Factorial, multifractal moments and short range correlation of shower particles at relativistic energies

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Abstract

We have computed factorial - F_q and the multifractal - G_q moments for the shower particles produced in the interactions of ^{197}Au -emulsion at 10.6A GeV and ^{28}Si -emulsion at 14.5A GeV in terms of a new scaled variable X suggested by Bialas and Gazdzicki. Power-law dependences of the moments on the number of bins M of the new variable (X) are observed in pseudorapidity η as well as in two dimensional product (η, ϕ) phase spaces. The exponents ϕ_q for F_q , and τ_q for G_q moments are determined and their relationships are discussed. The normalized factorial moments of the multiplicity distribution exhibit universal scaling behavior. When ordinary factorial moments are decomposed into cumulant moments, we find that for ^{197}Au ions, the cumulant moments of order three are non-zero, contrary to the observations of most of the relativistic heavy ion beams used so far.

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1 Introduction

Recently in multiparticle production, intermittency [1] has been tested for a variety of projectiles, targets and incident energies. In a parallel approach to address the aspect of self-similarity in nonstatistical fluctuations, the existence of fractal properties was speculated where each inelastic collision is considered as a geometrical object with fractional dimensions. The method involves the computation of multifractal moments G_q [2], where q is any real number. A power-law similar to that for factorial moments has been proposed for G_q moments, which ultimately leads to the spectrum of fractal indices τ_q . Recently a close connection [2] between the power-law exponents ϕ_q and τ_q of "intermittency" and "multifractality" techniques, respectively has been shown for lepton, hadron and nuclear interactions at relativistic en-

ergies revealing how the increasing power of $\langle F_q \rangle$ corresponds to the decreasing power of $\langle G_q \rangle$. It is important to mention that the observed increase in factorial moments has shown the presence of short range correlation which was calculated in terms of cumulant moments. The cumulant moments obtained so far for heavy ion like ^{16}O , ^{28}Si and ^{32}S beams vanish [3, 4] for orders higher than $q > 2$. However, for very heavy ions like ^{197}Au beam at 10.6A GeV, the cumulant moments (K_3) of order $q = 3$ do not vanish. As far as we know, this is for the first time that a heavy ion beam has shown departure from zero at the K_3 value, but for $q > 3$ the cumulant moments vanish and are discussed in the present paper along with F_q and G_q moments of the shower particles produced in the interactions with nuclear emulsion.

In nuclear emulsion, pseudorapidity (η) being the basic variable, the non-uniformity in η -distributions is a crucial factor. So we avoided using this variable but followed the new scaled variable X introduced by Bialas and Gazdzicki [5] which is related to the single particle density distribution $\rho(\eta)$ as,

$$X(\eta) = \int_{\eta_1}^{\eta} \rho(\eta') d\eta' / \int_{\eta_1}^{\eta_2} \rho(\eta') d\eta'. \quad (1)$$

Here, η_1 and η_2 are the two extreme points of pseudorapidity distribution. The variable X varies uniformly between $X_{min} = 0.0$ to $X_{max} = 1.0$, and $\rho(X) \approx$ constant within η_1 and η_2 . Thus in the present analysis the multifractal and factorial moments are computed from a single variable $X(\eta)$ distribution or $X(\phi)$ distributions which are highest for ^{197}Au beam and we show only $X(\eta)$ distribution for two beams ^{28}Si and ^{197}Au from AGS in Fig. 1. According to this technique, N singly charged particles in a given $X(\eta)$ interval $\Delta X = X_{max} - X_{min}$ are distributed into M equal bins. The data used in the present investigation are the ^{197}Au beam at 10.6A GeV and ^{28}Si beam at 14.5A GeV from BNL Expt. No. 875 and Expt. No. 847, respectively. For ^{197}Au beam by

scanning along-the-track, we observed about 1400 interactions. By putting the restrictions on the projectile fragments of charge $1 \leq Z \leq 17$ and the number of shower particles $N_s > 50$, we are presenting the analysis of 245 events for ^{197}Au beam and for ^{28}Si beam we are exhibiting the data of 172 central and semi-central events out of 860 minimum-bias events. The details of the data acquisition procedure and angle measurement techniques can be found elsewhere [4, 6]. The singly charged particles produced within a very narrow forward cone with a semivertex angle $\theta_c \approx 0.2/p_{beam}$, where p_{beam} is the beam momentum per nucleon, were eliminated from each event [6]. The power-law dependence of $\langle G_q \rangle$ of order q on M^d is expressed as

