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AND THE DYNAMICAL REDUCTION PROGRAM

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International Atomic Energy Agency
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**MACROSCOPIC REALITY
AND THE DYNAMICAL REDUCTION PROGRAM¹**

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With reference to recently proposed theoretical models accounting for reduction in terms of a unified dynamics governing all physical processes, we analyze the problem of working out a worldview accomodating our knowledge about natural phenomena. We stress the relevant conceptual differences between the considered models and standard quantum mechanics. In spite of the fact that both theories describe individual physical systems within a genuine Hilbert space framework, the nice features of spontaneous reduction theories drastically limit the class of states which are dynamically stable. This allows one to work out a description of the world in terms of a mass density function in ordinary configuration space. A topology based on this function and differing radically from the one characterizing the Hilbert space is introduced and in terms of it the idea of similarity of macroscopic situations is made precise. Finally it is shown how the formalism and the proposed interpretation yield a natural criterion for establishing the psychophysical parallelism. The conclusion is that, within the considered theoretical models and at the nonrelativistic level, one can satisfy all sensible requirements for a consistent, unified, and objective description of reality at the macroscopic level.

1 Introduction.

A quite natural question which all scientists who are concerned about the meaning and the value of science have to face is whether one can elaborate a worldview which can accommodate our knowledge about natural phenomena. As is well known, this desideratum does not raise particular problems within classical physics for various reasons which have been lucidly pointed out by J.S. Bell [1]: *Of course it is true that also in classical mechanics any isolation of a particular system from the world as a whole involves approximation. But at least one can envisage an accurate theory, of the universe, to which the restricted account is an approximation, ... and moreover ... even a human observer is no trouble (in principle) in classical theory - he can be included in the system (in a schematic way) by postulating a "psycho-physical parallelism" - i.e., supposing his experience to be correlated with some function of the coordinates.*

The situation is quite different in standard quantum mechanics (SQM), due to the specific peculiarities of the formalism. In particular, the linear nature of the Hilbert space description of physical systems gives rise to well known difficulties with macroscopic objects. The theory allows linear superpositions of macroscopically different states which render problematic to attribute definite properties to the systems. In particular, as is well known, this situation occurs in measurement-like processes in which, after the system-apparatus interaction is over, one has, in general, a linear superposition of macroscopically distinguishable apparatus states.

Various solutions to this puzzling situation have been proposed; for our purposes it turns out not to be relevant to discuss their specific features and/or to comment on their pros and cons. What has to be pointed out is that now everybody agrees that one needs a reinterpretation or a modification of the formalism

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(such as breaking the von Neumann chain, introducing hidden variables, limiting measurability or modifying the evolution law) which does not appreciably alter quantum predictions for microsystems but implies or makes legitimate to state that macro-objects have definite macroproperties.

All these attempts attribute to positions a privileged role in the description of the macroscopic world. This is quite natural since the definiteness, the particularity of the world of our experience, derives from our perceiving physical objects in definite places and this is also why the prescriptions for establishing the psycho-physical correspondence usually involve positions.

This paper is devoted to show how, within the context of the recently introduced nonrelativistic models [2,3] of spontaneous dynamical reduction, one can give a consistent description of the universe overcoming all the above mentioned difficulties.

2 The Dynamical Reduction Program.

The program we are interested in discussing in this paper is based on the consideration, within a genuinely Hilbert space description of physical processes, of nonlinear and stochastic modifications of Schrödinger's dynamics which imply, without entailing any violation of established experimental facts, wave packet reduction with fixed pointer positions in measurement processes and, more generally, forbid the persistence of linear superpositions of macroscopically distinguishable states.

2.1 Quantum Mechanics with Spontaneous Localization.

We begin by discussing a slightly modified version (in which mass has a privileged role) of the first and simplest model of this kind [2], Quantum Mechanics with Spontaneous Localization (QMSL). The model is based on the assumption that, besides the standard evolution, physical systems are subjected to spontaneous localizations occurring at random times and affecting their elementary constituents. Such processes, which we will call "hittings", are formally described in the following way.

We consider a system of N particles. We suppose that when the i -th particle of the system suffers a localization the wave function changes according to

$$\Psi(\mathbf{r}_1, \dots, \mathbf{r}_N) \rightarrow \Psi_{\mathbf{x}}(\mathbf{r}_1, \dots, \mathbf{r}_N) = \Phi_{\mathbf{x}}(\mathbf{r}_1, \dots, \mathbf{r}_N) \cdot \Phi_{\mathbf{x}}^{-1}(\mathbf{r}_i) \quad (2.1)$$

$$\Phi_{\mathbf{x}}(\mathbf{r}_1, \dots, \mathbf{r}_N) = (\alpha/\pi)^{3/4} e^{-\frac{\alpha}{2}(\mathbf{r}_i - \mathbf{x})^2} \Psi(\mathbf{r}_1, \dots, \mathbf{r}_N) \quad (2.1)$$

The probability density of the process occurring at point $\mathbf{x}$ is given by $|\Phi_{\mathbf{x}}|^2$. For what concerns the temporal features of the processes we assume that the hittings for the various particles occur independently and at randomly distributed times

with a mean frequency λ_m which depends on their mass. We choose $\lambda_m = (m/m_0)\lambda$, where m is the mass of the particle, m_0 is the nucleon mass and λ is of the order of $10^{-16} \text{ sec}^{-1}$. The localization parameter $1/\sqrt{\alpha}$ is assumed to take the value 10^{-5} cm .

Let a macroscopic system be in a state $|\Psi\rangle = |\Psi_1\rangle + |\Psi_2\rangle$ which is a superposition of two states $|\Psi_1\rangle$ and $|\Psi_2\rangle$ in which a given number of particles are well localized within two different (and far apart with respect to the characteristic distance $1/\sqrt{\alpha}$) spatial regions A and B , respectively. When one of these particles suffers a hitting that localizes it in the position corresponding to the state $|\Psi_1\rangle$ ($|\Psi_2\rangle$), the other term of the superposition is exponentially suppressed. Therefore, the macroscopic system jumps either to the state $|\Psi_1\rangle$ or to $|\Psi_2\rangle$ every time one of the particles differently located in the two states suffers a hitting. There follows that the rate at which one of the two terms of the superposition is suppressed increases proportionally to the number and the masses of the particles which are displaced.

