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**NON-ABELIAN FORMULATION
OF A VECTOR-TENSOR GAUGE THEORY
WITH TOPOLOGICAL COUPLING**

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WITH TOPOLOGICAL COUPLING**

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ABSTRACT

We obtain a non-Abelian version of a theory involving vector and tensor gauge fields interacting via a massive topological coupling, besides the nonminimum one. The new fact is that the non-Abelian theory is not reducible and Stuckelberg fields are introduced in order to compatibilize gauge invariance, nontrivial physical degrees of freedom and the limit of the Abelian case.

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1. The use of tensor fields as gauge fields was pointed out by Kalb and Ramond [1] with the motivation that they could carry the force among string interactions. In spite of this fact be true or not, the Maxwell-like theory for these fields constitutes an interesting subject by its own rights. They are antisymmetric and consequently in $D = 4$ exhibit six degrees of freedom. By virtue of the massless condition, the number of degrees of freedom goes down to four [2]. Since the gauge parameter is a vector quantity, this number would be zero if all of its components were independent. This is not the case because the system is reducible and the final number of physical degrees of freedom is one. This peculiar structure of constraints implies that quantization deserves some care a reasonable amount of work has been done on this subject [3, 4].

On the other hand, the presence of tensor gauge fields in $D = 4$ makes possible the introduction of a kind of Chern-Simons [5] term in the Lagrangian, where vector and tensor fields are coupled in a topological way. The presence of this term gives us an interesting and alternative mechanism of mass generation for vector fields without Higgs, where the remaining degree of freedom of the tensor field is absorbed by the vector one to acquire mass [6].

However, everything that has been done in this direction is in the Abelian case and consequently cannot have connection with the real world where massive vector fields are non-Abelian. The main purpose of our paper is to present a non-Abelian formulation for the vector-tensor gauge theory with the presence of a topological coupling. We shall see that the reducibility condition is lost and that the introduction of Stuckelberg fields redeem all the gauge freedom of theory, as well as the number of physical degrees of freedom. It is important to emphasize that our results are compatible with the Abelian limit and that the mass generation also naturally occurs in the non-Abelian case.

2. Let us briefly review the Abelian case. We shall see that mass generation for the vector field may directly suggest the existence of tensor ones. We also take this opportunity to introduce the main ingredients and ideas that will be useful to develop the non-Abelian counterpart. We start with the well-known Lagrangian of the Maxwell electromagnetic theory

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad (1)$$

where the tensor field $F_{\mu\nu}$ is defined as usual

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (2)$$

Let us now suppose that we would like to have massive photon fields. If we directly put a mass term into the Lagrangian like

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu, \quad (3)$$

we would have two problems (maybe more): The theory would lose the gauge invariance and would not be renormalizable any more. Of course, it is well-known that spontaneous symmetry breaking and the Higgs mechanism cure these two problems. But in the non-Abelian case there might appear residual fields (Higgs bosons). The existency of these fields in nature would explain why and how gauge fields acquire mass. However, there is no precise theoretical predictions on the mass scale where the Higgs fields must be found and experiences till now have shown no evidence about them.

As it was told above, it is possible to have another mass generation mechanism by means of tensor fields and that are not plagued with Higgs bosons. We present below in a general way how this mechanism works out.

Let us consider the Lagrangian (3) by rewriting the mass term with the help of an auxiliary field J_μ as

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} J_\mu J^\mu + m J_\mu A^\mu. \quad (4)$$

We notice that calculating the equation of motion for J_μ and replacing it back into (4), the previous Lagrangian (3) is obtained.

Let us now look at the Lagrangian (4) again, but considering that J_μ is a function of another field. In this case we cannot assume that (3) and (4) are equivalent, even classically. However, the interesting point is that the gauge invariance, lost in the Lagrangian (3), can be restored in (4) if the following properties over J_μ are assumed: off-shell divergenceless and gauge invariance.

It is necessary to be off-shell divergenceless in order to compensate the gauge transformation of A_μ , i.e.

$$\begin{aligned} \delta A_\mu J^\mu &= \partial_\mu \alpha J^\mu, \\ &\longrightarrow -\alpha \partial_\mu J^\mu, \\ &= 0. \end{aligned} \quad (5)$$

Concerning the gauge invariance of J_μ , it is an assumption that can always be done, in principle for the Abelian case.