$$\langle G_q \rangle = \frac{1}{N_{ev}} \sum_{j=1}^{N_{ev}} \sum_{i=1}^{M^d} (n_j/N)^q \propto (M^d)^{-\tau_q}, \quad (2)$$

where, n_j denotes the number of charged particles detected in the j th bin and q is a positive integer, N is the total number of particles in an event such that $N = \sum_{j=1}^M n_j$, N_{ev} stands for the number of central events in a given data sample and τ_q are determined unambiguously from the asymptotic behavior. In η phase space $d = 1$ and in the product (η, ϕ) phase space $d = 2$.

2 Experimental Results and Discussion

In Fig. 2(a) and 2(b) are presented the plots of $\ln G_q$ vs. $\ln M^d$ in η phase space while in Figs. 2(c) and 2(d) are shown the distributions for η, ϕ phase space for ^{197}Au and ^{28}Si beams, respectively. There is clearly a linear dependence of $\ln \langle G_q \rangle$ on $\ln M^d$ in η and the product (η, ϕ) phase spaces, which is a qualitative measure of the manifestation of the multifractal structures exhibited by all the data sets used here. The slopes (τ_q) obtained from the least square fit to the data points are given below in Table I. The τ_q values for ^{197}Au are slightly larger than for ^{28}Si beam but for either beam their individual values in η do not vary much from the product (η, ϕ) space values. For statistical contribution to $\langle G_q \rangle$, we randomly generated N pseudorapidity values within the given ΔX interval ($0 \leq X \leq 1$) by feeding the distribution of shower particles for each event from the real data sets in the Monte Carlo generator. For every event $\rho(X) = \text{const}$ and the G_q moments exhibit a power-law behavior (for $M^d \geq 0$), characteristic of a self-similar system. Points corresponding to the simulated events are always below the real data of their respective beams of each order of q . There are deviations in τ_q from the statistical τ_q^{st} and these deviations are a measure of real dynamics as shown in Table I.

To establish a link between the scaled factorial moments and the self similarity discussed above on the basis of multifractals, F_q moments are defined as, $\langle F_q \rangle =$

$$\frac{1}{N_{ev}} \sum_{i=1}^{N_{ev}} \sum_{j=1}^{M^d} \frac{n_j(n_j-1)\dots(n_j-q+1)}{N^q} \propto (M^d)^{\phi_q}. \quad (3)$$

Here ϕ_q are the intermittency exponents. The parameters N , M^d , n_j and N_{ev} are explained in Eq. (2) previously. Once again, a linear dependence of $\ln \langle F_q \rangle$ on $\ln M^d$ is observed for the ^{197}Au and ^{28}Si beams as shown in Figs. 3(a) and 3(b) for η space and in Figs. 3(c) and 3(d) for (η, ϕ) space, respectively. ϕ_q values determined from the least square fitting of the data points are given in Table I. ϕ_q values for ^{197}Au beam are lower than ^{28}Si beam in η as well as in η, ϕ phase spaces. Of course, two-dimensional values of the slopes ϕ_q are higher than one-dimensional values in each beam. To compare ϕ_q with the deviations of τ_q from $q-1$, a plot of ϕ_q and $\Delta\tau_q = \tau_q^{st} - \tau_q$ as a function of q from Table I is given in Figs. 4(a) and 4(b), respectively for η and (η, ϕ) phase spaces. The values of ϕ_q deviate from zero and are measures of nonstatistical fluctuations of dynamical origin of the reactions discussed here. Closed symbols in Fig. 4(a) represent the data corresponding to ϕ_q obtained from intermittency analysis, whereas open symbols denote $\Delta\tau_q$ determined from the multifractal analysis. The values of ϕ_q and $\Delta\tau_q$ differ after $q > 6$ for heavier ions like ^{197}Au while for light ion like ^{28}Si beam they differ at lower order of q i.e. for $q > 3$. This difference is due to their definitions of $\langle F_q \rangle$ and $\langle G_q \rangle$ moments, since the former is closely related to the correlation function, while the latter gives a measure of the fractal structures. For (η, ϕ) phase space, the magnitude of ϕ_q values are very close to $\Delta\tau_q$ values for ^{197}Au beam at all q s excepting for $q = 7$, while for ^{28}Si beam, the magnitude of these values are close to one another only up to $q \leq 2$, and beyond $q > 2$ they are different. The parameter ϕ_q characterizes the strength of generalized dimensions D_q as shown earlier [6] and they are commonly used for the description of fractals and multifractals and are related to anomalous dimensions D_q by $D_q = \phi_q/(q-1)$. These fractal dimensions describe how the distribution changes with increasing resolution in the range where $\phi_q < q-1$. Thus, a study of anomalous fractal dimensions may give some useful information [7]. Let us look at the behavior of D_q as a function of q . In Fig. 5(a) and 5(b) are plotted the values of D_q against q for the variables (i) η and (ii) (η, ϕ) , respectively. We observe that, in general, the D_q values are (a) $D_q(\eta) < D_q(\eta, \phi)$ at all values of q and (b) they have the tendency to grow almost linearly with the increase in q for ^{28}Si beam in η and η, ϕ phase spaces excepting the last point in ^{28}Si beam. For ^{197}Au data in η as well as in (η, ϕ) phase

