MODULI INFLATION WITH LARGE SCALE STRUCTURE PRODUCED BY TOPOLOGICAL DEFECTS

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ABSTRACT

It is tempting to inflate along one of the many flat directions that arise in supersymmetric theories. The required flatness of the potential to obtain sufficient inflation and to not overproduce density fluctuations occurs naturally. However, the density perturbations (in the case of a single moduli field) that arise from inflaton quantum fluctuations are too small for structure formation. Here we propose that topological defects (such as cosmic strings), which arise during a phase transition near the end of moduli inflation can provide an alternative source of structure. The strings produced will be ‘fat’, yet have the usual evolution by the time of nucleosynthesis. Possible models are discussed.
I. Introduction

The inflationary universe model was proposed [1] to solve several cosmological puzzles, namely the horizon, flatness, and monopole problems. During the inflationary epoch, the energy density of the universe is dominated by the vacuum energy, $\rho \simeq \rho_{\text{vac}}$, and the scale factor of the universe expands superluminally. In many models this expansion is exponential, $R(t) \propto e^{Ht}$, where the Hubble parameter $H = \dot{R}/R \simeq (8\pi\rho_{\text{vac}}/(3m_{\text{pl}}^2))^{1/2}$ during inflation and $m_{\text{pl}} \sim 10^{19}\text{GeV}$ is the Planck mass. If the interval of exponential expansion satisfies $\Delta t \gtrsim 60H^{-1}$, a small causally connected region of the universe grows sufficiently large to explain the observed homogeneity and isotropy of the universe. In addition, any overdensity of magnetic monopoles is diluted.

To satisfy a combination of constraints on inflationary models [2], in particular, sufficient inflation and microwave background anisotropy limits [3] on density fluctuations, the potential of the field responsible for inflation (the inflaton) must be very flat. It was shown in [7] that, for a general class of inflation models involving a slowly-rolling field (including new [4], chaotic [5], and double field [6] inflation), any potential satisfying these two constraints together with the condition of overdamping must also obey the following condition

$$\chi \equiv \Delta V/(\Delta \psi)^4 \leq O(10^{-6} - 10^{-8}).$$

Here $\chi$ is the ratio of the height to the width of the potential, i.e., $\Delta V$ is the change in the potential $V(\psi)$ and $\Delta \psi$ is the change in the inflaton field $\psi$ during the slowly rolling portion of the inflationary epoch. Thus, the couplings in the inflationary potential must be small; for example, if the inflationary potential is quartic, then the quartic coupling constant must satisfy $\lambda < O(\chi)$.

Introducing arbitrary small couplings at tree level in the inflationary potential is unnatural because a fine-tuning must be performed to cancel large radiative corrections. This procedure would simply replace a cosmological naturalness problem with unnatural particle physics. Instead, there are two different attitudes one can take to explain this required small number. One can simply resort to future physics: we know that there is a hierarchy problem (e.g., the mass of the electroweak Higgs is much smaller than the grand unified scale), and hopefully expect that whatever solves the hierarchy problem might someday explain the small ratio of scales required for inflation. Alternatively, one can look for small numbers in particle physics today. One possibility, that has been explored in the Natural Inflation model [8], is to identify the inflaton as a Nambu-Goldstone boson. Another possibility is to take advantage of supersymmetry and invoke the ‘technical naturalness’ argument, where small numbers once postulated at tree level in the superpotential, are protected by supersymmetry from receiving large radiative corrections [9].
Recently, there has been interest in trying to inflate along (nearly) flat directions in supersymmetric field theories [10-12]. Flat directions are directions in field space where the classical scalar potential exactly vanishes and are parametrised by complex scalar fields, referred to as moduli fields, \( \psi \). In the supersymmetric limit the potential along these flat directions vanishes identically (neglecting nonrenormalisable terms), i.e., \( V(\psi) = 0 \). However soft supersymmetry breaking terms will lift the scalar potential by an amount \( V(\psi) = m_W^2 |\psi|^2 \), where \( m_W \) must be of order the electroweak scale to solve the hierarchy problem associated with the electroweak Higgs mass (all numerical values in the paper are obtained with \( m_W \sim 1 \) TeV). In the inflationary context this potential is still very flat because \( m_W \ll m_{pl} \), where typically \( \Delta \psi \sim \mathcal{O}(m_{pl}) \) in the early universe. Thus the constraint in Eq. (1) on the ratio \( \chi \) is easily satisfied.