The QMSL mechanism does not respect the symmetry properties of the wave function in the case of identical constituents. Its generalization satisfying such a requirement, the Continuous Spontaneous Localization Model (CSL), has been presented and discussed in various papers [3].

2.2 The CSL Model.

The model is based on a linear stochastic evolution equation for the statevector. The evolution does not preserve the norm but only the average value of the squared norm. The equation is:

$$\frac{d|\Psi_w(t)\rangle}{dt} = \left[\frac{i}{\hbar} H + \sum_i A_i w_i(t) - \frac{\gamma}{2} \sum_i A_i^2 \right] |\Psi_w(t)\rangle \quad (2.2)$$

In eq. (2.2), the quantities $w_i(t)$ are c -number Gaussian stochastic processes satisfying

$$\langle\langle w_i(t) \rangle\rangle = 0$$

$$\langle\langle w_i(t) w_j(t') \rangle\rangle = \gamma \delta_{ij} \delta(t - t') \quad (2.3)$$

while the quantities A_i are commuting self-adjoint operators. Let us assume, for the moment, that these operators have a purely discrete spectrum and let us denote by P_σ the projection operators on their common eigenmanifolds.

The physical meaning of the model is made precise by the following prescription: if a homogeneous ensemble (pure case) at the initial time $t = 0$ is associated to the statevector $|\Psi, 0\rangle$, then the ensemble at time t is the union of homogeneous ensembles associated with the normalized vectors $|\Psi_w(t)\rangle / \|\Psi_w(t)\|$, where $|\Psi_w(t)\rangle$ is the solution of eq.(2.2) with the assigned initial conditions and for the specific stochastic process w which has occurred in the interval $(0, t)$.

The probability density for such a subensemble is the “cooked one”, i.e. it is given by:

$$P_{\text{Cook}}[w] = P_{\text{Raw}}[||\Psi_w(t)||^2] \quad (2.4)$$

where we have denoted by $P_{\text{Raw}}[w]$ the “raw” probability density associated, according to eq. (2.3), to the Gaussian process w in the interval $(0, t)$, i.e.:

$$P_{\text{Raw}}[w] = \frac{1}{N} e^{-\frac{1}{2} \sum_i \int_0^t dt v_i^2(t)}, \quad (2.5)$$

N being a normalization factor.

One can prove that the map from the initial ensemble to the final ensemble obeys the forward time translation semigroup composition law. It is also easy to prove that the evolution, at the ensemble level, is governed by the dynamical equation for the statistical operator

$$\frac{d\rho(t)}{dt} = \frac{i}{\hbar} [\rho(t), H] + \gamma \sum_i A_i \rho(t) A_i - \frac{\gamma}{2} \sum_i \{A_i^2, \rho(t)\}. \quad (2.6)$$

from which one immediately sees that, if one disregards the Hamiltonian evolution, the off-diagonal elements $P_\sigma \rho(t) P_\tau$ ($\sigma \neq \tau$) of the statistical operator are exponentially damped.

For our concerns, the relevant feature of the dynamical process (2.2) with the prescription (2.4) is that it drives [3] the statevector of each individual member of the ensemble into one of the common eigenmanifolds of the operators A_i with the appropriate probability.

This concludes the sketchy exposition of the general structure of the CSL model. Obviously, to give a physical content to the theory one must choose the so called preferred basis, i.e. the eigenmanifolds on which reduction takes place or, equivalently, the set of commuting operators A_i . The specific form that has been shown to possess all the desired features has been presented and discussed in refs. [3]. Recently, various considerations have led to a slightly modified version making the mass to play a specific role [4,5]. In this version one identifies the discrete index i and the operators A_i of the above formulae with the continuous index $\mathbf{r}$ and the operator

$$M(\mathbf{r}) = \sum_k m^{(k)} N^{(k)}(\mathbf{r}), \quad (2.7)$$

where $m^{(k)}$ is the mass of the particles of type k and $N^{(k)}(\mathbf{r})$ are the number density operators giving the average density of particles of type k in a volume of about 10^{-15}cm^3 around the point $\mathbf{r}$:

$$N^{(k)}(\mathbf{r}) = \left[\frac{\alpha}{2\pi} \right]^{\frac{3}{2}} \sum_s \int d\mathbf{q} e^{-\frac{\alpha}{2}(\mathbf{q} - \mathbf{r})^2} a_{(k)}^\dagger(\mathbf{q}, s) a_{(k)}(\mathbf{q}, s). \quad (2.8)$$

Here $a_{(k)}^\dagger(\mathbf{q}, s)$ and $a_{(k)}(\mathbf{q}, s)$ are the creation and annihilation operators of a particle at point $\mathbf{q}$ with spin component s , satisfying the canonical commutation or anticommutation relations. Correspondingly, one has a continuous family of stochastic Gaussian processes satisfying:

$$\begin{aligned} \langle \langle w(\mathbf{r}, t) \rangle \rangle &= 0, \\ \langle \langle w(\mathbf{r}, t) w(\mathbf{r}', t') \rangle \rangle &= \frac{\gamma}{m_0^2} \delta(\mathbf{r} - \mathbf{r}') \delta(t - t') \end{aligned} \quad (2.9)$$

In the above formula m_0 is the nucleon mass, the parameter α is assumed to take the same value (10^{10}cm^{-2}) as in the case of QMSL and γ is related to the frequency $\lambda = 10^{-16} \text{sec}^{-1}$ of that model according to $\gamma = \lambda(4\pi/\alpha)^{3/2}$. With the above choices the theory exhibits the same features of the standard CSL model of refs. [3] for what concerns the localization of nucleons. Obviously, also other massive particles suffer localizations but with a rate depending on their masses. The nonhamiltonian terms in the dynamical equation lead to the objectification of the position variables of massive particles.

It goes without saying that, due to the extremely low frequency of the localizations (for simplicity we use here the language of the QMSL model) of the elementary constituents of physical systems, all microscopic systems evolve in (practically) perfect agreement with the predictions of SQM. On the contrary, due to the amplification mechanism discussed above, macroscopic systems are strongly affected: a linear superposition of macroscopically different states like those corresponding to different locations of a macroscopic pointer cannot last for more than about $10^{-7} - 10^{-6}$ seconds.

