Simple supersymmetric solution
to the strong $CP$ problem

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Abstract

It is shown that the minimal supersymmetric left-right model can provide a natural solution to the strong $CP$ problem without the need for an axion, nor any additional symmetries beyond supersymmetry and parity.
1 Introduction

Quantum chromodynamics, which is extremely successful in describing strongly interacting phenomena both in the low as well as the high energy domain, has the well-known problem that it can lead to an uncontrolled amount of CP violation in the flavor conserving hadronic processes. This is the strong CP problem [1]. The parameter $\Theta$ which characterizes the strength of these CP-violating interactions is constrained by present upper limits on the electric dipole moment of the neutron to be less than $10^{-9} - 10^{-10}$. Presence of such a small number in a theory indicates the existence of new symmetries beyond the standard model of electroweak and strong interactions. Three classes of spontaneously broken symmetries have, in the past, been advocated as solutions to the strong CP problem: (i) Peccei-Quinn U(1) symmetry [2]; (ii) Parity (or left-right) symmetry of weak interactions [3] and (iii) softly broken CP symmetry [4]. There also exist other solutions which use less transparent symmetries to constrain the form of quark mass matrices into interesting forms thereby suppressing $\Theta$ to the desired level [5]. In the absence of any experimental evidence for or against any of these solutions, one can look for theoretical criteria to reduce the number of such possibilities. One criterion discussed in recent years is to use the lore that unlike local symmetries, all global symmetries are broken by non-perturbative gravitational effect such as black holes and wormholes. Since all our solutions involve new global symmetries, one must investigate whether in the presence of these effects, the solution to the strong CP problem remain viable. In Ref. [6] it was shown that the presently invisible axion models [7] are incompatible with the above non-perturbative effects essentially due to the fact that the PQ symmetry breaking scale in this case must be $\approx 10^{10} - 10^{12}$ GeV. On the other hand, it was shown in Ref. [8] that as long as the scales of $P$ or CP violation are less than some intermediate scale, the non-perturbative Planck scale effects do not destabilize the second and third solutions to the $\Theta$—problem. In this article, we will focus on the class of models which use parity symmetry to solve the strong CP problem [3]. We will show that in a class of minimal supersymmetric models recently dicussed [9, 10] in order to have automatic $R$-parity conservation prior to symmetry breaking, the strong CP parameter $\tilde{\Theta}$ naturally vanishes both at the tree and one-loop level, thus providing a solution to the strong CP problem. No additional symmetries are needed for the purpose. These models also have the virtue that including Planck scale effects leaves the solution unscathed, as in Ref. [8].
Furthermore, unlike the model of Ref. [10], R-parity is naturally conserved to all orders in $\frac{1}{M_{Pl}}$, when we incorporate such non-renormalizable operators. This therefore leaves the lightest supersymmetric particle (LSP) absolutely stable in this model (unlike the MSSM) which can then play the role of CDM [11].

This paper is organized as follows: in Sec. 2, we discuss how in generic left-right models one solves the strong $CP$ problem; in Sec. 3 we present the outline of the SUSY LR model and in Sec. 4 we discuss how $\bar{\Theta} = 0$ at the tree level of the supersymmetric left-right model (SUSY LR); in Sec. 5, we show the vanishing of the one loop contribution. In Sec. 6, we give the concluding remarks. In Appendix A, we show how non-perturbative Planck scale effects can induce the parity conserving minimum. In Appendix B, we discuss the minimization of the Higgs potential to show that the vacuum expectation values for the bidoublet fields are $CP$-conserving for arbitrary choice of parameters in the theory.

2 Solution to Strong $CP$ problem in generic left-right models

To see how parity symmetry really suppresses the $\bar{\Theta}$, let us start by noting that in an electroweak theory, there are two contributions to $\bar{\Theta}$ at the tree level:

$$\bar{\Theta} = \Theta + \text{Arg det}(M_u M_d),$$

where $\Theta$ is the coefficient of the $GG$ term in the QCD Lagrangian induced by instanton effects and the second term in (1) is self-explanatory with $M_u$ and $M_d$ denoting the up and down quark mass matrices. Since $GG$ is odd under parity, if the theory is required to be parity invariant, we must have $\Theta = 0$. The vanishing of the second term is however more tricky. In the conventional left-right models based on the gauge group $\text{SU}(2)_L \times \text{SU}(2)_R \times U(1)_{B-L}$ (as we consider here) [12] the quark masses arise from the following gauge invariant Lagrangian:

