Duality of a Supersymmetric Standard Model without R parity

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**Abstract**

Recently one of the authors proposed a dual theory of a Supersymmetric Standard Model (SSM), in which it is naturally understood that at least one quark (the top quark) should be heavy, i.e., almost the same order as the weak scale, and the supersymmetric Higgs mass parameter \(\mu\) can naturally be expected to be small. Unfortunately, the model cannot possess Yukawa couplings of lepton sector. In this paper, we examine a dual theory of a Supersymmetric Standard Model without R parity. In this scenario, we can introduce Yukawa couplings of lepton sector. In order to induce the enough large Yukawa couplings of leptons, we must introduce fairly large R parity breaking terms, which may be observed in the near future.

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Recently, it has become clear that certain aspects of four dimensional supersymmetric field theories can be analyzed exactly [1, 2, 3, 4]. By using the innovation, it has been tried to build models in order to solve some phenomenological problems [4, 5, 6, 7]. One of the most interesting aspects is “duality” [1, 3]. By using “duality”, we can infer the low energy effective theory of a strong coupling gauge theory. One of the authors suggested that nature may use this “duality”. He discussed a duality of a Supersymmetric Standard Model (SSM). Unfortunately, his model does not possess Yukawa couplings of lepton sector. One possibility to introduce them is to unify quarks and leptons by considering the Pati-Salam gauge group [8]. In this paper, we discuss the model with R-parity breaking terms in order to obtain the Yukawa couplings of the lepton sector.

First we recapitulate the previous model. Then we discuss the extension of the previous model and see how leptons acquire their mass. Then we give a summary and discussion.

To get the point of the previous idea first we review Seiberg’s duality. Following his discussion [1], we examine $SU(N_C)$ Supersymmetric (SUSY) QCD with $N_F$ flavors of chiral superfields,

<table>
<thead>
<tr>
<th>SU($N_C$)</th>
<th>SU($N_F$L)</th>
<th>SU($N_F$R)</th>
<th>U(1)$_B$</th>
<th>U(1)$_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_i^i$</td>
<td>$N_C$</td>
<td>$N_F$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\bar{Q}_i$</td>
<td>$\bar{N}_C$</td>
<td>1</td>
<td>$\bar{N}_F$</td>
<td>-1</td>
</tr>
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which has the global symmetry $SU(N_F)_L \times SU(N_F)_R \times U(1)_B \times U(1)_R$. In the case $N_F \geq N_C + 2$, Seiberg suggested [1] that at the low energy scale the above theory is equivalent to the following $SU(\bar{N}_C)$ SUSY QCD theory ($\bar{N}_C = N_F - N_C$) with $N_F$ flavors of chiral superfields $q_i$ and $\bar{q}^i$ and meson fields $T^i_j$,

<table>
<thead>
<tr>
<th>SU($\bar{N}_C$)</th>
<th>SU($N_F$L)</th>
<th>SU($N_F$R)</th>
<th>U(1)$_B$</th>
<th>U(1)$_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_i$</td>
<td>$\bar{N}_C$</td>
<td>$N_F$</td>
<td>1</td>
<td>$N_C/(N_F - N_C)$</td>
</tr>
<tr>
<td>$\bar{q}^i$</td>
<td>$N_C$</td>
<td>1</td>
<td>$N_F$</td>
<td>$-N_C/(N_F - N_C)$</td>
</tr>
<tr>
<td>$T^i_j$</td>
<td>1</td>
<td>$N_F$</td>
<td>$\bar{N}_F$</td>
<td>0</td>
</tr>
</tbody>
</table>

and with a superpotential

$$W = q_i T^i_j \bar{q}^j.$$

Then the idea of SSM is the following [5].

We introduce ordinary matter superfields besides Higgs doublets:

$$Q^i_L = (U^i_L, D^i_L) : (3, 2)_{i \frac{1}{6}}, \quad U^i_{\bar{R}i} : (\bar{3}, 1)_{-\frac{2}{3}}, \quad D^i_{\bar{R}i} : (\bar{3}, 1)_{\frac{1}{3}}$$

$$L^i = (N^i_L, E^i_L) : (1, 2)_{i \frac{1}{2}}, \quad E^i_{\bar{R}i} : (1, 1)_{i}, \quad i = 1, 2, 3.$$  

1
which transform under the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$.

