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Decay And Spectra Of Baryons Especially Beauty Baryons

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Abstract

Masses and decays of the baryons are considered. The entire spectroscopy of baryons containing u,d,s,c and b quarks is calculated using the five quark masses and only four additional parameters describing the potential between the baryons. This potential is taken to be a short-range Coulomb potential together with a long-range linear potential modified by a harmonic-oscillator potential. Decays are studied using the quark pair creation model of Le Yaouanc et. al. The pair strength γ is replaced by k' . This and the meson radius are the only parameters used in the calculation of the decays. Overall, we have a useful model, employing a small number of parameters, yet capable of yielding a description of the baryons in good accord with experimental data.

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1. INTRODUCTION

High-energy experiments reveal a richness in the spectrum of the strongly interacting particles. The vast amount of the of the experimental data in baryon spectrum and the complicated three-body problem would by itself justify the study of the spectroscopy. One major question of interest is the exact nature of the inter quark potential. Since baryon spectroscopy samples a different region of this potential than meson spectroscopy, it is of some interest to compare fits to both sets of data. Of even more interest would be an experiment starting with quarks placed fairly far apart and than moving them closer together. Let's see how we can accomplish this task. First we start with non-relativistic-quantum-mechanics. It is well established [1] that this will give a good account of spectroscopy. Extra relativistic energies simply accumulate in the mass of the *constituent* quarks which are situated at fixed distances from each other. The distance is varied by changing the quark flavour. Changing from light u, d and s quarks to the c and b quarks yields a wealth of information about the interquark potential.

Although most of the works on the baryon spectrum have accounted for the most important properties of baryons such as their masses, spin, etc., there is a serious drawback in comparing the spectrum with the experimental data without studying their decays. For most of the resonances, it is difficult to assign predicted baryons to experimentally known particles. This difficulty is compounded by the many resonances

given by the model which have thus far not been observed. In order to be useful in accounting for the spectroscopy, a decay model must have the following properties:

- i) It should be consistent with the model of the mass calculation. A change of some feature of the mass calculation in the decay model may compromise the purpose.
- ii) The result of the decay calculation should be mainly determined by the mass calculation. If there are too many parameters in the decay model, it is difficult to relate the results obtained to the spectroscopy described by the mass calculation.

Generally there are two types of strong-decay models: the meson emission model, which treats the emitted meson as a point like model and the pair creation model which takes into account the quark components and the size of the meson. The main advantage of the former model is its simplicity. One may use relativistic kinematics for the meson, while considering the quark components of the meson restricted to a nonrelativistic approximation. In this paper the quark pair creation model of Le Yaouanc et. al. is utilized. It seems to be the natural choice to relate to the spectroscopy developed using a mass calculation based on a nonrelativistic quark model. Moreover the quark pair creation model of Le Yaouanc et. al. takes into account the quark components and the size of the particles involved, which is more realistic than the meson emission model. On the other hand, this model is still a simple model, lacking a fully developed theoretical background. One should keep this in mind during the process of studying the decays.

2. THE MASS CALCULATION

The model for this calculation by Kalman and Tran [2], based on non-relativistic-quantum-mechanics, uses fewer parameters than any other model; five parameters comprising the masses of the five u,d,s,c and b quarks and three parameters corresponding to the inter quark potential. At small distances, QCD suggests a Coulomb-type potential $-\alpha_s/r$, where α_s is the quark-gluon fine-structure constant. At large distances one expects a confining potential. Although the exact form is unknown, lattice gauge theory [3] and string models [4] lead one to expect a linear confining potential. Such a combination was shown long ago to be yield a good fit to meson mass spectra. [5] For baryons it has been suggested that the potential might not be in the form of a direct quark-quark interaction, but instead in the form of a branched string emanating from some central point in between the three quarks. It is probably not necessary to give special consideration to such models since it has been shown in an appropriate treatment of lattice gauge theory [6] that a sum of linear two-body forces has the same effect as such a three-body force. Gromes and Stamatescu [7] made a general study of baryons using a harmonic-oscillator basis by writing:

$$\begin{aligned} V'(r_{ij}) &= \frac{1}{2} \mu \omega^2 r_{ij}^2 + [V(r_{ij}) - \frac{1}{2} \mu \omega^2 b r_{ij}] \\ &= V_0(r_{ij}) + U(r_{ij}) \\ V(r_{ij}) &= -C_F \alpha_s / r_{ij} + a r_{ij} + C \end{aligned} \quad (1)$$