We will consider an inflationary epoch where the inflaton is identified with a moduli field, \( \psi \), and the inflationary potential is given by the soft-supersymmetry breaking term \( V(\psi) = m_W^2 |\psi|^2 \). The moduli field has an initial value \( \psi_0 \sim 4 - 5 m_{pl} \) (as in chaotic inflation [5]) and the universe inflates as the field \( \psi \) rolls down the potential. Moduli inflation using soft terms was previously discussed in Refs [11,12]. An interesting consequence of moduli inflation, pointed out by Randall and Thomas [12], is that one can avoid the ‘cosmological moduli’ problem [13]. Normally, weakly coupled scalar fields with masses \( m \ll H \) and initial values of \( \mathcal{O}(m_{pl}) \) that are displaced far from their minima either overclose the universe, or decay so late that they destroy the predictions of nucleosynthesis. This problem is resolved by a period of moduli inflation because typical scalar masses \( m \sim m_W \sim H \) and the offending scalar fields are quickly driven to their minima. Possible caveats to this solution have been addressed by [14] (e.g., there may still be a residual moduli problem if the potential minima do not coincide before and after inflation), but scenarios exist where this approach could work.

However, a problem that arises during inflation with a single moduli field is that the magnitude of the density perturbations produced is too small. This can easily be seen by considering the equation of motion for the scalar field during inflation,

\[
|\ddot{\psi}| + 3H |\dot{\psi}| = -\frac{dV}{d|\psi|}.
\]

In the overdamped approximation known as ‘slowly rolling’ one may neglect the acceleration term (\(|\ddot{\psi}|\)) during inflation. In general the density fluctuations scale with the height of the potential and for a model of inflation driven by the potential \( V = m_W^2 |\psi|^2 \), we obtain

\[
\frac{\delta \rho}{\rho} \sim \frac{1}{10} \frac{H^2}{|\dot{\psi}|} \sim \frac{m_W^2}{m_{pl}^3} |\psi|^2.
\]

In the early universe, a typical value for the scalar field is \( \psi \sim m_{pl} \), and so the density
fluctuations produced are roughly

\[ \frac{\delta \rho}{\rho} \sim \frac{m_W}{m_{pl}} \sim 10^{-16}. \]  

This value is too small to explain the observed large scale structure. Recent COBE measurements of microwave background anisotropies obtain a value [3]

\[ \frac{\delta \rho}{\rho} \big|_{obs} = \text{few} \times 10^{-5}. \]  

This general problem of producing large enough density perturbations for moduli inflation occurs because the known scales in particle physics do not coincide with the scale needed for density perturbations. In fact, in a recent moduli inflation model by Thomas [10], a dynamical supersymmetry breaking scale is introduced at \( \Lambda \sim 10^{16}\text{GeV} \) solely for the purpose of producing the correct density perturbations. Unfortunately, supersymmetry breaking at \( 10^{16}\text{GeV} \) has no relevance for the physical particle spectrum and supersymmetry needs to be restored at the end of inflation. If we do use relevant soft terms for the inflationary potential, then the density fluctuations are too small. This is because the height of the potential is too small. In Ref. [12], the lack of sizeable density perturbations is avoided by assuming that moduli inflation is preceded by an earlier inflationary epoch that produces the correct magnitude of density perturbations. In order not to wipe out these density perturbations the subsequent moduli inflationary period can only last for at most 30 e-folds. In recent work Randall and Guth [21] have been working on coupling two scalar fields (with a potential we describe in section IIC) to obtain a hybrid inflation model [26,27] with adequate density fluctuations.