Let’s examine the dual theory of this theory with respect to the gauge group $SU(3)_C$. In this case the number of the flavor is 6 and the global symmetry in the sense of Seiberg’s is $SU(6)_{Q_L} \times SU(6)_{Q_R} \times U(1)_B \times U(1)_R$. We can assign $Q = (U^1_L, D^1_L, U^2_L, D^2_L, U^3_L, D^3_L)$ and $\bar{Q} = (U^1_R, \tilde{D}^1_R, U^2_R, \tilde{D}^2_R, U^3_R, \tilde{D}^3_R)$ in the table. Since $N_F = 6$, the dual gauge group is also $SU(3)_C$ ($\tilde{N}_C = N_F - N_C$), which we will assign to the ordinary QCD gauge group. A subgroup, $SU(2)_L \times U(1)_Y$, of the global symmetry group $SU(6)_{Q_L} \times SU(6)_{Q_R} \times U(1)_B \times U(1)_R \times SU(6)_L \times SU(3)_{ER} \times U(1)_L \times U(1)_ER$ is gauged. $SU(6)_L \times SU(3)_{ER} \times U(1)_L \times U(1)_ER$ is the global symmetry of the lepton sector. For example, $SU(6)_L \times U(1)_L$ acts on the multiplet $L \equiv (N^c_L, E^c_L, N^c_R, E^c_R)$. $SU(2)_L$ generators are given by

$$I^a_L = I^a_{Q_L1} + I^a_{Q_L2} + I^a_{Q_L3} + I^a_{L1} + I^a_{L2} + I^a_{L3}, \quad a = 1, 2, 3,$$

where $I^a_{Q_Li}$, $[I^a_L]$, are generators of $SU(2)_{Q_Li}$ [$SU(2)_Li$] symmetries which rotate $(U^i_L, D^i_L)$ $([N^c_L, E^c_L])$. The generator of hypercharge $Y$ is given by

$$Y = \frac{1}{6} B - (I_{R1}^3 + I_{R2}^3 + I_{R3}^3) - \frac{1}{2} L + ER$$

where $I^a_{Ri}$ are generators of $SU(2)_{Ri}$ symmetries which rotate $(U^c_{Ri}, D^c_{Ri})$. In this theory, the global symmetry group is $SU(3)_{Q_L} \times SU(3)_{Q_R} \times SU(3)_D \times U(1)_B \times U(1)_R \times SU(6)_L \times SU(3)_{ER} \times U(1)_L \times U(1)_ER$. Then we can write down the quantum numbers of dual fields;

$$q_{Li} = (d_{Li}, -u_{Li}) : (3, 2)_\frac{1}{2}, \quad u^i_{Ri} : (3, 1)_{-\frac{2}{3}}, \quad d^i_{Ri} : (3, 1)_{\frac{1}{3}}, \quad M^i_j : (1, 2)_{-\frac{1}{2}}, \quad N^i_j : (1, 2)_{\frac{1}{2}}, \quad L^i = (N^c_L, E^c_L) : (1, 2)_{-\frac{1}{2}}, \quad E^c_{Ri} : (1, 1)_1, \quad i = 1, 2, 3$$

under the standard gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. Here $M^i_j \sim Q^i_L U^c_{Rj}$ and $N^i_j \sim Q^i_L D^c_{Rj}$ are the meson fields and we assign $q = (d^1_L, -u^2_L, d^2_L, -u^3_L, d^3_L, -u^3_L)$ and $\bar{q} = (d^1_R, -u^2_R, d^2_R, -u^3_R, d^3_R, -u^3_R)$. Because leptons don’t have color indices they exist as they were. It is interesting that the matter contents of both theories are almost the same. The difference is the existence of nine pairs of Higgs superfields $M^i_j$ and $N^i_j$ and their Yukawa terms coupling to ordinary matter superfields,

$$W = -q_{Li}^i N^i_j u_{Rj}^j + q^i_{Li} M^i_j d_{Rj}^i.$$

If one combination of these Higgs scalar fields $^4$

$$\tilde{H} = \sum_{i,j}^3 a^i_j \tilde{N}_i^j + b^j_i (\tilde{M}^j)^c_k,$$

\footnote{Strictly speaking two of four global $U(1)'s$ are broken by $SU(2)_L$ and $U(1)_Y$ anomalies.}

\footnote{In this paper we denote scalar component by tilde, e.g. $\tilde{A}$ means the scalar component of the superfield $A$.}
where $M^c$ denotes the charge conjugated field of $M$, has a VEV $\langle H \rangle = (v, 0)$, the $SU(2)_L \times U(1)_Y$ symmetry is broken to the electromagnetic gauge group $U(1)_Q$. In this case, ordinary quark mass matrices are determined by the mixing of the Higgs scalar field. You should notice that at least one quark has a heavy mass, which is almost the order of the weak scale $v$, if the Yukawa coupling can be taken to be of order one because of the strong dynamics. Namely the heavi ness of the top quark can be naturally understood.

There is, however, a serious problem that the leptons are massless in this theory.

We break $R$ parity in the above model in order to induce the Yukawa couplings of leptons. We introduce the superpotential

$$W = \lambda_{ijk} L^i \bar{L}^j E_R^k,$$

which breaks the global symmetry $SU(3)_L \times SU(3)_E \times U(1)_L \times U(1)_{ER}$ for leptons and $R$ parity. Because leptons are singlet under the $SU(3)_C$, these terms survive in the dual theory. Though to introduce all of such terms causes dangerous phenomena which are experimentally excluded, we can introduce only one term among them[9, 10].