2.3 Some Historical Remarks.

Before concluding this Section we consider it useful to look at the proposed modifications of SQM from the point of view taken by Einstein in one of his last writings on quantum theory [6]: *Reply to Criticisms*. Contrary to what is usually referred as his firm position about the Copenhagen interpretation of the theory, in the paper under consideration he states clearly that one could accept the peculiar features which, according to this interpretation, characterize the microscopic processes, provided one could save the definiteness, the particularity of our experience with macro-objects. He remarks that in the case of the decay of a radioactive atom, the orthodox quantum theorist would claim that *within the framework of our theoretical total construction* it is not reasonable even to posit the existence of a definite point of time for the transformation of a single atom. He plainly declares that such a position is *certainly by no means absurd*. However he also stresses that Schrödinger taught us that the question about the time of decay can be straightforwardly transformed (by simply coupling the atom to an appropriate apparatus) into the question of the presence of a macroscopic registration mark on a paper strip. And he states: *the location of the*

mark on the registration-strip is a fact which belongs entirely within the sphere of macroscopic concepts ... and ...there is hardly likely to be anyone who would be inclined to accept that the existence of that location is essentially dependent upon the carrying out of an observation In fact: in the macroscopic sphere it simply is considered certain that one must adhere to the program of a realistic description in space and time; whereas in the sphere of microscopic situations one is more readily inclined to give up, or at least to modify, this program. Few pages later he comes back to this point and he stresses that his realistic attitude does not derive from a philosophical prejudice: the “real” in physics is to be taken as a type of program, to which we are, however, not forced to cling a priori. No one is likely to be inclined to attempt to give up this program within the realm of the “macroscopic” (location of the mark on the paper strip “real”). And then he draws his conclusion: But the “macroscopic” and the “microscopic” are so inter-related that it appears impracticable to give up this program in the “microscopic” alone.

We have chosen to devote so much space to this quotation to call attention to the fact that, with reference to the previous analysis, the crucial problem of our best theory is to succeed in accounting for the smooth merging of the cloudy microworld with the definite macroworld; stated differently, to make precise where and how acts occur which, in J.A. Wheeler’s words, can be unambiguously identified as [7] *irreversible acts of amplification*. Quite appropriately, in our opinion, J. A. Wheeler maintained that locating the shifty split cannot be related to acts of empirical assertions. On the contrary, the occurrence of the irreversible acts of amplification must be part of the physical description and one should not ascribe to them any exceptional position above the rest.

The Dynamical Reduction Program represents an attempt to answer the puzzling questions we have discussed by identifying in a precise way within a strictly quantum context (i.e., by assuming completeness of the Hilbert space description of the states of individual physical systems) the shifty split between micro and macro, reversible and irreversible. The very existence of precise dynamical reduction models shows that, contrary to Einstein’s expectations, the program of building up a consistent and unified theory allowing the microsystems to be wavy and nevertheless implying the definiteness of the macroscopic world is viable. It is important to stress that the emergence of such definiteness is implied and precisely described by the formalism so that, to account for it, there is no need to invoke (as many other attempts do) the practical impossibility of revealing interference effects at the macroscopic level. We leave to the reader the evaluation of the potential relevance of the approach and to identify its advantages and drawbacks with respect to the many interesting recent proposals. At any rate, it seems to us that having the explicit proof that a theory exhibiting such features is possible has, by itself, a certain relevance for the investigations about the foundations of quantum mechanics.

3 How Macroscopic Reality is Described.

A scientific theory is a conceptual scheme dealing with formal elements which are supposed to embody some information about something existing and to account for physical processes *which take place* “out there”. Among these processes there are experiments performed by observers and also what have been called measurement-like events which [8] *we are obliged to admit are going on more or less all the time, more or less everywhere*.

To be something more than a simple list of facts and to allow one to go beyond a purely instrumentalistic position, i.e. *to explain* rather than simply *to describe*, a theory must be supplemented with some sort of interpretation. We are interested in interpretations which allow one to make claims about properties (related to specific variables) possessed by individual physical systems. With reference to this crucial point, which is the core of the debate about the foundations of quantum mechanics, we start by making precise the statement that such properties are objectively possessed or accessible.

Definition: we will claim that a property corresponding to a value (a range of values) of a certain variable in a given theory is *objectively possessed or accessible* when, according to the predictions of that theory, there is an experiment (or a physical process) yielding reliable information about the variable that would, if performed (or taking place), give an *outcome* corresponding to the claimed value. Thus, the crucial feature characterizing *accessibility* (as far as statements about individual physical systems are concerned) is the matching of the claims and the *outcomes* of physical processes testing the claims.

3.1 Introductory Remarks.

At this point we are ready to tackle our central problem, i.e. the one of giving an acceptable and appropriate description of the macroscopic world. Before doing that we would like to recall once more the lucid position [9] of J.S. Bell about what one has to require from a theoretical model in order that it can be considered as satisfactory. He points out that a theory is a mathematical framework related to empirical facts. And he states: *you must identify some things as being really there, as distinct from the many mathematical concepts that you can easily devise - like the projection of the side of a triangle to infinity and so on. We must decide that some things are really there and that you are going to take them seriously*. This requires a neat distinction between what he calls the kinematical and the dynamical aspects of the theory. The kinematics should list the possibilities one is envisaging, the dynamics should give the probabilities to be attached to the possibilities. And he puts forward strict requirements for the theoretician: *I would want the kinematics of your theory tell me what it is you are talking about before you tell me what about it*. We are perfectly aware that this position could be considered as quite naive from a philosophical standpoint,

but it expresses in very simple terms our firm convictions about reality and the reasons which have motivated our interest in the foundations of science. Having stated this we can now go on with our analysis.