$$\mathcal{L}_Y = h^i_{ab} \bar{Q}_{L,a} \Phi_i Q_{R,b} + h.c.,$$

where $Q_a = (u_a, d_a)$ ($a = 1, 2, 3$ for three generations) and $\Phi_i$ are bidoublets $(2,2,0)$. In the minimal non-supersymmetric model, one usually considers one $\Phi$ so that there exists
another bidoublet $\tilde{\Phi} \equiv \tau_2 \Phi^* \tau_2$ leading to two yukawa matrices $h^{(1)}$ and $h^{(2)}$. Under
left-right (P) symmetry, one assumes that

$$
Q_{L,a} \rightarrow Q_{R,a}.
$$

$$
\Phi_i \rightarrow \Phi_i^*.
$$

It is then easy to show that parity invariance demands that

$$
h^{(i)} = h^{(i)*}.
$$

Now, if the ground state had the property that $<\Phi_i>$ is real (i.e. the ground state is $CP$-conserving) then one would have hermitean mass matrices and this would lead to
the second term in the Eq. 1 being zero. One would then have obtained $\bar{\Theta}_{\text{tree}} = 0$.

Unfortunately, without extra symmetries, the most general Higgs potential in non-supersymmetric left-right model has complex couplings and therefore the vacuum state is necessarily $CP$-violating. As an example consider the Higgs system $\Phi$, $(\Delta_L, \Delta_R)$ [13],
where $\Delta_L$ and $\Delta_R$ are left and right SU(2) triplets respectively with $B - L = 2$. In this
model, all but one scalar coupling in the Higgs potential are real but the complex one

$$
|\lambda| \det \Phi (e^{ia} \Delta_L^\dagger \Delta_L + e^{-ia} \Delta_R^\dagger \Delta_R) + h.c.,
$$

which induces a complex vacuum expectation value (VEV). Note now that in the presence
of complex VEVs $<\Phi>$, the mass matrix is not hermitean and at the tree level $\bar{\Theta} \neq 0$
despite the theory being parity invariant. One therefore needs new symmetries that
forbid the above term [3]. An alternate mechanism suggested in the third reference in
Ref. [3] is to avoid the bidoublet - but then to get non-zero quark masses one needs
heavy vectorlike quarks and leptons.

While technically the above model with an extra symmetry provides solutions to the
strong $CP$ problem, it would have been preferable if the new symmetry invoked to solve
the strong $CP$ problem was also independently motivated. The heavy vectorlike fermion
models may have a slight advantage from this point of view, since questions such as
fermion mass hierarchies or small neutrino masses do provide independent motivation for
such model.
In this paper, we show that supersymmetry, which is very strongly motivated by considerations such as stability of electroweak scale or radiative generation of W boson mass, when coupled with left-right symmetry does solve the strong \( CP \) problem without additional symmetry assumptions. Of course, it has been discussed before that SUSY LR models also stop uncontrollable baryon violation [14] present in the MSSM.

### 3 The model

As already mentioned, the gauge group of the theory is \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) with quarks and leptons transforming as doublets under \( SU(2)_{L,R} \) depending on their chirality. As is usual in supersymmetry, we take all superfields to be left chiral. In Table 1, we denote the quark, lepton and Higgs superfields in the theory along with their transformation properties under the gauge group. Note that we have chosen two bidoublet fields to obtain realistic quark masses and mixings (one bidoublet implies a KM matrix proportional to unity, because supersymmetry forbids \( \tilde{\Phi} \) in the superpotential).