spaces, D_q values are almost constant, but the constant value for η phase space is relatively less than in (η, ϕ) phase space. However, from the work of Refs. [8] and [9], one can state that the behavior of D_q as a function of q can not be used as an indication of whether or not the system undergoes a second order phase transition. But in an independent study of projectile fragments of charge $Z \geq 2$ has indicated [10] that there is a collective flow and hence there is some compression produced in such collisions with the ^{197}Au beam at 10.6A GeV.

For a simple scale invariant cascade model [11], it was found that the factorial moments F_q of higher orders are related to the second order moments F_2 by a power-law in a good approximation as

$$\ln \langle F_q \rangle = (\phi_q / \phi_2) \ln \langle F_2 \rangle + c. \quad (4)$$

where c is a constant. We tested this scaling behavior of ^{197}Au and ^{28}Si beams in η and (η, ϕ) variables from the values of ϕ_q given in Table I for both the beams and these values seem to confirm the existence of scaling in the nuclear system within their errors. The order scaling is necessary for the presence of intermittency. As for the ^{28}Si beam, the data are relatively less than for ^{197}Au beam, and the errors in the slope determination are relatively large in ^{28}Si beam and hence we show in Fig. 6 the distribution of $\ln F_q$ vs $\ln F_2$ for ^{197}Au beam only. The values of r_q are determined from the straight line fits in Figs. 6(a) and 6(b) for η and (η, ϕ) variables, respectively and are given in Table II.

The relationship observed in Fig. 6 between the moments of different orders expressed by $F_q \propto F_2^{r_q}$ (power-law behavior) may be due to the dynamics of heavy ion collisions by Ginzburg-Landau [12] formalism for phase transition in the second order [2]. In Figs. 7(a) and 7(b) are shown the plot of r_q against $(q-1)$ for η and (η, ϕ) variables for the ^{197}Au beam. The relation $r_q = (q-1)^\nu$ satisfies the data with $\nu = 1.69 \pm 0.07$ and 1.94 ± 0.06 for η and (η, ϕ) variables and we denote it with (ν) as the scaling exponent in the dynamics of heavy ion collisions. The use of hadron multiplicity fluctuations, in heavy ion central collisions, may offer a viable scheme to detect evidence of quark-hadron phase transition. The values of ν observed here are different from Ginzburg-Landau formalism for the phase transition of second order as given in Ref. [2].

In the above discussion we have experimentally measured intermittency and factorial moments, but these observations raises the issue of whether the observed increase in factorial moments is due to the presence of the short range correlation between the produced particles. We may point out that true correlation of any specific order is revealed only when contributions from lower orders (called cumulant moments) are subtracted out of the higher order correlation functions. We expand factor-

ial moments here into cumulants. The use of cumulants does not prejudice the case either for or against the existence of the scaling behavior sought in experiments and theory. It merely extracts the statistically significant correlations from the background of trivial terms consisting of lower-order correlations. For hadronic data, it has been found that, the largest parts of factorial moments of all orders come from two-particle correlations [3]. To obtain the true correlations of higher order correlations ($q > 2$), we express the factorial moments in terms of the factorial cumulants,

$$F_2 = K_2 + 1, \quad F_3 = K_3 + 3K_2 + 1,$$

$$F_4 = K_4 + 4K_3 + 3\overline{(K_2)^2} + 6K_2 + 1,$$

$$F_5 = K_5 + 5K_4 + 10K_3 + 10\overline{K_3 K_2} + 15\overline{(K_2)^2} + 10K_2 + 1 \quad (5)$$