In order to fulfill these two conditions, tensor fields emerge naturally by writing J_μ as a topological quantity

$$J_\mu = \frac{1}{2} \epsilon_{\mu\nu\rho\lambda} \partial^\nu B^{\rho\lambda}, \quad (6)$$

The initial factor one half is just for convenience. We assume that $B^{\mu\nu}$ is independent of the gauge transformation of the vector gauge field, characterized by the parameter $\alpha(x)$. Consequently, the gauge invariance condition for the topological current J_μ is directly verified. On the other hand, the antisymmetric tensor field

can have its own gauge transformation, that we use a vector parameter $\xi_\mu(x)$ to characterize it

$$\delta B^{\rho\lambda}(x) = \partial^\rho \xi^\lambda(x) - \partial^\lambda \xi^\rho(x). \quad (7)$$

From the expression above, we see the origin of the reducibility of the tensor gauge theory. We notice that $\delta B^{\rho\lambda} = 0$ if ξ_ρ is replaced by the derivative of some scalar quantity. We also verify that J_μ remains invariant for this kind of gauge transformation. If we assume that A_μ does not depend on it, the Lagrangian (4) will be invariant for these two gauge transformations.

Summarizing all the above results, we have

$$\begin{aligned} (i) \quad & \text{Vector gauge transformations :} \\ & \delta_V A_\mu = \partial_\mu \alpha, \\ & \delta_V B_{\mu\nu} = 0, \\ & \delta_V J_\mu = 0, \end{aligned} \quad (8)$$

$$\begin{aligned} (ii) \quad & \text{Tensor gauge transformations :} \\ & \delta_T B_{\mu\nu} = \partial_\mu \xi_\nu - \partial_\nu \xi_\mu, \\ & \delta_T A_\mu = 0, \\ & \delta_T J_\mu = 0, \end{aligned} \quad (9)$$

where we have used different subscripts to denote both transformations. Making the replacement of J_μ , given by (6), into the Lagrangian (4), we get

$$\begin{aligned} -\frac{1}{2} J_\mu J^\mu &= -\frac{1}{8} \epsilon_{\mu\nu\rho\lambda} \epsilon^{\mu\zeta\alpha\beta} \partial^\nu B^{\rho\lambda} \partial_\zeta B_{\alpha\beta}, \\ &\equiv -\frac{1}{72} \epsilon_{\mu\nu\rho\lambda} \epsilon^{\mu\zeta\alpha\beta} H^{\nu\rho\lambda} H_{\zeta\alpha\beta}, \\ &= \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho}, \end{aligned} \quad (10)$$

where the tensor $H_{\mu\nu\rho}$ is defined by

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\rho B_{\mu\nu} + \partial_\nu B_{\rho\mu}. \quad (11)$$

We write down the final expression of the Lagrangian as it usually appears in literature

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{2} m \epsilon_{\mu\nu\rho\lambda} A^\mu \partial^\nu B^{\rho\lambda}. \quad (12)$$

It is important to emphasize that the Lagrangian above, although gauge invariant, effectively describes a massive vector gauge field. This can be seen both classically and quantumly [4, 6].

2. Let us now pass to consider the non-Abelian version of this theory. To fix the notation and convention, we consider that the generators T^a of the $SU(N)$ symmetry group satisfy

$$\begin{aligned} [T^a, T^b] &= i f^{abc} T^c, \\ \text{tr}(T^a T^b) &= \frac{1}{2} \delta^{ab}, \\ (T^a)^{bc} &= -i f^{abc}. \end{aligned} \tag{13}$$

The remaining ingredients involving the non-Abelian vector field are

$$\begin{aligned} D_\mu &= \partial_\mu - ig A_\mu^a T^a, \\ \Rightarrow F_{\mu\nu} &= \frac{1}{g} [D_\mu, D_\nu], \\ &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c, \end{aligned} \tag{14}$$

$$\begin{aligned} \delta_V A_\mu^a &= -\frac{1}{g} (D_\mu \alpha)^a, \\ &= -\frac{1}{g} \partial_\mu \alpha^a + f^{abc} \alpha^b A_\mu^c, \\ \Rightarrow \delta_V F_{\mu\nu}^a &= f^{abc} \alpha^b F_{\mu\nu}^c. \end{aligned} \tag{15}$$