The superpotential for this theory is given by (we have suppressed the generation index):

\[
W = h^{(i)}_q Q^T \tau_2 \Phi^c_i \tau_2 Q^c + h^{(i)}_l L^T \tau_2 \Phi^c_i \tau_2 L^c \\
+ i(f^T \tau_2 \Delta L + f^T c^T \tau_2 \Delta^c L^c) \\
+ \mu_\Delta \text{Tr}(\Delta \tilde{\Delta}) + \mu_{\Delta^c} \text{Tr}(\Delta^c \tilde{\Delta}^c) + \mu_{ij} \text{Tr}(\tau_2 \Phi^T \tau_2 \Phi_j).
\] (6)
At this stage all couplings $h^{(i)}_{ij}, \mu_{ij}, \mu_{i\Delta}, \mu_{\Delta'}, f, \ell$ are complex with $\mu_{ij}, f$ and $\ell$ being symmetric matrices. The part of the supersymmetric action that arises from this is given by

$$S_W = \int d^4x \int d^2\theta W + \int d^4x \int d^2\bar{\theta} W^\dagger. \quad (7)$$

The terms that break supersymmetry softly to make the theory realistic can be written as

$$\mathcal{L}_{\text{soft}} = \int d^4\bar{\theta} \sum_i m_i^2 \phi_i^\dagger \phi_i + \int d^2\theta \theta^2 \sum_i A_i W_i + \int d^2\bar{\theta} \bar{\theta}^2 \sum_i A_i^* W_i^\dagger + \int d^2\theta \theta^2 \sum_p m_p^\ast \bar{W}_p\bar{W}_p^\dagger. \quad (8)$$

In Eq. 8, $\bar{W}_p$ denotes the gauge-covariant chiral superfield that contains the $F_{\mu\nu}$-type terms with the subscript going over the gauge groups of the theory including SU(3)$_C$. $W_i$ denotes the various terms in the superpotential, with all superfields replaced by their scalar components and with coupling matrices which are not identical to those in $W$. Eq. 8 gives the most general set of soft breaking terms for this model.

### 4 Left-Right Symmetry and Vanishing of $\bar{\Theta}$ at the tree level

In Sec. 2 we saw that left-right symmetry implies that the first term in Eq. 1 is zero. Let us now see how supersymmetric left-right symmetry also requires the second term in this equation to vanish naturally. We choose the following definition of left-right transformations on the fields and the supersymmetric variable $\theta$

$$Q \leftrightarrow Q^\dagger$$
$$L \leftrightarrow L^\dagger$$
$$\Phi_i \leftrightarrow \Phi_i^\dagger$$
$$\Delta \leftrightarrow \Delta^\dagger$$
$$\bar{\Delta} \leftrightarrow \bar{\Delta}^\dagger$$
\[ \theta \leftrightarrow \bar{\theta} \]
\[ \bar{W}_{SU(2)_{L}} \leftrightarrow \bar{W}_{SU(2)_{R}}^{\dagger} \]
\[ \bar{W}_{B-L,SU(3)_{C}} \leftrightarrow \bar{W}_{B-L,SU(3)_{C}}^{\dagger} \]

With this definition of L-R symmetry, it is easy to check that

\[ h_{q,i}^{(i)} = h_{q,i}^{(i)^{\dagger}} \]
\[ \mu_{ij} = \mu_{ij}^{*} \]
\[ \mu_{\Delta} = \mu_{\Delta}^{*} \]
\[ f = f^{*} \]
\[ m_{\lambda_{SU(2)_{L}}} = m_{\lambda_{SU(2)_{R}}}^{*} \]
\[ m_{\lambda_{B-L,SU(3)_{C}}} = m_{\lambda_{B-L,SU(3)_{C}}}^{*} \]

(10)

From Eq. 10 we see that all terms involving Higgs fields are real.

Now we are ready to look for minima of the Higgs potential to see whether \( \langle \Phi_{i} \rangle \) have phases or not. In discussing this, we must recall the result of Ref. [10] that in order for the ground state to respect electromagnetic gauge invariance, one must break R-parity, i.e. \( \langle \bar{\nu}^{c} \rangle \neq 0 \) for at least one generation. Secondly, the \( \langle \bar{\nu}^{c} \rangle \) VEV will always induce the VEV of \( \langle \bar{\nu} \rangle \), but since these involve leptonic Yukawa couplings, its magnitude will be small. Because of these sneutrino VEVs the minimum equations become very complicated and it is not obvious that a small phase in the bidoublet VEVs will not be generated. Although we believe these contributions to \( \bar{\Theta} \) will be rather small, they will depend on details beyond the scope of this paper, such as the structure of the leptonic Yukawa couplings.