where μ is the reduced mass, r_{ij} is the separation between quark i and quark j , C_F is the colour factor of baryons taken to be $2/3$ as suggested by QCD, a is a constant describing the string strength and C is an overall constant. (In Gromes and Stamatescu $b=1$). $U(r_{ij})$ is treated perturbatively. Such a treatment takes advantage of the fact that the harmonic-oscillator potential is the only potential that can be solved exactly for any number of particles. [8] Indeed it is a standard method in theoretical chemistry to use Gaussian wave functions to calculate molecular energies. [9,10,11] Isgur and Karl [12] used an unspecified $U(r_{ij})$ in a highly successful calculation of the excited positive-parity baryons containing u,d and s quarks. In this model different parameter sets have to be applied to analyze the positive- and

negative- parity baryons. Kalman [13] in a similar approach was able to describe a broad spectrum of baryons containing c and b quarks in addition to u,d and s quarks.

In this paper, the combination of Coulomb and linear potential given by eq. (1) is modified by a harmonic-oscillator potential. As seen in fig. 1 this combination at short and medium range is similar to the combination of a Coulomb and a logarithmic potential used by Gromes and Stamatescu. [7] It should be noted that as Gromes and Stamatescu pointed out that the logarithmic potential only describes the interaction at medium energy and the potential may have a different form at long range. Furthermore the energy of the states (mass of the baryons) corresponds only to the short- and medium-range behaviour of the chosen potential. Thus any modifications of the potential at long-range needed to produce complete confinement of the quarks is inconsequential to the mass calculation.

Fig. 1. The graphs of the potentials:
1) $V(r) = -0.667/r + 0.08r - 0.004r^2$ and
2) $V(r) = -0.667/r + 0.191 \ln r$

In this work a single set of parameters is used for all baryon states. The mixing between the ground-state and first-excited positive parity baryons as well as the mixing between the first-excited and second-excited -negative parity baryons are taken into account. The importance of this interband mixing was shown by Isgur and Karl. [14]

Until now no true relativistic potential model exists. The approach taken by many model builders is to adopt relativistic kinematics. [15,16] A study of the unequal-mass quarkonium spectra by Kalman and D'Souza [17] using only a few undetermined parameters and a nonrelativistic approach

yields results which are extremely close to experimental data and which are similar to those obtained by Godfrey and Isgur [18] in a model based on relativistic kinematics. A better attempt to incorporate relativity to calculate the spectra of baryons containing u,d or s quarks is by Dong et al. [19] Their potential is derived from the exact three dimensional relativistic equation. In view of the earlier remarks found in this paper, it is interesting to note that Dong et al. end up incorporating a logarithmic term coming from the retardation expression. Dong et al. use their final section, discussions and conclusions to compare their results to the mass calculation of Kalman and Tran. [2] They point out that they have a somewhat better agreement with experiment than Kalman and Tran. Such a comparison is however difficult as Kalman and Tran use only 7 parameters to cover this part of the spectra of baryons containing u,d or s quarks compared to the 12 parameters used by Dong et al. Their slightly better agreement might be due to this feature alone.

The full Hamiltonian employed by Kalman and Tran has the form;

$$H = H_0 + \sum_{i < j} U(r_{ij}) + H_{hyp}^{ij} \text{ where}$$

$$H_{hyp}^{ij} = \frac{2\alpha_s}{3m_i m_j} \left\{ \frac{8\pi}{3} \mathbf{S}_i \cdot \mathbf{S}_j \delta^3(r_{ij}) \right. \quad (2)$$

$$\left. + \frac{1}{r_{ij}} \left[\frac{3(\mathbf{S}_i \cdot \mathbf{r}_{ij})(\mathbf{S}_j \cdot \mathbf{r}_{ij})}{r_{ij}^2} - \mathbf{S}_i \cdot \mathbf{S}_j \right] \right\},$$

H_0 is the harmonic-oscillator potential and $U(r_{ij})$ is found in eq. (1). As with the U term, the hyperfine interaction is treated perturbatively. In accordance with the previous remark, the mixings by the U term and the hyperfine interaction within each band and between different bands are also taken into account. The fit to the baryon spectra yields $a=0.08 \text{ GeV}^2$, $b=1.68$, $C=-0.6 \text{ GeV}$, $\alpha_s=1.0$, $m_u=m_d=0.23 \text{ GeV}$, $x_1=m_u/m_s=0.38$, $x_2=m_u/m_c=0.1237$, $x_3=m_u/m_b=0.0455$.