3.2 A Short Digression about Macroscopic Similarity.

Before tackling the problem of property attribution at the macroscopic level it is useful to point out the inappropriateness of the Hilbert space topology to describe the similarity or difference of macroscopic situations. Suppose a physical system S is an almost rigid body and let us consider the following three states: $|\varphi^A\rangle$, $|\varphi^B\rangle$ and $|\tilde{\varphi}^A\rangle$. The state $|\varphi^A\rangle$ corresponds to a definite internal state of S and to its centre of mass being well localized around A ; the state $|\varphi^B\rangle$ is obtained by simply displacing $|\varphi^A\rangle$ so that it is well localized in a far region B ; the state $|\tilde{\varphi}^A\rangle$ differs from $|\varphi^A\rangle$ simply by the fact that one or a microscopic number of its “constituents” are in states which are orthogonal to the corresponding ones in $|\varphi^A\rangle$.

It is obvious that, under any reasonable assumption about similarity or difference of the states of the macrosystem, $|\tilde{\varphi}^A\rangle$ must be considered very close (identical) to $|\varphi^A\rangle$ while $|\varphi^B\rangle$ must be considered very different from $|\varphi^A\rangle$. On the other hand, according to the Hilbert space topology

$$\|(|\varphi^A\rangle - |\tilde{\varphi}^A\rangle)\| = \|(|\varphi^A\rangle - |\varphi^B\rangle)\| = \sqrt{2}. \quad (3.1)$$

This shows with striking evidence that the “natural” topology of SQM is totally inadequate for the description of the macroscopic world. In Subsection 3.6, taking advantage of its dynamical features, we will show how CSL suggests a natural topology which allows one to overcome the now mentioned difficulty.

3.3 The Mass Density Distribution.

Let S be a physical system which constitutes our universe, and let H^S be its associated Hilbert space. Let us make an attempt to describe the physical reality of *what exists out there* by giving the status of objectively possessed properties to the mean values of the average (over the characteristic volume of CSL) mass density operator (2.7):

$$\mathcal{M}(\mathbf{r}, t) = \langle \Psi(t) | M(\mathbf{r}) | \Psi(t) \rangle. \quad (3.2)$$

If one assumes, as one can consistently do within a non relativistic quantum framework, that the system S contains a fixed and finite number of particles, Eq. (3.2) defines, for a given t , a mapping of the unit sphere of H^S into the space of positive and bounded functions of $\mathbf{r}$.

A question naturally arises: within the conceptual framework we are interested in does $\mathcal{M}(\mathbf{r}, t)$ represent an accessible property of physical systems?

Before answering this question a comparison with the case of SQM is appropriate.

3.4 Mass Density and SQM.

The map (3.2) is obviously many to one. To better focus on this point as well as for future purposes it turns out to be useful to compare two statevectors $|\Psi^\oplus\rangle$ and $|\Psi^\otimes\rangle$ defined as follows. Let us consider a very large number N of particles and two space regions A and B with spherical shape and radius R . The state $|\Psi^\oplus\rangle$ is the linear superposition, with equal amplitudes, of two states $|\Psi_N^A\rangle$ and $|\Psi_N^B\rangle$ in which the N particles are well localized with respect to the characteristic length (10^{-5} cm) of the model and uniformly distributed in regions A and B , respectively, in such a way that the density turns out to be of the order of 1 gr/cm^3 . On the other hand, $|\Psi^\otimes\rangle$ is the tensor product of two states $|\Phi_{N/2}^A\rangle$ and $|\Phi_{N/2}^B\rangle$ corresponding to $N/2$ particles being uniformly distributed in region A and $N/2$ in region B , respectively:

$$|\Psi^\oplus\rangle = \frac{1}{\sqrt{2}} [|\Psi_N^A\rangle + |\Psi_N^B\rangle], \quad |\Psi^\otimes\rangle = |\Phi_{N/2}^A\rangle \otimes |\Phi_{N/2}^B\rangle. \quad (3.3)$$

It is easy to see that the two considered states give rise to the same function $\mathcal{M}(\mathbf{r})$.

Let us now investigate the status of the quantity $\mathcal{M}(\mathbf{r})$ within SQM. To this purpose let us suppose that one can use quantum mechanics to describe the gravitational interaction between massive bodies and let us consider the following gedanken experiment: a test mass is sent through the middle point of the line joining the centres of regions A and B with its momentum orthogonal to it. In the case of the state $|\Psi^\otimes\rangle$ quantum mechanics predicts that the test particle will not be deflected. Thus the outcome of the considered test agrees with the value for the mass density in regions A and B given by eq.(3.2); we can legitimately state that such a value is objectively possessed. On the other hand, in the case of $|\Psi^\oplus\rangle$, it is quite obvious that $\mathcal{M}(\mathbf{r})$ cannot be considered as an “accessible” mass density function. In fact, if the same test is performed when the state is $|\Psi_N^A\rangle$ ($|\Psi_N^B\rangle$) quantum mechanics predicts an upward (downward) deviation of the test particle. Then, the linear nature of the evolution implies that if one would be able to prepare the state $|\Psi^\oplus\rangle$ the final state would be

$$|\phi\rangle = \frac{1}{\sqrt{2}} [|\Psi_N^A\rangle \otimes |\phi^{\text{UP}}\rangle + |\Psi_N^B\rangle \otimes |\phi^{\text{DOWN}}\rangle], \quad (3.4)$$

with obvious meaning of the symbols. If one includes the test particle into the “universe” and considers the mass density operator in regions corresponding to the wave packets $|\phi^{\text{UP}}\rangle$ and $|\phi^{\text{DOWN}}\rangle$, one discovers that nowhere in the universe is there a density corresponding to the density of the test particle. In a sense, if one would insist in giving a meaning to the density function he

would be led to conclude that the particle has been split by the interaction into two pieces of half its density so that the test does not give a definite outcome. According to our criterion this shows that the mass density distribution $\mathcal{M}(\mathbf{r})$ in regions A and B , in the case of the state $|\Psi^+\rangle$, is inaccessible.