We choose a different approach. In studying the strong \( CP \) problem we will work with the minimum where \( \langle \bar{\nu}^{c} \rangle = 0 \). So how does one evade the theorem of Ref. [10]? Let us recall that the result of Ref. [10] is valid for the most general renormalizable superpotential of the model. However, if one assumes that non-perturbative Planck scale effects can induce operators with dimension 4 or higher, the result of Ref. [10] can be avoided as we show in Appendix A. The simplest operator that is helpful is

\[ \frac{\lambda}{M_{Pl}} \text{Tr}(\Delta^{c} \tau_{m} \bar{\Delta}^{c})^{2} \].
In this case (when the sneutrino VEVs are zero) we have made a detailed analysis of the Higgs potential and find that, at the minimum of the potential, the $<\Phi_i>$ are real. This result is not at all trivial because of large number of VEVs that enter and one might naively think that there is spontaneous $CP$ violation. However, a recent analysis [15] has shown that a general supersymmetric model with two pairs of Higgs doublets (of which SUSY LR is a special case) cannot break $CP$ spontaneously. We give the details of this calculation in Appendix B. It is now clear that the quark mass matrices are hermitean and therefore $\Theta = 0$ naturally at the tree level.

An interesting point to note is that the B-L gaugino and gluino mass terms are $CP$-conserving. As a result, the problem of large neutron electric dipole moment does not exist in this model.

5 Vanishing of $\tilde{\Theta}$ at the one-loop level

While vanishing of $\tilde{\Theta}$ at the tree level is a necessary condition for solving the strong $CP$ problem, it is not sufficient since if the quark mass matrices lose their hermiticity at the one loop they will induce a too large $\tilde{\Theta}$. Let us therefore investigate the one loop contribution to $M_u$ and $M_d$. There are both Higgs and gaugino mediated diagrams (Figs. 1 and 2 respectively). The higgs mediated graph contributes as follows

$$\delta M^H_q = [A_{ij} h^{(i)} M_q^{(0)} h^{(j)}].$$

Here $M_q^{(0)}$ denotes the tree level contribution. Due to the symmetry property $\mu_{12} = \mu_{21}$ and reality of $\mu_{ij}$, it follows that $\delta M^H_q$ is hermitean. As far as the gauge mediated contribution is concerned, $\delta M^G_q \propto M_q^{(0)}$. Turning to gaugino contributions, since $m_\lambda$ for the $SU(2)_{L,R}$ can be complex, a careful analysis is needed to see what their contribution to $\tilde{\Theta}$ is. We find these contributions come always in pairs for both left and right gauginos, and because of (10) their complex parts cancel out when the diagrams are summed up. Two typical graphs are shown in Figure 2. Therefore the gauge mediated contribution is also automatically hermitean. Thus, the total one loop contribution to $\tilde{\Theta}$ vanishes $^1$.

Thus the lowest order contribution if any can arise only at the two loop level. Its

$^1$Relations (10) hold only above the right hand breaking cale. However, this are loop effects and do not affect our conclusions.
contribution to $\bar{\Theta}$ can be crudely estimated to be:

$$\bar{\Theta} \sim \left(\frac{m_t m_b}{V_{WW}^2}\right) \frac{1}{(16\pi^2)^2} \left(\frac{\mu_f^2}{M^2}\right)^f.$$  \hspace{1cm} (12)$$

For $\mu_{ij} \sim 10^{-1} M$, this “primitive” estimate gives $\bar{\Theta} \sim 4 \times 10^{-9} f$. In a more careful estimate there are also small mixing angles present, which will further suppress $\bar{\Theta}$.

6 Conclusion

In summary, we have shown that minimal models that combine supersymmetry and parity invariance provide a simple solution to the strong $CP$ problem without the need to invoke any additional symmetries. A key element in our proof is the transformation of supersymmetry coordinate $\theta \leftrightarrow \bar{\theta}$ under parity. We show by a detailed analysis that the vacuum expectation values of the bidoublet fields are real for arbitrary values of the parameters consistent with the above symmetry. The non-perturbative Planck scale effects imply that the solution holds as long as $v_R \leq 10^{10} \text{ GeV}$ or so.

Note added in proof: After this work was completed, we came across a paper by R. Kuchimanchi [16] which also arrives at the same result, under the assumption that all gaugino masses are same at the Planck scale and that there are new symmetries beyond parity and supersymmetry. Our goal has been to avoid having to add new symmetries and our result is therefore more general.