Here,

$$K_q(\delta y) = \frac{1}{M(\delta y)^q} \sum_m \int_{\Omega_m} \prod_i dy_i \frac{C_q(y_1, \dots, y_q)}{(\bar{\rho}_m)^q} \quad (6)$$

is the cumulant of q th order and, $C_2(y_1, y_2) = \rho_2(y_1, y_2) - \rho_1(y_1)\rho_1(y_2)$, $C_3(y_1, y_2, y_3) = \rho_3(y_1, y_2, y_3) - \rho_1(y_1)\rho_2(y_2, y_3) - \rho_1(y_2)\rho_2(y_3, y_1) - \rho_1(y_3)\rho_2(y_1, y_2) + 2\rho_1(y_1)\rho_1(y_2)\rho_1(y_3)$ etc. are the two and three particle correlation functions. Here, $\rho_1(y_1)$, $\rho_2(y_1, y_2)$ and $\rho_3(y_1, y_2, y_3)$ are one, two and three particle densities, respectively. In the above expressions of K_4 and K_5 , the bar averaging is done in the following way: $\overline{K_2 K_3} = \frac{1}{M} \sum_{m=1}^M K_2 K_3$, and so on. The relations between vertically averaged factorial moments of different order and the corresponding cumulant moments $K_q^v(M)$, as are given by Eq. (5), are exact.

The cumulants moments thus obtained will be vanishing in the absence of any true dynamical correlation. The results on cumulant moments have been reported on hadronic and nuclear collisions [3]. We have measured in the present investigation the values of cumulant moments of lowest orders (i.e. $q = 2, 3, 4$ and 5) on shower particle data obtained from interactions of ^{197}Au and ^{28}Si beams. In Fig. 8, the values of cumulant moments of orders $q = 2, 3, 4$ and 5 are plotted in η space against M for ^{197}Au beam. The values of second and third order cumulants are significantly different from zero and in each case with increasing M , the values undergo a saturation following an initial rise. For $q = 4$ and 5, the values are consistent with zero within the errors. The same is the situation with cumulant moments in (η, ϕ) phase space as shown in Fig. 9 for ^{197}Au ion and the values are zero for $q = 4$ and 5. This indicates, for the first time, genuine three-body correlations among the shower particles. For ^{28}Si beam in η space, the cumulant moments for $q > 3$ are zero as shown in Fig. 10 and the

same is true in (η, ϕ) space for $q > 3$ as exhibited in Fig. 11. According to the suggestion made in Ref. 3, we have parametrized our experimental data in terms of the algebraic expression

$$K_2 = 2\gamma\xi^2[(M/\xi) - 1 + e^{-M/\xi}]/M^2 \quad (7)$$

In all cases, the fitting process yields reasonably low χ^2 values. The values of the parameters γ , the strength of the correlation length ξ , and χ^2 per degrees of freedom (dof) for ^{197}Au and ^{28}Si are: $\gamma = 0.335 \pm 0.020$, $\xi = 2.33 \pm 0.15$ and $\chi^2/\text{dof} = 0.33$; $\gamma = 0.124 \pm 0.016$, $\xi = 1.18 \pm 0.09$ and $\chi^2/\text{dof} = 0.36$, respectively. These values for ^{28}Si beam are not far away from those calculated earlier [6]. For ^{197}Au beam the zero values for the order $q > 3$ may still be due to two-particle correlations like Bose-Einstein interference, but it does not seem to be the main source of intermittency which is observed in Fig. 3. If strong space-time fluctuations occur, which may be associated with the transition to a quark-gluon plasma phase [3], the cumulant moments shall still obey the identity of Eq. (5). In that case one may expect the higher cumulant moments than order two to suddenly increase to reflect the presence of more violent bulk fluctuations preceding hadronization. In the present experiment, the establishment of three-body correlation ($K_3 \neq 0$) among the shower particles, indicates that there may be some kind of collective phenomenon present among the particles produced in ^{197}Au -Emulsion interactions. This has already been established recently through an independently study of the projectile fragments of charge $Z \geq 2$ among nuclear interactions for ^{197}Au beam at 10.6 A GeV [10]. We find that the collective flow increases with the charge of the fragments and we also observed the collective-flow correlation among the azimuthal angles of these fragments [13]. For the phase transition, one is led to address the question of whether the matter produced in these collisions can be considered at least locally, to be in thermal equilibrium. Local equilibrium implies that the matter behaves collectively, which has been observed in ^{197}Au beam [10]. Therefore, it is justified to use the inclusive data from a thermalized nuclear system to determine the critical exponents at the phase transition. We have recently extracted the critical exponents γ , β and τ , which are very close to those for a liquid-gas system [14].