3.5 CSL and Objectively Possessed Properties.

Coming back to CSL, it should be obvious that the situation is radically different from the one of SQM since such a theory does not allow the persistence for more than a split second of states like $|\Psi^+\rangle$ which are strictly analogous to those giving rise to the measurement problem. To analyze this point it is useful to introduce a precise mathematical criterion embodying, for the mass density function, the accessibility requirement put forward at the beginning of this Section. To this purpose, let us consider the mass density variance at point $\mathbf{r}$ at time t defined by the following map from the unit sphere of $H^{(N)}$ into $\mathbb{R}^+$:

$$\mathcal{V}(\mathbf{r}, t) = \langle \Psi(t) | [M(\mathbf{r}) - \langle \Psi(t) | M(\mathbf{r}) | \Psi(t) \rangle]^2 | \Psi(t) \rangle \quad (3.6)$$

and let us evaluate the ratio

$$\mathcal{R}^2(\mathbf{r}, t) = \mathcal{V}(\mathbf{r}, t) / \mathcal{M}^2(\mathbf{r}, t). \quad (3.7)$$

We then claim that at time t the mass density at point $\mathbf{r}$ is accessible if

$$\mathcal{R}(\mathbf{r}, t) \ll 1. \quad (3.8)$$

This criterion is clearly reminiscent of the probabilistic interpretation of standard quantum mechanics. Actually, within such a theory, Eq.(3.7) corresponds to the fact that the spread of the operator $M(\mathbf{r})$ is much smaller than its mean value. Even though we take a completely different attitude with respect to the mean value of $M(\mathbf{r})$, it turns out to be useful to adopt the above criterion also within the new context. In fact, as we will discuss in what follows, when one has a space region such that for all its points relation (3.7) holds, all reliable tests aimed to ascertain the mass density value give outcomes corresponding to $\mathcal{M}(\mathbf{r})$.

With reference to the previous example we stress that in the case of $|\Psi^+\rangle$ all points within regions A and B are such that criterion (3.7) is very well satisfied. In the case of $|\Psi^-\rangle$ for the same points one would have:

$$\begin{aligned} \mathcal{M}(\mathbf{r}) &\simeq \frac{d}{2} m_{\pm}, \\ \mathcal{V}(\mathbf{r}) &\simeq \frac{d^2}{4} m_{\pm}^2, \end{aligned} \quad (3.9)$$

where d represents the average density in a volume of 10^{-10}cm^3 around $\mathbf{r}$. Then follows

$$\mathcal{R}(\mathbf{r}) \cong 1, \quad (3.9)$$

so that, as expected, the average mass density in regions A and B is nonaccessible.

Obviously for a microsystem, exception made for the case in which its wavefunction is appreciably different from zero only within a region of linear extension smaller than 10^{-7}cm , its average mass density turns out to be almost always nonaccessible since CSL does not forbid superpositions of arbitrarily far apart microscopic states. This fact does not constitute a difficulty for the proposed interpretation, on the contrary, it embodies the fact that nature has compelled us to allow [10] *electrons* (in general microsystems) *to enjoy the cloudiness of waves, while* our experience of the world requires *tables and chairs, and ourselves, and black marks on photographs, to be rather definitely in one place rather than another, and to be described in "classical terms"*. The possible nonaccessibility of the mass density distribution does not entail that it is not real. Actually even regions in which the mass density does not represent an objectively possessed property according to our mathematical criterion may play an important physical role¹. We point out that within a theory like CSL which implies the definiteness of the macroscopic world, the request of being able to speak of objectively possessed macroproperties and the desire, we will discuss in the next subsection, to identify a topology which is appropriate for describing the similarity and the difference of macroscopic situations, lead to consider "two levels of reality". The attribution of a real status to $\mathcal{M}(\mathbf{r})$ even when it is not accessible does not give rise to the difficulties one meets when one tries to interpret the wavefunction as giving the charge or the mass density of the elementary constituents of nature. Actually this corresponds to plainly

A typical example is represented by an intergalactic cloud with a very low mass density and wavefunctions for its particles with a large spread. According to our strict criterion, the mass density of the cloud is inaccessible, while, if we adopt the physically meaningful criterion put forward at the beginning of this Section and to test it we resort to the consideration of its gravitational effects, e.g., on the trajectory of a far asteroid, we would be led to state that our claims about the mass density are objectively true since they agree with the "physical outcome". Here, a comment is appropriate. The theory contains a specific dynamical mechanism and specific parameters which allow a precise mathematical formulation of the accessibility criterion. Obviously, when one is interested in a particular class of physical processes the requirement that the outcomes match the claims about properties could be satisfied even though the strict mathematical one does not. This does not raise any problem. It finds its counterpart in the fact that to analyze a specific physical situation one can use a coarser graining than the one corresponding to the fundamental characteristic length of the theory. Accordingly, if we adapt the graining to the problem of interest, we can assert that the mass of the cloud is accessible. Obviously, while *for practical purposes* one can change the graining according to the problem he is interested in, adopting a graining which is finer than the fundamental one of the theory makes no sense. In fact, if one takes the theory seriously as describing the laws governing the evolution of the universe, mass density values would almost never be objectively possessed when such a graining is chosen.

accept the well-known fact that [8] *however far the wavefunction has extended, the reaction of a detector ... remains spotty.*

3.6 The Appropriate Topology for the CSL Model.

We come back to the problem of giving a precise mathematical criterion which allows one to consider “similar” or “indistinguishable” various macroscopic situations. Let us denote by $\mathcal{U}^{(S)}$ the unit sphere in $H^{(S)}$ and let us consider the non linear map $\mathcal{M}$ associating to the element $|\varphi\rangle$ of $\mathcal{U}^{(S)}$ the element $\mathcal{M}(\mathbf{r}, |\varphi\rangle) = \langle \varphi | \mathbf{M}(\mathbf{r}) | \varphi \rangle$ of $L^{(2)}$, the Hilbert space of the real square integrable functions on $\mathbb{R}^{(3)}$.

On $\mathcal{U}^{(S)}$ we define a topology by introducing a mapping $\Delta: \mathcal{U}^{(S)} \times \mathcal{U}^{(S)} \rightarrow \mathbb{R}$ according to:

$$\Delta(|\varphi\rangle, |\psi\rangle) = \left\{ \int d\mathbf{r} |\mathcal{M}(\mathbf{r}, |\varphi\rangle) - \mathcal{M}(\mathbf{r}, |\psi\rangle)|^2 \right\}^{1/2}. \quad (3.10)$$

Such a mapping is not a distance since it may happen that $\Delta(|\varphi\rangle, |\psi\rangle) = 0$ even though $|\varphi\rangle \neq |\psi\rangle$. However Δ meets all other properties of a distance:

$$\Delta(|\varphi\rangle, |\psi\rangle) = \Delta(|\psi\rangle, |\varphi\rangle) \geq 0,$$

$$\Delta(|\varphi\rangle, |\psi\rangle) \leq \Delta(|\varphi\rangle, |\chi\rangle) + \Delta(|\chi\rangle, |\psi\rangle), \quad (3.11)$$

as one can easily prove.