In the theory with $SU(3)_C$, we introduce the following $R$-violating superpotential,

$$W = \lambda L^2 L^3 E_R^{c3},$$

which breaks the global symmetry of the lepton sector $SU(3)_L \times SU(3)_E \times U(1)_L \times U(1)_E$ to $SU(2)_{L23} \times U(1)_{L1} \times SU(2)_{ER} \times U(1)_{E \{E_1 + E_2 \}} \times U(1)_{L_2 + L_3 - 2E_3}$, where $SU(2)_{L23}$ acts on the multiplet $(L_2, L_3)$, $U(1)_{L_1}$ means that the charge of $L_1$ is 1 and those of others' are 0 and so on. The magnitude of the coupling is expected to be $O(0.1)[9]$.

If the scalar component of $L_i$'s ($\equiv \tilde{L}_i$) get a VEV, then at least one lepton acquire mass. When we consider SUSY breaking terms in the theory it is naturally understood that $L_i$'s acquire VEVs. We will see this in the following.

First of all, the global symmetry group $SU(3)_{QL} \times SU(3)_{UR} \times SU(3)_{DR} \times U(1)_R$ must be broken explicitly because otherwise when Higgs fields $N^j_i$ and $M^j_i$ have VEVs, that is the global symmetry is spontaneously broken, massless Nambu-Goldstone bosons appear. One possibility of breaking the global symmetry explicitly is to introduce soft SUSY breaking terms[11, 12] which also break all the global symmetry except $U(1)_B$:

$$\sum_{i,j,k} (A_{ijk} \tilde{Q}_i \bar{U}_j \tilde{L}_k + B_{ijk} \tilde{Q}_i \bar{D}_j \tilde{L}_k + C_{ijk} \tilde{L}_i \bar{\tilde{L}}_j \tilde{E}_k) + \text{mass term}. \quad (10)$$

Thus there appear mixing terms between the Higgs doublets, $\tilde{N}^j_i$ and $\tilde{M}^j_i$, and the scalar components of lepton doublets $\tilde{L}_i$:

$$m^2_{N^j_i} \tilde{N}^j_i \tilde{L}_k + m^2_{M^j_i} \tilde{M}^j_i \tilde{L}_k,$$
Then after Higgs fields get VEVs, VEVs of the scaler leptons are induced. We assume transition masses are smaller than self masses because the former breaks not only the global flavor symmetry but also the lepton global symmetry while the latter breaks only some part of the lepton global symmetry. In this case the induced VEVs are roughly given by \( \langle \tilde{L}_i \rangle \sim (m_{N_j}^2 / m_{N_k}^2) \langle N_k \rangle \sim (m_N^2 / m_{\tilde{H}}^2) v \), where \( m_N^2 \) is a typical transition mass and \( v \) is the VEV of \( \tilde{H} \). Graphically they are expressed in Fig. 1(a).

Naively the magnitude of the scalar lepton VEVs is \( O(1) \) GeV.

By these VEVs lepton masses arise at tree level according to (9) (see Fig 1(b)). The form of the mass matrix for leptons is

\[
M_l = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & m_2 \\
0 & 0 & m_3
\end{pmatrix}
\]  

(13)

where \( m_2 = -\lambda \langle \tilde{L}_3 \rangle \), \( m_3 = \lambda \langle \tilde{L}_2 \rangle \sim O(1) \) GeV\(^6\). Thus we can understand \( \tau \) mass naturally.

Through radiative corrections like Fig. 2 [13] the other will be induced. These are quite dependent on \( C_{ijk} \)\(^7\).

In summary, we examine duality of a SUSY model with R-Parity breaking terms. In this model, the Yukawa couplings of the lepton sector as well as the quark sector are

\(^6\)Because there is \( SU(2)_{L23} \) global symmetry, we can assume, without loss of generality, that among \( \tilde{L}_2 \) and \( \tilde{L}_3 \) only \( \tilde{L}_2 \) acquires a VEV and hence only \( m_3 \) is not zero.

\(^7\)In the case that R-Parity is broken, in general, neutrino masses are also induced[10]. These are also dependent on SUSY breaking parameters and here we do not touch the detail.
Figure 2: a typical graph which induces a mass term for lepton at the loop below the electroweak scale.

induced. Moreover since all the global symmetries except $U(1)_B$ are broken by SUSY breaking terms, we can avoid the appearance of Nambu-Goldstone bosons when Higgs fields acquire VEVs. The fact that the nature doesn’t respect the R-Parity, which might be observed in near future, may suggest that the nature uses the duality.

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References