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APPENDIX A

In this Appendix we will show that if in the minimal SUSY LR model one includes non-renormalizable Planck scale induced terms, the ground state of the theory can be $Q_{em}$ conserving even for $<\bar{\nu}> = 0$. For this purpose, let us briefly recall the argument of Ref. [10]. The part of the potential containing $\bar{t}_c$, $\Delta_c$ and $\bar{\Delta}_c$ fields only has the form (see Appendix B or [10] ).
\[ V = V_0 + V_D, \]  

where

\[ V_0 = \text{Tr} i f^i L^c L^{cT} \tau_2 + \mu_4^2 \bar{\Delta}^c \]
\[ + \mu_2^2 \text{Tr}(\Delta^c \Delta^{c\dagger}) + \mu_2^2 \text{Tr}(\bar{\Delta}^c \bar{\Delta}^{c\dagger}) \]
\[ + \mu_2^2 \text{Tr} \bar{\Delta}^c \bar{\Delta}^{c\dagger} \]

and

\[ V_D = \frac{g^2}{8} \sum_m [\bar{L}^{c\dagger} \tau_m \bar{L}^c + \text{Tr}(2\Delta^{c\dagger} \tau_m \Delta^c + 2\bar{\Delta}^{c\dagger} \tau_m \bar{\Delta}^c)]^2 \]
\[ + \frac{g^2}{8} |\bar{L}^{c\dagger} \bar{L}^c - 2 \text{Tr}(\Delta^{c\dagger} \Delta^c - \bar{\Delta}^{c\dagger} \bar{\Delta}^c)|^2. \]  

Note that if \( < \bar{\nu}^c > = 0 \) then the vacuum state for which \( \Delta^c = \frac{1}{\sqrt{2}}v \tau_1 \) and \( \bar{\Delta}^c = \frac{1}{\sqrt{2}}v' \tau_1 \) is lower than the vacuum state \( \Delta^c = v \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \) and \( \bar{\Delta}^c = v' \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \). However, the former is electric charge violating. The only way to rescue the situation is to have \( < \bar{\nu}^c > \neq 0 \). On the other hand, if we have non-renormalizable terms included in the theory, the situation changes: for instance, let us include non-renormalizable gauge invariant terms of the form:

\[ W = \frac{\lambda_1}{M} [\text{Tr}(\Delta^c \tau_m \bar{\Delta}^c)]^2. \]  

This will change \( V \) to the form:

\[ V = V_0 + V_1 + V_D, \]  

where \( V_0 \) and \( V_1 \) are given before and \( V_1 \) is given by

\[ V_1 = \frac{\lambda_1 \mu}{M} [\text{Tr}(\Delta^c \tau_m \bar{\Delta}^c)]^2 + \frac{4\lambda_1 \mu}{M} [\text{Tr}(\Delta^c \tau_m \bar{\Delta}^c)][\text{Tr}(\Delta^{c\dagger} \tau_m \Delta^c)] \] + \Delta^c \leftrightarrow \bar{\Delta}^c + \text{etc.} \]  

For the charge violating minimum above, this term vanishes but the charge conserving minimum receives a nonzero contribution. Note that the sign of \( \lambda_1 \) is arbitrary and
therefore, by appropriately choosing \( \text{sgn} \lambda_i \) we can make the electric charge conserving vacuum lower than the \( Q^m \)-violating one. In fact, one can argue that, since we expect \( v^2 - v'^2 = \frac{\alpha^2}{16\pi^2} \) in typical Polonyi type models, the charge conserving minimum occurs for \( f < 4\pi \left( \frac{4\lambda \mu_\Delta}{M_{Pl}} \right)^\frac{1}{2} \). For \( \lambda \approx 1, \mu_\Delta \approx 1\text{TeV}, \) and \( f \leq 10^{-3} \). The constraint of course becomes weaker for larger values of \( \mu_\Delta \). We wish to note that a possible non-renormalizable term of the form \( \text{Tr}(\Delta^c \tau_m \bar{\Delta}^c)\text{Tr}(\Phi_i \tau_m \Phi_j) \frac{v^2}{M_{Pl}} \) does induce a complex effective mass for the bidoublets but its magnitude is given by \( \frac{v^2}{M_{Pl}} \) which for \( v_R \leq 10^{10} \text{ GeV} \) is of order \( 10^{-9} \) and will therefore not affect the solution to the strong \( CP \) problem.