From now on we will limit our considerations to the proper subset $\mathcal{A}^{(S)}$ of $\mathcal{U}^{(S)}$ of those states which are allowed by the CSL dynamics². For any element $|\varphi\rangle$ of $\mathcal{A}^{(S)}$ we consider the set of states of $\mathcal{A}^{(S)}$ for which $\Delta(|\varphi\rangle, |\psi\rangle) \leq \varepsilon$. Here the quantity ε has the dimensions of a mass and is chosen of the order of $10^{17} m_n$, with m_n the nucleon mass. From the properties of the map Δ it follows that:

i. $\{\Delta(|\varphi\rangle, |\psi\rangle) \leq \varepsilon \text{ and } \Delta(|\varphi\rangle, |\chi\rangle) \leq \varepsilon\}$ implies $\Delta(|\chi\rangle, |\psi\rangle) \leq \varepsilon$.

ii. $\{\Delta(|\varphi\rangle, |\psi\rangle) > \varepsilon \text{ and } \Delta(|\varphi\rangle, |\chi\rangle) \leq \varepsilon\}$ implies $\Delta(|\chi\rangle, |\psi\rangle) > \varepsilon$.

We have introduced the parameter ε in such a way that it turns out to be sensible (see Section 4) to consider similar to each other states whose “distance” Δ is smaller than (or of the order of) ε . More specifically, when

$$\Delta(|\varphi\rangle, |\psi\rangle) \leq \varepsilon \quad (3.12)$$

we will say that $|\varphi\rangle$ and $|\psi\rangle$ are “macroscopically undistinguishable”.

We now discuss the “distorted” (with respect to the Hilbert space one) topology associated to the “distance” Δ . First of all we stress that the two states

²For a more precise identification of such a set see ref. [5].

$|\varphi^A\rangle$ and $|\tilde{\varphi}^A\rangle$ which are maximally distant in the Hilbert space topology, turn out to be undistinguishable, i.e. to satisfy condition (3.12), in the new topology. This is an example showing how such a topology takes more appropriately into account the fact that, under any sensible assumption, the “universes” associated to the considered states are very similar.

Obviously, one problem arises. Criterion (3.12) leads to identify states which may be quite different from a physical point of view, even at the macroscopic level. To clarify this statement we take into account two states $|\varphi\rangle$ and $|\psi\rangle$ corresponding to an almost rigid body located, at time $t = 0$, in the same position but with macroscopically different momenta, let us say $P = 0$ and $P = P^*$, respectively. Even though the two states are physically quite different, their distance at $t = 0$ is equal to zero. However, if one waits up to the time in which the state $|\psi\rangle$ has “moved away” from $|\varphi\rangle$, the “distance” $\Delta(|\varphi(t)\rangle, |\psi(t)\rangle)$ becomes large and the two states are no longer undistinguishable.

3.7 A Worldview Based on CSL.

As we have pointed out at the beginning of this Section, we are interested in describing something “existing and evolving out there” in such a way that the resulting physical picture is compatible with our experience of the world.

For the reader who has followed the above analysis, it should be clear that, within CSL, physical systems are fully described by the statevector which evolves according to equations embedding genuine elements of chance. However, even though [11] *there is nothing in this theory but the wavefunction*, due to the fact that as we have extensively discussed the dynamics itself tends (on the appropriate scale) to make objectively possessed just the values of the mass density distribution, it turns out to be correct to identify the “basic kinematical elements” in Bell’s sense with such a quantity. This is what we have done in the previous Subsections by putting forward explicit criteria allowing one to recognize its being accessible and by showing the appropriateness of resorting to the mass density function to define the idea of similarity and difference between macroscopic situations.

In a sense we could legitimately state that the elements of the theory *to be taken seriously* are the values of the mass density function. All other properties which emerge as a consequence of the dynamical evolution derive from the basic feature of the theory of making this function *accessible*. For example in the case analyzed at the end of the previous subsection the macroscopic momentum of the system can, after a certain time, be related to an objectively possessed property just as a consequence of the objectification of the mass density.

With reference to the position we have just taken we feel the necessity of mentioning two other possible attitudes about the formal elements which have *to be taken seriously*. One has been mentioned by David Albert in his comments to our talk and coincides with the one taken by Bell himself in his last years. It

amounts simply to consider as the only physically meaningful quantity of the theory the modulus square of the wavefunction in the $3N$ -dimensional configuration space: such a quantity should be interpreted as representing the density of stuff of which the world is made⁴. The second one has been discussed in a recent preprint [12] by P. Pearle. He denotes, as we did in ref.(5), as *objective reality* the properties we have preferred to call *accessible* in this paper. But, besides defining objective reality, this author introduces two other concepts that he considers useful: projective reality and subjective reality. Moreover he takes into account all these three “levels” of reality both for macro and for microsystems. Projective reality is described by the mean values of all self-adjoint operators of the physical system under consideration. On the other hand, since any attempt to get a relativistic generalization of the dynamical reduction models implies that the mean values of local observables of microsystems depend, in general, on the space like surface one is considering to evaluate them⁵ such mean values can be different for different observers, a fact justifying the use of the expression “subjective reality” to describe them. The basic reason to take into account all these levels of reality is that them all have precise dynamical consequences for the system and may play an important role in determining the properties which will become accessible at subsequent times.

