\phi_2(\omega_l \pm \omega_g) = (\omega_l \pm \omega_g)L_2/c \simeq \omega_l L_2/c$, the expressions for γ_- is,

$$\gamma_- = 1 - r_2 t e^{2i\phi_2(\omega_l)} e^{-i\psi(\omega_g^o)}. \quad (4.14)$$

Since we want to enhance the sideband at $\omega_l + \omega_g$, the condition for which the sideband at $\omega_l - \omega_g$ gets considerably suppressed is when γ_- takes its largest value, i.e. when,

$$\gamma_- = 1 + r_2 t. \quad (4.15)$$

This implies that maximum narrow banding occurs when,

$$e^{i\psi(\omega_g^o)} = i. \quad (4.16)$$

This also implies through Eq.(4.10),

$$\omega_g^o \simeq (1 - r_o)/\tau. \quad (4.17)$$

This can be understood in the following manner. $\tau/(1 - r_o)$ is like the storage time of the F-P cavities and therefore it must be comparable to the period of the gravitational wave for resonance to occur. This condition fixes the reflectivity r_o depending on the frequency of the gravitational wave that we are most interested in. The above resonance condition can also be explained in terms of phasors. The complex vector $e^{i\psi(\omega_g^o)}$ rotates the vector $r_2 t e^{i2\phi_2(\omega_l)}$ to orient it along the negative x-axis which makes the resultant vector $1 - r_2 t$ very small. For the negative sideband the vector $r_2 t e^{i2\phi_2(\omega_l)}$ gets rotated in the opposite direction by the angle $-\psi(\omega_g^o)$ which makes the resultant $1 + r_2 t$ quite large in comparison. Since $\gamma_- \gg \gamma_+$ we can neglect the sideband at $\omega_l - \omega_g$ for further analysis. If we now make a Taylor expansion for γ_+ around the resonant frequency ω_g^o we get,

$$\gamma_+ = 1 - r_2 t - i(\omega_g - \omega_g^o) r_2 t \psi'(\omega_g^o), \quad (4.18)$$

where,

$$\psi'(\omega_g^o) = \frac{2X}{\omega_g^o(1 + X^2)}, \quad (4.19)$$

and, $X = \omega_g \tau / (1 - r_o)$. We have already seen that maximum narrow banding occurs when $X = 1$, hence $\psi'(\omega_g^o) = 1/\omega_g^o$. With the above expansion of γ_+ we now use Eq.(4.1) and Eq.(4.2) to obtain an expression for the power spectral density $S_h(\omega_g)$. Since in most circumstances r_1, r_2 and t are close to unity, we write $t = 1 - \alpha, r_1 = 1 - \beta_1, r_2 = 1 - \beta_2$ (where α, β_1 and β_2 are small compared to unity). Keeping terms linear in α, β_1 and β_2 , the expression for the sensitivity becomes,

$$1/S_h(\omega_g) = \frac{16e^{2\epsilon}\beta_1\beta_2}{(\alpha + \beta_1)^2(\alpha + \beta_2)^2} \frac{1}{[1 + \Delta\omega_g^2/((\omega_g^o)(\alpha + \beta_2))^2]} \frac{S_o}{[1 + \omega_g^2 r_o^2/(\omega_g^o)^2]}, \quad (4.20)$$

where,

$$S_o = \frac{P_o t_o^4 \omega_l \tau^2}{\hbar(1 - r_o)^4}, \quad \Delta\omega_g = \omega_g - \omega_g^o. \quad (4.21)$$

In the above expression, the laser power P_o has been introduced through the relation, $\bar{n}\delta\omega = P_o/\hbar\omega_l$. As seen from Eq.(4.20), the sensitivity is increased by the factor e^{2s} assuming that the rest of the parameters remain unchanged. The optimum choice of β_1 and β_2 for which the sensitivity is maximised is $\alpha = \beta_1 = \beta_2$. We consider the following parameters for the numerical value of the sensitivity. $P_o=5\text{W}$ and $\omega_l = 3.6 \times 10^{15}$ rad/sec. The arm length of the interferometer is 3 km, for which $\tau = 2 \times 10^{-5}$ sec. The value of the reflectivity of the front mirror(r_o) of the F-P cavities is determined by Eq.(4.17). We choose the resonant frequency to be $\omega_g = 1000$ rad/sec, for which $r_o=.98$. We consider 1% power losses in the interferometer, which corresponds to $\alpha = .5 \times 10^{-2}$. For these parameters, the sensitivity evaluated at the resonant frequency $\omega_g^o = 1000$ rad/sec can be written as, $1/S_h(\omega_g^o = 1000) = e^{2s} 1.38 \times 10^{49}$. It is clear that for squeezed light with $s > 0$, the sensitivity increases. With a squeeze factor of $s = .45$, which corresponds to 60% squeezing [5], the sensitivity is enhanced by a factor of 2.5. A graph of sensitivity (in units of S_o) plotted verses frequency (see fig. 3) has been displayed for three different squeeze factors. The figure shows that squeezed light clearly enhances the sensitivity. The bandwidth is approximately given by $\Delta\omega \simeq (\alpha + \beta_2)\omega_g^o$ which for the parameters we have chosen is 10 rad/sec. The crucial point to notice is that squeezing does not change the bandwidth of the interferometer. The reason as to why squeezing is useful in this case can be understood by recalling that the signal recycling cavity has been tuned to one of the sideband frequencies ($\omega_l + \omega_g^o$). The choice of $r_2 = t$ implies that the signal recycling cavity has end mirrors of equal reflectivity. Hence the sideband generated by the gravitational wave in the F-P cavities is transmitted through completely. Whereas the squeezed light which is not resonant with the signal recycling cavity is partly reflected. This reflected light then interferes with the laser light to reduce the output photon number fluctuations.