**APPENDIX B**

Here we show that the VEVs of the bidoublet Higgs fields in the supersymmetric left-right model are real. The scalar potential is given by

\[
V = V_F + V_{\text{soft}} + V_D, \tag{19}
\]

where

\[
V_F = \sum_p |h_q^{(i)}_{pr} \tau_2 \Phi_i \tau_2 Q^c_r|^2 + \sum_r |h_q^{(i)}_{pr} Q_p \tau_2 \Phi_i \tau_2|^2 \\
+ \sum_i \text{Tr}[h_q^{(i)} Q Q^c_T + h_i^{(i)} L L^c_T + 2\mu_{ij} \Phi_j]^2 \\
+ \sum_p |h_i^{(i)}_{pr} \tau_2 \Phi_i \tau_2 L^c_r|^2 + 2i f_{pr} \tau_2 \Delta L r|^2 + \sum_r |h_i^{(i)}_{pr} L_p \tau_2 \Phi_i \tau_2 + 2i f_{pr} L_p \tau_2 \Delta^c|^2 \\
+ \text{Tr}[i f^{\dagger} L L^T \tau_2 + \mu_\Delta \bar{\Delta}]^2 + \text{Tr}[i f^{\dagger} L^c L^{cT} \tau_2 + \mu_\Delta \bar{\Delta}^c]^2 \\
+ |\mu_\Delta|^2 \text{Tr}(\Delta^+ + \Delta^+ \Delta^c) \tag{20}
\]

\[
V_{\text{soft}} = m^2_\Phi(\bar{Q}^{\dagger} \bar{Q} + \bar{Q}^{c\dagger} \bar{Q}^c) + m^2_\Phi(\bar{L}^{\dagger} \bar{L} + \bar{L}^{c\dagger} \bar{L}^c) + m^2_\Phi \Phi^\dagger \Phi_i \\
+ m_\Delta^2 \text{Tr}(\Delta^+ + \Delta^{c+} + \Delta^c) + m_\Delta^2 \text{Tr}(\bar{\Delta}^+ \bar{\Delta} + \bar{\Delta}^{c+} \bar{\Delta}^c) \\
+ [ A_{q_i} h_i^{(i)} \bar{Q}^{\dagger} \tau_2 \Phi_i \tau_2 \bar{Q}^c + A_{l_i} h_i^{(i)} \bar{L}^{\dagger} \tau_2 \Phi_i \tau_2 \bar{L}^c \\
+ A_{l_i} (f \bar{L}^{\dagger} \tau_2 \Delta \bar{L} + f^{\dagger} \bar{L}^{cT} \tau_2 \Delta^c \bar{L}^c) \\
+ A_{\Delta}(\mu_\Delta \text{Tr}(\Delta \bar{\Delta}) + \mu_\Delta^* \text{Tr}(\Delta^c \bar{\Delta}^c) + A_{\Phi i j} \text{Tr}(\tau_2 \Phi_i \tau_2 \Phi_j) + h.c.), \tag{21}
\]
\[ V_D = \frac{g^2}{8} \sum_m |\bar{L}^\dagger \tau_m \bar{L} + \text{Tr}(2\Delta^\dagger \tau_m \Delta + 2\bar{\Delta}^\dagger \tau_m \bar{\Delta} + \Phi^\dagger \tau_m \Phi)|^2 \]
\[ + \frac{g^2}{8} \sum_m |\bar{L}^\dagger c \tau_m \bar{L}_c + \text{Tr}(2\Delta^\dagger c \tau_m \Delta^c + 2\bar{\Delta}^\dagger \tau_m \bar{\Delta}^c + \Phi^\dagger \tau_m \Phi^c)|^2 \]
\[ + \frac{g^2}{8} |\bar{L}^\dagger c - \bar{L}^\dagger + 2\text{Tr}(\Delta^\dagger \Delta - \Delta^\dagger \Delta^c - \bar{\Delta}^\dagger \bar{\Delta} + \bar{\Delta}^\dagger \bar{\Delta}^c)|^2. \] (22)

We assume the following fields get the VEVs:

\[ \Delta^c = \begin{pmatrix} 0 \\ \Delta^0 e^{-i\beta_\Delta} \end{pmatrix}, \quad \bar{\Delta}^c = \begin{pmatrix} 0 \\ \delta^0 \end{pmatrix}, \] (23)

and

\[ \Phi_1 = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 e^{i\delta_2} \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} v_3 e^{i\delta_3} & 0 \\ 0 & v_4 e^{i\delta_4} \end{pmatrix}, \] (24)

where we have rotated away the nonphysical phases.