Now, the non-Abelian current J_μ^a is charged. It is then a question of consistency to replace the common derivative by a covariant one in the current definition [7].

$$J_\mu^a = \frac{1}{2} \epsilon_{\mu\nu\rho\lambda} (D^\nu B^{\rho\lambda})^a. \tag{16}$$

It might be opportune to mention that there are no inconsistencies with the fact that vector and tensor fields have two different couplings in the same theory. The one in $m J_\mu^a A^{a\mu}$ is related to a bilinear term, while the ones coming from the covariant derivative are trilinear and quartic. We also observe that J_μ^a is not divergenceless, even in terms of covariant derivative. However, it still keeps the topological condition.

Our next step is to try to construct a non-Abelian gauge invariant Lagrangian. We deal with the vector transformation first.

4. Now, the non-Abelian tensor field might also transform because it is charged. Let us assume that it transforms as a vector in the colour space, i.e.

$$\delta_V B_{\mu\nu}^a = f^{abc} \alpha^b B_{\mu\nu}^c. \quad (17)$$

Considering (17) into the transformation of J_μ^a , we obtain that the non-Abelian topological current also transforms as a vector in the colour space, namely

$$\delta_V J_\mu^a = f^{abc} \alpha^b J_\mu^c. \quad (18)$$

Consequently, the quantity $J_\mu^a J^{a\mu}$ is invariant for the vector gauge transformation. However, the mass term $m A_\mu^a J^{a\mu}$ is not because

$$\delta_V (m A_\mu^a J^{a\mu}) = -\frac{m}{g} \partial_\mu \alpha^a J^{a\mu} \quad (19)$$

and the fact that the topological current is not divergenceless in the non-Abelian case.

To circumvent this problem, we follow the analogue procedure of obtaining the non-Abelian formulation of the Chern-Simons theory [5], that is to say, we look for another noninvariant term to add to the Lagrangian in order that both can restore the gauge invariance. It is then just a matter of algebraic calculation to show that

$$\delta_V (m A_\mu^a J^{a\mu} - \frac{1}{4} g m \epsilon_{\mu\nu\rho\lambda} f^{abc} A^{a\mu} A^{b\nu} B^{c\rho\lambda}) = 0 \quad (20)$$

regardless of total derivatives.

Then, the non-Abelian Lagrangian that is invariant under vector gauge transformation reads

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{2} J_\mu^a J^{a\mu} + m A_\mu^a J^{a\mu} - \frac{1}{4} g m \epsilon_{\mu\nu\rho\lambda} f^{abc} A^{a\mu} A^{b\nu} B^{c\rho\lambda}. \quad (21)$$

5. The final step is to see if the Lagrangian above is also invariant for the tensor gauge transformation or what we have to do in order to be so. Of course, the non-Abelian version of the tensor gauge transformation (7) must be [7]

$$\delta_T B_{\mu\nu}^a = (D_\mu \xi_\nu)^a - (D_\nu \xi_\mu)^a. \quad (22)$$

We can directly show that the topological terms in (21) are actually invariant, i.e.

$$\delta_T (m A_\mu^a J^{a\mu} - \frac{1}{4} g m \epsilon_{\mu\nu\rho\lambda} f^{abc} A^{a\mu} A^{b\nu} B^{c\rho\lambda}) = 0, \quad (23)$$

if one takes

$$\delta_T A_\mu^a = 0. \quad (24)$$

It is important to emphasize that the extra term previously introduced to achieve the vector transformation is crucial in the obtainment of the invariance above.

Concerning the nontopological term $\frac{1}{2} J_\mu^a J^{a\mu}$, there is a point in the transformation (22) that makes this term significantly different from the Abelian case. We notice that if one replaces the gauge parameter ξ_μ^a by a derivative (even covariant) of some (spacetime) scalar quantity, we do not get zero as in the Abelian case. So, the reducibility condition does not occur in the non-Abelian formulation.