Here we propose, instead, that the density fluctuations responsible for the formation of large scale structure are produced from cosmic topological defects such as cosmic strings [15]. Near the end of the \( \psi \) moduli field driven inflation (or after inflation), a phase transition is induced in another complex scalar field \( \phi \), which creates cosmic defects. The term in the Lagrangian that drives the phase transition is of the form \( H^2 |\phi|^2 \); such a term is necessarily always present in the early universe. Cosmic strings arise when a \( \text{U}(1) \) symmetry is spontaneously broken, which occurs when the mass squared \( (m_\phi^2) \) term changes sign. In general, the radius of the string core is given by [16]

\[ \delta_\phi \sim m_\phi^{-1}. \]  

As we will show, the cosmic strings produced in this model are fatter than usual by a factor of \( 10^{10} - 10^{13} \). However, it turns out that by the time the strings play any role in physics that might be observable, such as during nucleosynthesis or at recombination, the universe is sufficiently large that the thickness of the strings is again negligible. The size
of the fat strings is roughly $10^{-20} - 10^{-17}$ cm, while the horizon size at nucleosynthesis is $\sim 10^{10}$ cm. Thus, the strings behave as usual for any observables (and for the formation of cosmic structure).

The only parameter that enters into the formation of cosmic structure is the mass per unit length of the string $\mu$, which must have a value $\mu \sim 10^{-6} m_{\text{pl}}^2$. There are two types of cosmic strings possible depending on whether the U(1) symmetry is local or global. For local strings (e.g., for a cosmic string potential of the form $V_\phi = \lambda(\phi^3 - \eta^2)^2$) the string has an inner core with linear mass density [16]

$$\mu \sim \eta^2,$$  \hspace{1cm} (7)

where $\eta$ is the minimum of the cosmic string potential. Requiring $\mu \sim 10^{-6} m_{\text{pl}}^2$ then determines the minimum of the string field potential to be $\eta \sim 10^{16}$ GeV. In the case of global strings, one obtains instead

$$\mu \sim 2\pi \eta^2 \ln(R/\delta_\phi),$$  \hspace{1cm} (8)

where $R$ is a cutoff given either by the radius of the string loop or by the distance to the neighboring string. For global strings parametrised by (8), the location of the minimum is roughly $\eta \sim 10^{15}$ GeV for fat strings. Hereafter for simplicity, we will only consider examples of string potentials with a global U(1) symmetry. We will impose the condition that after the phase transition the string field sits at a minimum $\langle \phi \rangle \sim 10^{15}$ GeV, so that global cosmic strings can explain the observed density fluctuations.

We should comment that many authors have been working on a comparison of predictions from cosmic strings and textures with various observations, including the microwave background and the power spectrum for large scale structure. For example, Crittenden and Turok [25] have pointed out that textures will produce a Doppler peak in the microwave background at scale $l \sim 400$ (whereas inflation should produce a peak at $l \sim 200$.) Whether or not cosmic defects will prove to be in concordance with upcoming observations and will consequently provide the explanation for the origin of large scale structure is of course at present unclear.

Note that the idea of cosmic string production during or near an inflationary era is not new and has been considered by a number of authors [15]. Early work on this subject includes a paper by Shafi and Vilenkin [15] who showed that the spontaneous breaking of a global U(1) symmetry in minimal SU(5) grand unification can produce topologically stable strings at the end of an inflationary era. Various scenarios for coupling the string field to the inflaton such as via a direct coupling of the two fields or via the spacetime curvature scalar have also been considered [15]. However, in this previous work the formation of topological defects was considered in the context of inflation with a Hubble
constant $H \gg m_W$. In the present work we are considering topological defects in the interesting context of moduli inflation where $H \sim m_W$.

The plan for the rest of the paper is as follows: In Section II we consider a model of moduli inflation in which the large scale structure is formed by cosmic defects. We then discuss the various constraints any such model must satisfy, and illustrate the resulting requirements for parameters in the model. We will present three different examples of the cosmic string potential and comment on the better motivated scenarios. Further discussion and our conclusion will be given in Section III.