The VEV of the scalar potential is

\[ \langle V \rangle = |2\mu_{11}v_1 + 2\mu_{12}\nu_3^{i\delta_3}|^2 + |2\mu_{11}v_2^{i\delta_2} + 2\mu_{12}\nu_4^{i\delta_4}|^2 \]
\[ + |2\mu_{21}v_1 + 2\mu_{22}\nu_3^{i\delta_3}|^2 + |2\mu_{21}v_2^{i\delta_2} + 2\mu_{22}\nu_4^{i\delta_4}|^2 \]
\[ + |\mu_\Delta|^2(\Delta^0 + \delta^0) \]
\[ + m_\Delta^2 \Delta^0 + m_\bar{\Delta}^2 \delta^0 \]
\[ + m_{\Phi_1}(v_1^2 + v_2^2) + m_{\Phi_2}(v_3^2 + v_4^2) + A_\Delta |\mu_\Delta| \Delta^0 \delta^0 \cos(\beta_\Delta + \text{Arg}(\mu_\Delta)) \]
\[ + A_{\Phi_{\mu_1}} |2v_1 v_2 \cos \delta_2 + A_{\Phi_{\mu_2}} (v_1 v_4 \cos \delta_4 + v_2 v_3 \cos(\delta_2 + \delta_3)) \]
\[ + A_{\Phi_{\mu_3}} |2v_3 v_4 \cos(\delta_3 + \delta_4)\rangle < V_D \rangle, \] (25)

where \( \langle V_D \rangle \) is the VEV of the D-term

\[ \langle V_D \rangle = \frac{g^2}{8} |v_1^2 + v_2^2 - v_3^2 - v_4^2|^2 \]
\[ + \frac{g^2}{8} [2(\Delta^0 + \delta^0) + v_1^2 + v_3^2 - v_2^2 - v_4^2] \]
\[ + \frac{g^2}{8} [2(\Delta^0 + \delta^0)]^2. \] (26)
Note that the phases of the bidoublets $\delta_i$, $i = 2, 3, 4$ come in the following terms

\begin{align*}
    &v_1 v_i \cos \delta_i, \ i = 2, 3, 4 \\
    &v_2 v_3 \cos (\delta_2 + \delta_3) \\
    &v_2 v_4 \cos (\delta_2 - \delta_4) \\
    &v_3 v_4 \cos (\delta_3 + \delta_4)
\end{align*}

(27)

Also, powers of the bidoublet VEVs which are higher than two come only in the D-term, and there in one only combination $g(v) = v_1^2 + v_3^2 - v_2^2 - v_4^2$. This is exactly the situation in general four Higgs doublet supersymmetric models with real mass parameters. In Ref. [15] it was shown, by using a simple geometrical interpretation for the minimum equations for the three phases, that the minimum in such a model is $CP$ conserving. Thus we conclude that in the SUSY LR model the VEVs of the doublets are real. This conclusion holds for general $A_{\phi_{ij}}$, which can be different for different $i, j$.

The phase of the VEV of the triplet $\beta_\Delta$ is induced by the phase of the coupling $\mu_\Delta$ but it does not couple to the VEVs of the doublets. Thus it is irrelevant since it does not enter the calculation of $\tilde{\Theta}$ at the tree level or one loop.

References


Figure 1: Higgs contribution to one loop calculation of $\bar{\Theta}$. 
Figure 2: Examples of gaugino contributions to one loop calculation of $\tilde{\Theta}$. $V_{L,R}$ are left and right gauginos, respectively. The gaugino mass $m_{\lambda_L}$ is in general complex. There is an analogous graph to (b) that involves right-handed gauginos.