Using the transformations (22) and (24), we get

$$\begin{aligned} \delta_T J_\mu^a &= \frac{1}{2} \epsilon_{\mu\nu\rho\lambda} (D^\nu \delta_T B^{\rho\lambda})^a \\ &= \epsilon_{\mu\nu\rho\lambda} (D^\nu D^\rho \xi^\lambda)^a \\ &= \frac{1}{2} \epsilon_{\mu\nu\rho\lambda} ([D^\nu, D^\rho] \xi^\lambda)^a \\ &= \frac{1}{2} g \epsilon_{\mu\nu\rho\lambda} f^{abc} F^{b\nu\rho} \xi^{c\lambda} \end{aligned} \quad (25)$$

and we can verify that the nontopological term is not invariant for the tensor gauge transformations. Further, the nature of this term is different of the massive ones, and there is no term that can be add to the Lagrangian in order to get the invariance. We could also try to generalize the previous transformations or try to modify the definition of the non-Abelian tensor quantity. But nothing of these procedures can lead to the tensor gauge invariance to the nontopological term.

This result appears to make sense. If one could get a invariant nontopological term and being the theory nonreducible, the tensor gauge field would have zero degrees of freedom.

A way we envisage to circumvent this problem is to extend the configuration space by introducing some convenient auxiliary field. The interesting point is that a Stuckelberg field can be introduced for this purpose, but without spoiling the Abelian limit. To do this we define a new tensor field as follows

$$\tilde{B}_{\mu\nu}^a = B_{\mu\nu}^a + (D_\mu \phi_\nu)^a - (D_\nu \phi_\mu)^a \quad (26)$$

We actually notice that the non-Abelian Stuckelberf field ϕ_μ^a disappears in the Abelian limit of the new topological current

$$\tilde{J}_\mu^a = \frac{1}{2} \epsilon_{\mu\nu\rho\lambda} (D^\nu B^{a\rho\lambda} + 2 D^\nu D^\rho \phi^\lambda)^a. \quad (27)$$

Considering that the tensor gauge transformation of the Stuckelber field is

$$\delta_T \phi_\mu^a = -\xi_\mu^a, \quad (28)$$

we get

$$\delta_T \tilde{J}_\mu^a = 0 \quad (29)$$

and the gauge invariance of the nontopological term is restored. We easily see that the transformation above does not spoil the previous transformation for the mass terms because the topological current appears linearly. Our final Lagrangian then reads

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{2} \tilde{J}_\mu^a \tilde{J}^{a\mu} + m A_\mu^a \tilde{J}^{a\mu} - \frac{1}{4} g m \epsilon_{\mu\nu\rho\lambda} f^{abc} A^{a\mu} A^{b\nu} \tilde{B}^{c\rho\lambda}. \quad (30)$$

and the vector and gauge transformations are

(i) Vector gauge transformations :

$$\begin{aligned} \delta_V A_\mu^a &= -\frac{1}{g} (D_\mu \alpha)^a, \\ \delta_V \phi_\mu^a &= 0, \\ \delta_V \tilde{B}_{\mu\nu}^a &= f^{abc} \alpha^b \tilde{B}_{\mu\nu}^c, \\ \delta_V \tilde{J}_\mu^a &= f^{abc} \alpha^b \tilde{J}_\mu^c, \end{aligned} \quad (31)$$

(ii) Tensor gauge transformations :

$$\begin{aligned} \delta_T A_\mu^a &= 0, \\ \delta_T \phi_\mu^a &= -\xi_\mu^a, \\ \delta_T B_{\mu\nu}^a &= 0, \\ \delta_T \tilde{J}_\mu &= 0. \end{aligned} \quad (32)$$

6. In conclusion, we have obtained a non-Abelian formulation of a theory with vector and tensor fields interacting via usual nonminimum coupling and topological one. We have seen that the non-Abelian theory is not reducible and the compatibility between gauge invariance and nontrivial number of degrees of freedom was achieved with the introduction of Stuckelberg fields. The obtained theory is compatible with the Abelian limit, where the Stuckelberg fields disappear. This shows the consistency of our procedure. We think important to emphasize that the free non-Abelian propagator exhibits also a massive pole in a effective theory without tensor fields [4, 8].

There are two points we would like to comment. The first one concerns to the covariant quantization, like BFV and BV [9], of this non-Abelian formulation. Here,

ghost-of-ghosts have to be introduced to take care of the reducibility condition and it is an interesting task how to do this in order to work just in the Abelian limit. Second is the possible application of this mechanism in the Salam-Weinberg theory in replacement of the Higgs one. The problem we have to solve is the mass generation for fermions. Both of these problems are present under study and possible results shall be reported elsewhere.

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