II. Models of moduli inflation with cosmic strings

Consider two complex scalar fields $\psi$ and $\phi$. We assume that the field responsible for inflation is a moduli field, $\psi$, which has a soft supersymmetry-breaking potential. The second field, $\phi$, undergoes a phase transition near the end of inflation and gives rise to cosmic defects; for definiteness, we will take cosmic strings as an example. The potential for these two fields is assumed to have the form

$$V = m_W^2 |\psi|^2 + cH^2 |\phi|^2 + V_\phi.$$  \hspace{1cm} (9)

The last term, $V_\phi$, is the potential for the cosmic string field and is responsible for producing the symmetry breaking minima. The second term is always present in the early universe for any scalar field and arises from considering the full scalar potential of N=1 supergravity. This contribution to the $\phi$ scalar field mass may in general be of either sign. For example, as discussed by \cite{17} a negative contribution will arise from the Kahler potential term $\delta K \sim (1/m_{pl}^2) \psi^\dagger \psi \phi^\dagger \phi$. However, we will assume that the value of the coupling $c$ is positive and of order one ($c=3$ for a minimal Kahler potential). Note that a similar term, $H^2 \psi^2$ arises for the inflaton field \cite{27,28}, but since $H \sim m_W$ for moduli inflation as noted in the introduction, this term is comparable to the soft-breaking terms already present in Eq.(9).

Note that one could consider an additional interaction term in the Lagrangian $g^2 |\phi|^2 |\psi|^2$, which would contribute an effective mass term for $\phi$. This would typically dominate over the $H^2 |\phi|^2$ term, and become responsible for the symmetry breaking of the string field. The details of the string production in this scenario depend on the values of the parameters and requires a more thorough investigation. It is also possible that if $g \sim O(1)$, thermal effects during the reheating stage of the universe generate $T^2 \phi^2$ terms which will trap the string field at the origin leading to a thermal inflation phase \cite{18}. In this case the cosmic string production occurs after the universe cools to a temperature $T \sim m_\phi$. Cosmic strings could then form quite late, e.g., at the electroweak scale. However this thermal effect can be avoided if the coupling, $g$ is too weak to allow thermalisation. For the remainder of this paper we do not consider this interaction term further.
The basic evolution of both fields is as follows: The inflaton field $\psi$ starts out at a value $\geq m_{pl}$ and is assumed to dominate the energy density of universe. An inflationary epoch commences as $\psi$ rolls down towards its minimum at the origin. Since the vacuum energy during inflation $\rho_V \sim m_W^2 m_{pl}^2$, the Hubble constant $H = [8\pi\rho_V/(3m_{pl}^2)]^{1/2} \sim m_W$. The mass of the string field $\phi$ is assumed to be dominated by the contribution $H^2|\phi|^2$. During inflation this field will be quickly driven towards the origin. As inflation proceeds, $H$ will slowly decrease and at some point near the end of inflation, negative mass squared terms in $V_\phi$ will begin to dominate. This causes a phase transition and $\phi$ falls towards its new minimum (assumed to be at $m_{GUT}$). Cosmic strings (or other defects) are created in the process and will then become responsible for the formation of structure. Note that the density fluctuations produced directly from the inflaton quantum fluctuations are too small to play any role. The solution to the cosmological moduli problem as well as reheating proceed in the same way as discussed in [12].

Now we present three different possible potential terms, $V_\phi$ for the string field and discuss the constraints on each possibility.

IIA. Consider first the scalar potential

$$V_\phi = \lambda(\phi^\dagger \phi - \eta^2)^2,$$

which is similar to a supersymmetric GUT Higgs potential. The radius of the resultant strings follows from Eq. (6) and is given by $\delta_\phi \sim m^{-1}_\phi \sim \lambda^{-1/2}\eta^{-1}$; the mass per unit length is $\mu \sim \eta^2$. As mentioned in the Introduction, requiring $\mu \sim 10^{-6} m_{pl}^2$ determines $\eta \sim 10^{15}$ GeV. The constraint on the model are as follows.

1. The energy density must be dominated by the inflaton field $\psi$. Thus the vacuum energy density of the string field must satisfy

$$\lambda \eta^4 < m_W^2 |\psi|^2$$

during the inflationary epoch. Since $\psi \sim m_{pl}$, this means that $\lambda < 10^{-16}$. Although such a small number may be ‘technically natural’, the potential (10) with an extremely small $\lambda$ lacks motivation.