3 Conclusions

We may conclude that we have observed the power-law dependence in the scaled variable X bin width for factorial- F_q and multifractal- G_q moments. With this variable, the results can be easily compared with those from other experiments, where the basic measurable

quantities are different from the present emulsion experiments. The exponents τ_q and ϕ_q , which give a better measure of the dynamical fluctuations, are determined. They may be approximately related to each other, and they also reveal how the decreasing power of $\langle G_q \rangle$ corresponds to the increasing power of $\langle F_q \rangle$. The ratios r_q are found to be almost independent of the reaction type. When we plot r_q against $(q-1)$ in η -space, we observed a scaling behavior but the value of ν is different from the critical value of $\nu = 1.3$ as given in Ref. [2]. From the observations on cumulant moments K_q of lowest orders, both in η and (η, ϕ) dimensions we conclude that (i) correlation of order higher than three for ^{28}Si and ^{197}Au do not survive in our data beyond the statistical uncertainties. The observed power-law behavior in factorial moments with $q \geq 3$ for ^{28}Si and $q \geq 4$ for ^{197}Au beams may be due to two-particle dynamical correlation. It is the first time any heavy ion beam (like ^{197}Au) has shown departure from zero in its K_3 values. This may also be partially due to the occurrence of space-time fluctuations due to the creation of a highly chaotic phase in ^{197}Au ion interactions.

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- Fig. 6** Variation of $\ln \langle F_2 \rangle$ with $\ln \langle F_q \rangle$ for ^{197}Au beam: (a) in η phase space and (b) in $\eta.\phi$ phase space. Dotted lines are the best fits through the data points.
- Fig. 7** A variation of $\ln r_q$ with $\ln(q-1)$ for ^{197}Au beam: (a) in η phase space and (b) in $\eta.\phi$ phase space. Straight lines are the best fits to the data.
- Fig. 8** Variation of cumulant moments $K_q = 2, 3, 4$ and 5 with M for ^{197}Au ion. The solid curve in (a) represents the best fitted values of K_2 according to Eq. (7) with experimental data in η phase space.
- Fig. 9** Same as in Fig. 8 for the ^{197}Au data but in $\eta.\phi$ phase space.
- Fig. 10** Variation of cumulant moments $K_q = 2, 3, 4$ and 5 with M for ^{28}Si ion. The solid curve in (a) represents the best fitted values of K_2 according to Eq. (7) with experimental data in η phase space.
- Fig. 11** Same as in Fig. 10 for the ^{28}Si data but in $\eta.\phi$ phase space.

Figure Captions

Fig. 1 Single-particle density distributions $\rho(X)$ of the scaled variable $X(\eta)$ for ^{197}Au and ^{28}Si ions.

Fig. 2 $\langle \ln G_q \rangle$ as a function of $\ln M^d$ for: (a) ^{197}Au at 10.6A GeV for $q = 2 - 7$ and (b) ^{28}Si at 14.5A GeV for $q = 2 - 6$ with best fitted straight lines (dotted line) in η phase space. (c) and (d) are the same as in (a) and (b) but for $\eta.\phi$ phase space. Here, $d = 1$ for one-dimensional and $d = 2$ for two-dimensional phase spaces.

Fig. 3 $\ln \langle F_q \rangle$ vs. $\ln M^d$ for the ions as in Fig. 2.