3. THE QUARK PAIR CREATION MODEL OF LE YAOUANC et. al. [20]

The decay $A \rightarrow B + M$, where A and B are baryons and M is a meson is shown in fig. 2. During the process a quark-antiquark pair is created and combine with the original baryon B and the meson M. The initial quarks remain unaffected by the process.

Fig. 2. - Diagram of the decay $A \rightarrow B + M$

The $q\bar{q}$ pair has the quantum numbers of the vacuum. It is a colour singlet and flavour singlet and has zero momentum and zero total angular momentum. The pair must also be of positive parity for parity conservation. Finally a fermion-antifermion pair has $P = (-1)^{l+1}$, $C = (-1)^{l+s}$, so the pair must

be in a 3P_0 state. In the QPC model by Le Yaouanc et al. the pair creation is the same for all processes. In Kalman and Tran [20], the creation strength γ is replaced by k_M^γ to include other effects as discussed in Kalman and Tran. To have the correct unit for the decay widths, it should be understood that there is a constant $a = 1 \text{ GeV}^{-1}$ attached to k_M . Since each state in the spectroscopy is the result of the mixing of several states, the decay amplitude from $|A\rangle$ to $|B\rangle$ is the sum of all individual amplitudes between the components of $|A\rangle$ and $|B\rangle$. The mixings of the states are given in Kalman and Tran [20] and the exact form of the wave functions are found in Kalman and Tran. [2]

4. DISCUSSION OF RESULTS AND CONCLUSIONS.

Of the two major analyses, that of Koniuk and Isgur [21] does not cover the $N = 3$ band baryons, and the one by Forsyth and Cutkosky [22]. does not cover the Λ and Σ sectors. The $N = 3$ band in the Λ and Σ sectors has not been considered before Kalman and Tran [20]. the results of Kalman and Tran [20] show a generally good agreement with the experimental data. A large number of resonances are essentially decoupled from the studied channels, a result first noted by Koniuk and Isgur [21]. Koniuk and Isgur have suppressed the dependence of the amplitudes on the structure of the wave functions, which has an important affect on the decay amplitudes, and replaced two constants in the forward and recoil terms by four independent constants. Similarly Forsyth

and Cutkosky allow these two constants to be different in different spin and orbital states of the $q\bar{q}$ pair. In effect they used eight parameters in addition to the baryon and meson radii. In this work, beside the meson radius, γ and a are the only parameters of the decay model. (The parameter " a " is the parameter found in eq. (1) and its value is determined by the fit to the mass spectra.) The main purpose of this work is to establish the baryon spectrum, namely identifying the resonances with the experimental data. We are confident that with the results obtained, this goal is indeed achieved despite the fact that only the nonrelativistic quark model and the simple QPC model are used. We believe this is the most consistent quark model to-date describing the baryon spectroscopy in the sense that a single set of parameters is used for the whole spectra and then subjecting all of their properties namely the masses, radii and their wavefunction structures to a strict test by a decay model which has only two parameters. Finally the point should be stressed that the model here uses far less parameters than used by any other model. Not counting the masses of the c and b quarks, the model here uses six parameters as compared to thirteen in the relativized quark model. Forsyth's model only considers the nonstrange sector and employs nine parameters compared to five employed to calculate the nonstrange sector in this model. Finally note that this model uses only 7 parameters to cover this part of the spectra of baryons containing u, d or s quarks compared to the 12 parameters used by Dong et al. [19] For this conference I am presenting a new version of a portion of the tables found in Kalman and Tran [2] for baryons containing a single b quark. This new version is based on taking the mass of the ground state of Λ_b to be 5640 MeV based on the observed value of 5641 ± 50 MeV. We then note:

$$\begin{aligned} M(\Sigma_b^*) - M(\Lambda_b) &= 256 \text{ MeV This calculation} \\ &= 253 \pm 20 \text{ MeV Experiment} \\ &= 230 \pm 20 \text{ MeV Romagliu \& Lichtenberg[24]} \end{aligned}$$

$$\begin{aligned} M(\Sigma_b^*) - M(\Lambda_b) &= 250 \text{ MeV This calculation} \\ &= 200 \pm 20 \text{ MeV Experiment} \\ &= 197 \pm 20 \text{ MeV Romagliu \& Lichtenberg[24]} \end{aligned}$$