Unequal losses in the arms have a detrimental effect on the sensitivity, as has been shown in earlier work [5]. There it was stressed that for the squeezed state technique to be really useful, the visibility must be as close to unity as possible. We will therefore consider only small departures from equal losses. The effects of unequal losses is more pronounced in the case of narrow band (where the signal is made resonant) as compared to the broad band. We therefore concentrate on only the narrow band case. The losses in the two arms are $t_{l1} = t + \Delta t$ and $t_{l2} = t - \Delta t$, where Δt is extremely small. The approximate expression for the sensitivity is,

$$1/S_h(\omega_g) = \frac{16e^{2s}\beta_1\beta_2}{(\alpha + \beta_1)^2(\alpha + \beta_2)^2} \frac{2}{[(1 - \Delta\omega_g/\omega_g^o(\alpha + \beta_2))^2 + (1 + (\Delta t^2 + \Delta\omega_g/\omega_g^o)/(\alpha + \beta_2))^2]} \frac{S_o}{[1 + \omega_g^2 r_o^2/(\omega_g^o)^2]} \quad (4.22)$$

With respect to the two variables $\Delta\omega_g$ and Δt^2 , the above function is a skewed Lorentzian. This means that for the case of an interferometer with a small asymmetry in the arm losses ($\Delta t^2 \neq 0$), the sensitivity maximum shifts slightly to a different frequency. We can estimate by how much this shift is by maximising the sensitivity with respect to the frequency. The maximum sensitivity now appears at,

$$\omega'_g \simeq \omega_g^o - \omega_g^o \Delta t^2 / 2 \quad (4.23)$$

For $\omega_g = 1000$ rad/sec and the asymmetry in losses $\Delta t = .005$, the shift in frequency is by 1.25×10^{-2} rad/sec. This shift in frequency, although small, arises in the following manner. Previously, when we considered equal losses in the arms of the interferometer, the sideband amplitudes on emerging from the F-P cavities propagated towards the signal recycling mirror M2. The 'signal' therefore was made to resonate in the signal recycling cavity. With unequal losses, a small fraction of the sidebands escape towards the power recycling mirror M1, and hence into the power recycling cavity(in which they are not resonant). This leads to a coupling between the power recycling cavity and the signal recycling cavity. The coupled composite cavities if viewed as a single resonator, will be resonant at some frequency ω'_g which will be slightly different from ω_g^o . The sensitivity will therefore shift its maximum.

An experimental setup for using squeezed light in an interferometer with F-P cavities in the arms might be realised by using polarisation techniques [10]. In a dual recycled interferometer, Figure 4 shows how that might be achieved. The polarising beam splitter serves to separate spatially the ingoing squeezed light from the outcoming light which will fall on the photodetector.

V. CONCLUSIONS

We have analysed the usefulness of using squeezed light in a dual recycled interferometer. For the two cases of interest namely the broad band and the narrow band modes of operation, expressions for the sensitivity have been derived. In each case the sensitivity was maximised with respect to the recycling mirror reflectivities. This was done keeping in mind that the losses in the interferometer are the only constraints. In the broad band case where the signal recycling cavity is made resonant with the laser light, squeezing does not enhance the sensitivity. In the narrow band case squeezing increases the sensitivity in addition to that gained by recycling, while not changing the bandwidth of

the interferometer. We also pointed out that for small asymmetry in the arm losses, the resonant frequency of the interferometer shifts by a small amount. The squeezed state technique and dual recycling are therefore compatible. However we believe that further analysis needs to be carried out to take into account practical issues, one among such is the broad band nature of the squeezed light.

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FIG. 1. The electric fields in a Fabry Perot cavity with front mirror reflectivity r_0 and end mirror reflectivity equal to 1.

FIG. 2. The optical arrangement with Fabry Perot cavities in the arms of the interferometer as well as the signal recycling mirror M2 and the power recycling mirror M1. Laser light (E_a) enters from the input port and squeezed light (E_b) enters through the output port. The beam splitter (BS) is assumed to be 50:50. E_c is the electric field detected by the photodetector.

FIG. 3. The graph of Sensitivity in units of S_0 plotted v/s frequency for various values of the squeeze parameter s . The curves a, b, c, d are for $s=0, .25, .46, 0.80$ respectively.

FIG. 4. The optical arrangement showing various components. PBS stands for the polarising beam splitter, QWP stands for the quarter wave plate.

Figure 1

Figure 2

FIG. 3

FIG 4