Let us discuss these two proposals. Concerning the first one, we perfectly agree (this firm request has guided our investigations from the very beginning) that the statevector of an individual physical system contains the most detailed information one can have about it. But we cannot avoid stressing various important facts. Here we are primarily concerned in accounting for the world as we perceive it, a problem on which J.S. Bell has made some cute remarks: *when Von Bortkevitch collected statistics on the kicking of soldiers to death by horses in the Prussian Army, in different years, he found a Poisson distribution. Now,*

We feel the desire to mention that we had a series of discussions with J.S. Bell on this point and to call attention on his firm position about the matter. On October 3, 1989, answering to repeated questions I had raised in previous correspondence, he wrote: *As regards Ψ and the density of stuff, I think it is important that this density is in the $3N$ -dimensional configuration space. So I have not thought of relating it to ordinary matter or charge density in 3 space. Even for one particle I think one would have problems with the latter. So I am inclined to the view that you mention after “... as it is sufficient for an objective interpretation...” And it has to be stressed that the “stuff” is in $3N$ space - or whatever corresponds in field theory.* The reader can imagine how reluctant we are in taking a position differing to some extent from the one suggested by this great scientist and deep thinker. However, we would like to mention that, in a sense, the above letter strengthens our position. J.S. Bell is perfectly right in pointing out that “for one particle one would have problems” in passing to real space. In fact we have repeatedly stressed that the microscopic mass density is almost never accessible. But, at the macroscopic level, such density is dynamically objectified in real 3 space, and accounts for our definite perceptions.

⁴A detailed discussion of this feature and a precise analysis of the reasons for which it does not give rise to any conceptual difficulty has been presented in refs. [13]. The nice fact is that, why such a dependence is unavoidable at the microscopic level, once more, no ambiguity arises when macrosystems are considered.

you don't go into the world looking for the Poisson distribution, you go out looking for soldiers and horses and kicks. We could rephrase this sentence by stating that you do not go out in the $3N$ -dimensional configuration space looking for a density of stuff, you go out in 3-dimensional space and look for masses in definite places. But there are other reasons to consider the mass density distribution as the fundamental element of the theory: if the theory is taken seriously such a quantity represents precisely the only property that nature makes accessible, just as particle positions represent precisely the only real and objective properties of physical systems within the de Broglie-Bohm theory⁵. The mass density is a quantity whose meaning is quite clear and which is directly related to our experience. Stressing its prominent role has not to be taken as a specific ontological choice, but simply as the appropriate way to call attention on the basic physical features of the theory which make the “stat is of the world” consistent with our perceptions: the dynamics running all natural processes strives to make definite the positions of massive objects and this mechanism embodies the new features of nature which the theory attempts to grasp.

There are also some obvious and trivial practical reasons to assign a prominent role to the mass density function. For instance, looking at the wavefunction in the $3N$ -dimensional space does not yield any immediate information about the fact that the system you are dealing with is, e.g., a moving pointer. On the contrary, the changes of the mass density give precisely the relevant information. Obviously, the same argument could be used to claim the unappropriateness of the phase-space description of the state of a classical system. But one should not forget that within a quantum context the situation is much more peculiar since incompatible properties (which exist also for a macro-object) cannot become simultaneously accessible. Among all conceivable properties, the theory makes precisely accessible the mass density distribution, all the others (if they emerge) are a consequence of this fundamental dynamical feature of the theory.

Finally, as we have discussed in Subsections 3.2 and 3.3, if one is interested in the relevant problem of giving a precise formal status to the vague idea of similarity at the macroscopic level one needs a topology. As we have shown the appropriate topology cannot be defined simply in terms of the wavefunction or of the density of stuff in the $3N$ -dimensional space, it requires in an essential way the consideration of the mass density distribution, an accessible and objectively possessed feature of our universe.

⁵Obviously there are important differences. The pilot wave theory puts on the same footing the positions of micro and macroscopic systems. The wave function is an additional and independent fundamental element of the theory. But, leaving aside the technical points, we think that, if one takes into account that our primary interest is in the macroscopic level of reality, one can legitimately claim that just as [14] *the moral* (of the pilot wave theory) *is that in physics the only observations we must consider are position observations*, in a completely analogous way one can assert that the moral of CSL is that the only observations we must consider are those ascertaining the value of the accessible mass density distributions, typically those corresponding to the positions of macroscopic objects.

We come now to the second attitude mentioned above. Concerning this we stress that the consideration of inaccessible properties (mean values of observables referring to microsystems or to nonobjectively possessed properties) is uninteresting for the description of reality as we perceive it. It is true that such quantities play some role in determining the properties which will become accessible in the future. But taking them into account seems to us redundant. In fact, as it is well known, the assignment of the mean values of all bounded self adjoint operators is equivalent to assigning the wavefunction itself. Thus, since the quantities under discussion do not have a direct physical significance, the information they give is simply a substitute of the knowledge of the statevector. This being the case it would be more natural to take the position suggested by Bell. Analogous remarks, with the due changes, apply to subjective reality. For lack of space we cannot analyze here this interesting aspect which would require a detailed discussion of the features of “possible” relativistic reduction models. We limit ourselves to mention that any such model must unavoidably make accessible appropriate properties at the macroscopic level. These properties are those which should play, within such theoretical schemes, the role we have given here to the mass density function in accounting for physical reality.

4 Accessible Reality and Definite Perceptions.

The most characteristic and appealing feature of CSL and of the interpretation of it we have proposed consists in the fact that it allows one to give a satisfactory account of reality, to take a realistic view about the world, to talk about it as if it is really there even when it is not observed. However, one cannot avoid raising the problem of including also conscious observers into the picture, for *1: what is interesting if not experienced?* Thus one is led to consider the problem of the psycho-physical parallelism within the considered theory.

The previous analysis should have already given clear indications about the lines to be followed to reach this goal. In this Section we will first of all make clear how to relate the “different states of the universe” as characterized by the formalism according to the lines of the previous sections to the specificity of conscious perceptions. Secondly we will show how the theory itself supports the proposed correspondence. To get this we will perform a very sketchy analysis, from the point of view of CSL, of the current ideas about the physical processes leading to perceptions.

4.1 External World and Internal Perceptions.

As we have seen the CSL dynamics, at the macroscopic level, makes objectively possessed the values taken by the mass density function, and consequently suggests to give to this function the status of the basic element accounting for

reality, as we perceive it. According to this picture, “what is out there” is identified with a precise and accessible mass distribution in real space. The reality of a massive object in front of us corresponds to the fact that in the region it occupies the value of the mass density which characterizes it is an objectively possessed property.