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TABLE I. - *The spectrum of Λ_b .*

State	Calculation
$^2D_{,,}$ $3^{+}/2$	6489
$^2D_{\lambda\lambda}$ $3^{+}/2$	6533
$^4S_{,,}$ $3^{+}/2$	6632
$^2D_{,\lambda}$ $3^{+}/2$	6707
$^4D_{,\lambda}$ $3^{+}/2$	6716
$^2P_{,\lambda}$ $3^{+}/2$	6792
$^4P_{,\lambda}$ $3^{+}/2$	6796
2S $1^{+}/2$	5640
$^2S_{\lambda\lambda}$ $1^{+}/2$	6100
$^2S_{,,}$ $1^{+}/2$	6347
$^2S_{,\lambda}$ $1^{+}/2$	6618
$^4S_{,\lambda}$ $1^{+}/2$	6694
$^4D_{,\lambda}$ $1^{+}/2$	6789
$^2P_{,\lambda}$ $1^{+}/2$	6819
$^4P_{,\lambda}$ $1^{+}/2$	
$^2P_{\lambda}$ $1^{-}/2$	6032
$^4P_{\lambda}$ $1^{-}/2$	6144
$^2P_{,\lambda}$ $1^{-}/2$	6227
$^2P'_{,\lambda\lambda}$ $1^{-}/2$	6387
$^4P'_{,\lambda\lambda}$ $1^{-}/2$	6474
$^2P_{\lambda\lambda\lambda}$ $1^{-}/2$	6545
$^4P_{,\lambda\lambda}$ $1^{-}/2$	6612
$^2P_{,\lambda\lambda}$ $1^{-}/2$	6744
$^2P_{,\lambda\lambda}$ $1^{-}/2$	6909
$^2P'_{,\lambda\lambda}$ $1^{-}/2$	7051
$^4D_{,\lambda\lambda}$ $1^{-}/2$	7267
$^2P_{,\lambda\lambda}$ $1^{-}/2$	7300
$^4P_{,\lambda\lambda}$ $1^{-}/2$	7438
$^2P_{,\lambda\lambda}$ $1^{-}/2$	

TABLE II. - *The spectrum of Σ_b .*

State	Calculation
4S $3^{+}/2$	
$^4S_{\lambda\lambda}$ $3^{+}/2$	5905
$^2D_{,,}$ $3^{+}/2$	6332
$^4D_{,,}$ $3^{+}/2$	6485
$^4S_{,,}$ $3^{+}/2$	6503
$^4S_{,,}$ $3^{+}/2$	6565
$^2D_{,\lambda}$ $3^{+}/2$	6721
$^2D_{\lambda\lambda}$ $3^{+}/2$	6726
$^4D_{\lambda\lambda}$ $3^{+}/2$	6729
$^2P_{,\lambda}$ $3^{+}/2$	6786
2S $1^{+}/2$	
$^2S_{\lambda\lambda}$ $1^{+}/2$	5890
$^4D_{,,}$ $1^{+}/2$	6303
$^4D_{,,}$ $1^{+}/2$	6485
$^2S_{,,}$ $1^{+}/2$	6557
$^2S_{,\lambda}$ $1^{+}/2$	6641
$^4D_{\lambda\lambda}$ $1^{+}/2$	6724
$^2P_{,\lambda}$ $1^{+}/2$	6786
$^2P_{\lambda}$ $1^{-}/2$	
$^2P_{\lambda}$ $1^{-}/2$	6185
$^2P_{\lambda}$ $1^{-}/2$	6339
$^4P_{\lambda}$ $1^{-}/2$	6372
$^2P'_{,\lambda\lambda}$ $1^{-}/2$	6429
$^2P_{\lambda\lambda\lambda}$ $1^{-}/2$	6634
$^4P_{\lambda\lambda\lambda}$ $1^{-}/2$	6690
$^4P_{,\lambda\lambda}$ $1^{-}/2$	6718
$^2P_{,\lambda\lambda}$ $1^{-}/2$	7057
$^4D_{,\lambda\lambda}$ $1^{-}/2$	7196
$^2P'_{,\lambda\lambda}$ $1^{-}/2$	7213
$^4P_{,\lambda\lambda}$ $1^{-}/2$	7254
$^2P_{,\lambda\lambda}$ $1^{-}/2$	7296
$^4P'_{,\lambda\lambda}$ $1^{-}/2$	7257
$^2P_{,\lambda\lambda}$ $1^{-}/2$	