Fig. 4 (a) A comparison of the exponents of G_q and F_q moments, open and closed symbols corresponds to $\Delta\tau_q$ and ϕ_q for ^{197}Au ($\circ, \bullet$), and ^{28}Si (open triangles, closed triangles) in η phase space. (b) the same as in (a) but in $\eta.\phi$ phase space.

Fig. 5 (a) D_q versus q for ^{197}Au ($\bullet$) and ^{28}Si (closed triangles) in η phase space. (b) the same as in (a) but in $\eta.\phi$ phase space.

Table I. The values of slopes ϕ_q , τ_q , τ_q^{st} and $\Delta\tau_q$ for the data on 10.6A GeV ^{197}Au and 14.5A GeV ^{28}Si events with $N_h \geq 0$ in η and $\eta.\phi$ phase spaces. Error in ϕ_q and τ_q is the standard deviation in a given parameter.

^{197}Au in η phase space				
q	ϕ_q	τ_q	τ_q^{st}	$\Delta\tau_q$
2	0.0055 ± 0.0005	0.9012 ± 0.0061	0.9061	0.0049 ± 0.0003
3	0.0136 ± 0.0015	1.7443 ± 0.0127	1.7604	0.0161 ± 0.0010
4	0.0231 ± 0.0033	2.5499 ± 0.0162	2.5763	0.0264 ± 0.0017
5	0.0329 ± 0.0063	3.3230 ± 0.0166	3.3543	0.0313 ± 0.0020
6	0.0410 ± 0.0106	4.0565 ± 0.0178	4.1327	0.0762 ± 0.0049
7	0.0447 ± 0.0238	4.7658 ± 0.0256	4.9421	0.1763 ± 0.0113
^{28}Si in η phase space				
q	ϕ_q	τ_q	τ_q^{st}	$\Delta\tau_q$
2	0.0093 ± 0.0017	0.8653 ± 0.0065	0.8705	0.0052 ± 0.0004
3	0.0333 ± 0.0065	1.6727 ± 0.0112	1.7059	0.0332 ± 0.0025
4	0.0712 ± 0.0159	2.4132 ± 0.0176	2.5322	0.1190 ± 0.0091
5	0.1639 ± 0.0421	3.0674 ± 0.0441	3.4253	0.3579 ± 0.0273
6	—	3.6460 ± 0.0859	4.2205	—
^{197}Au in $\eta.\phi$ phase space				
q	ϕ_q	τ_q	τ_q^{st}	$\Delta\tau_q$
2	0.0080 ± 0.0003	0.8992 ± 0.0071	0.9067	0.0105 ± 0.0007
3	0.0224 ± 0.0010	1.7404 ± 0.0155	1.7649	0.0245 ± 0.0016
4	0.0444 ± 0.0023	2.5397 ± 0.0232	2.5936	0.0539 ± 0.0034
5	0.0743 ± 0.0052	3.3136 ± 0.0311	3.4161	0.1026 ± 0.0064
6	0.1097 ± 0.0106	4.1151 ± 0.0247	4.2342	0.1191 ± 0.0076
7	0.1439 ± 0.0196	4.8942 ± 0.0230	5.1481	0.2539 ± 0.0162
^{28}Si in $\eta.\phi$ phase space				
q	ϕ_q	τ_q	τ_q^{st}	$\Delta\tau_q$
2	0.0128 ± 0.0016	0.8586 ± 0.0071	0.8712	0.0126 ± 0.0010
3	0.0556 ± 0.0066	1.6504 ± 0.0133	1.6889	0.0385 ± 0.0029
4	0.1903 ± 0.0286	2.3777 ± 0.0196	2.4780	0.1003 ± 0.0076
5	0.5036 ± 0.0866	2.9707 ± 0.0392	3.2699	0.2992 ± 0.0228
6	—	3.4457 ± 0.0719	4.2026	0.7569 ± 0.0577

Table II. The values of r_q for the data of ^{197}Au ion in η and $\eta.\phi$ phase spaces. Error in r_q is determined from the number of events in ^{197}Au data sample.

q	η phase space	$\eta.\phi$ phase space
3	2.527 ± 0.161	2.834 ± 0.181
4	4.599 ± 0.294	5.734 ± 0.366
5	7.380 ± 0.471	9.990 ± 0.638
6	11.099 ± 0.709	15.949 ± 1.019
7	16.393 ± 1.047	24.038 ± 1.536

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11