On the other hand it is a fundamental feature of our perceptions that they correspond to objects having precise locations and extensions. The problem of establishing a map between reality and perceptions is then naturally solved by correlating our perceptions to mass density distributions⁶.

After these simple remarks we can pass to discuss how one can account, within CSL, for the emergence of perceptions.

4.2 Describing the Perceptive Process.

To make clear how CSL is able to “describe” the occurrence of definite perceptions of conscious beings, it turns out to be quite appropriate to start by discussing a criticism to this theory which has been put forward in some recent papers [15]. The idea is quite simple but raises a problem which deserves a detailed investigation. One considers the following process: a neutral microsystem with spin is sent through a Stern-Gerlach apparatus. The spin state is such that the system ends up in the superposition of being deflected, with equal probabilities, upwards or downwards, respectively. The two “potential trajectories” cross a fluorescent screen in two macroscopically far apart regions A and B, respectively. The particle-screen interaction is such as to lead to the excitation of a small number, e.g. of about ten atoms, which subsequently undergo a transition to the ground state accompanied by the emission of photons.

The argument of ref. [15] goes as follows. Since only few atoms of the screen are excited during the process, since their excitations involve displacements which are much smaller than the characteristic localization length of the model, since photons are not localized, there is no way for the CSL mechanism to yield the suppression of one of the two superposed states. Thus, the superposition of states corresponding to ten photons emerging from the different space regions A and B will persist for extremely long times. On the other hand, since the visual perception threshold is quite low (about 7 photons) there is no doubt that the naked eye of a human observer is sufficient to detect whether the luminous spot on the screen is at A or at B. This raises an interesting question: in the situation considered how can it happen that a definite perception about the location of the spot on the screen emerges? Are we compelled to accept that, at least in some circumstances, also within CSL the conscious observer and its perceptions play a (peculiar) role analogous to the one attached to them in Wigner’s views?

⁶Obviously our perceptions are much richer than those (corresponding to position and shape) we have listed here. In the next Subsection we will make clear how also more complex perceptions (such as colour etc.) can be naturally included in the picture we are presenting.

That this is not the case has been discussed in great detail in a recent paper [16]. If one takes, as one must, the above remark seriously one is compelled to consider the actual systems which enter into play and to analyse the implications of the CSL dynamics for them. We simply sketch here the argument of ref. [16]. One takes into account what we know about the transmission of nervous signals from, let us say, the retina and the higher visual cortex. Such a transmission requires, among other changes, the displacement of ions along the axons involved in the process. A very rough estimate of the mass associated to these ions and of the displacements they have to perform to flow through the ion channels which open at Ranvier's nodes to transmit the electric pulse can be given. This estimate makes perfectly plausible that the conditions which are sufficient, according to CSL, for the suppression of one of the two states (nervous signals) of the superposition within the visual perception time (which is of the order of 10^{-2} secs), are satisfied⁷.

We do not want to be misunderstood; this analysis does by no means amount to attribute a special role to the conscious observer or to his perception. The observer's brain is simply the only system among those which are present in which a superposition of two states involving different locations of a large number of particles occurs. As such it is the only place where the reduction can and actually must take place according to the theory. If, in place of the eye of a human observer, one puts in front of the photon beams a spark chamber or any device leading to the displacement of a macroscopic pointer or producing ink spots on a computer output, reduction will take place. In the considered example, the human nervous system is simply a physical system, a specific assembly of particles, which makes the same function as any one of these devices, if no such device is interacting with the photons before the human being does⁸.

This example should be sufficient by itself to clarify our problem. Our perceptions are triggered by our sensory apparatuses. In many cases, such as for the hearing or tactual perceptions, the stimulus itself cannot be ambiguous, for it cannot correspond to superpositions of different perceptions, since it requires

⁷The reader will now easily understand the appropriateness of the choice we have made in section 3 for the parameter ϵ , marking the transition between similar and different macroscopic situations. According to CSL the suppression of one of the two states in a superposition is governed by the expression $\exp[-(\lambda/2)(n - n')^2]$, n and n' representing the nucleon occupation number of the "elementary cells" of volume 10^{-15}cm^3 in the two states appearing in the superposition. In the worst case $n - n'$ equals one for the only cells which have different occupation numbers, so that, taking into account that $\lambda = 10^{-16}\text{sec}^{-1}$ and that the typical perception times are of the order of 10^{-3} - 10^{-2} sec, one sees that within such times superpositions of "macroscopically distinguishable" situations are dynamically suppressed.

⁸We consider appropriate a specification. The above analysis could be taken as indicating that we adopt a very naive and oversimplified attitude about the deep problem of the brain-mind correspondence. We do not claim and we do not pretend that CSL yields a physicalist explanation of consciousness. We simply point out that, for what we know about the purely physical aspects of the perceptual process, the conditions guaranteeing that superpositions of different perceptions cannot occur are satisfied.

macroscopic displacements (of the hearing membrane or of the skin). In other cases, as the one discussed above, the nervous signal can be triggered by a microscopic system, which can very well be in a superposition of states capable to induce different perceptions. But in all cases the nervous transmission of the signal involves a "macroscopic" (on the appropriate scale) displacement of mass in the brain. And, as discussed above, the CSL dynamics suppresses in a time of the order of the perception time superpositions characterized by different (in the precise mathematical sense we have given to this term in subsection 3.6) macroscopic mass distributions. In a sense the analysis shows that the problem of the psycho-physical correspondence admits a simple solution quite similar to the one which is usually assumed to hold for classical theories. Reality and perceptions involve the same fundamental quantity: the values of the mass density function. On one side "macroscopic reality" is, quite in general, related to accessible mass density distributions. On the other, definite perceptions are related to accessible macroscopic mass density distributions within the brain.

We do not want to spend further time in elaborating on this point. Our aim here is not that of performing a detailed technical analysis of any conceivable situation but simply of making plausible that the problem of the psycho-physical parallelism admits, within CSL, a solution which is quite analogous to the one of the unproblematic classical case. Thus CSL can be claimed, according to J.S. Bell's definition [1], to be an "exact theory" in the precise and limited sense that *it neither needs nor is embarrassed by an observer*.

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