2. There must be symmetry breaking of the U(1) associated with the string field $\phi$ in order to generate the strings. This happens when the mass squared term of the $\phi$ field changes sign, i.e., when $\lambda \eta^2 \sim H^2$. Since $H \sim m_W/m_{pl}|\psi|$ decreases during inflation, this criterion can be eventually reached. Strings can be produced any time after 50 e-folds before the end of inflation [15]; then the strings are not diluted too much by the subsequent inflation to be of relevance for structure formation. [Note that the $\lambda \eta^4$ term does not affect when the phase transition occurs.] Thus, the coupling must satisfy $\lambda \leq 10^{-24}$, where the upper bound corresponds to cosmic strings forming near the end.
of inflation. This value is even smaller than that required by the first constraint, and as discussed above, such a small number is unmotivated.

In the next two examples we study two scenarios with potentials qualitatively similar to Eq. (10) but not requiring extremely small parameters.

IIB. Here we follow Lyth and Stewart [18] and consider

\[ V_\phi = V_0 - m_W^2 |\phi|^2 + b \frac{|\phi|^{n+4}}{m_{pl}^n}, \]  

(12)

where \( n > 1 \) and \( b \) is a constant. The minimum of the potential (12) occurs at

\[ \langle \phi \rangle = \left[ \frac{2m_W^2 m_{pl}^n}{(n+4)b} \right]^{1/(n+2)}. \]  

(13)

In order to obtain \( \langle \phi \rangle \sim 10^{15} \) GeV, as required for cosmic string formation with the correct mass per unit length, we need \( n \approx 6 \), assuming \( b \) to be of order one. This requires all nonrenormalizable terms with \( n < 6 \) to be suppressed; otherwise the minimum will be too low in energy. This could be possible if one identifies \( \phi \) with a flat direction which is lifted by a dimension 4 superpotential term [19]. Alternatively the situation may not be quite as extreme if there is a reason to obtain \( b \ll 1 \). Then \( n \) need not be as large. This may, for example, happen in string theory if one imposes discrete symmetries which only allow specific couplings of the last term in Eq. (12) with remnant string fields, \( S \) [20] (note that \( S \) does not refer to cosmic string fields). For example, one may have \( b \sim \langle S \rangle^p / m_{pl}^p \) where \( p \) is some integer and \( \langle S \rangle / m_{pl} \sim 0.1 \) at the string scale. In this way one hopes to get a minimum for the potential at the GUT scale.

The constant term \( V_0 \) must be added to obtain the right value of the cosmological constant today, \( \Lambda \sim 0 \). Requiring \( V_\phi = 0 \) at the potential minimum \( \langle \phi \rangle \sim 10^{15} \) GeV gives \( V_0 \sim (10^9 \text{GeV})^4 \) (for \( n = 6 \)). The mass per unit length of the cosmic strings produced will then be \( \mu \sim 10^{-6} m_{pl}^2 \) as required. The thickness of the strings is \( \delta_\phi \sim m_W^{-1} \sim 10^{13} m_{GUT}^{-1} \) where \( m_{GUT} \sim 10^{16} \) GeV, i.e., \( 10^{13} \) times as large as usual. Indeed these are fat strings.

The required constraints for inflation followed by cosmic string production to work can be satisfied. Indeed the constraint that the inflaton potential dominate the energy density of the universe is satisfied: the vacuum energy of the string field \( V_0 \) is smaller than that of the inflaton, i.e., \( V_0 < m_W^2 m_{pl}^2 \sim (10^{11} \text{GeV})^4 \). The phase transition in the string field occurs when \( H^2 \sim m_W^2 \), i.e., when \( \psi \sim m_{pl} \). Thus, one can have moduli inflation with cosmic string production near the end of inflation, where both \( \psi \) and \( \phi \) can be identified with flat directions in a supersymmetric theory.

IIIC. The third possibility we consider is

\[ V_\phi = M^4 \cos^2 \frac{|\phi|}{f}, \]  

(14)

8
where $M$ is some as yet unspecified mass scale and the minimum of the potential must be at $f \sim 10^{15}$ GeV in order to obtain the correct $\mu$ for the cosmic strings. Unfortunately such a value for $f$ is not well-motivated. The same form of the potential is considered by Randall and Guth [21] in constructing a hybrid inflation model with moduli (they do not require the same value of $f$, however). For $|\phi| \ll f$, we can expand the string potential so that

$$V_\phi = M^4 - \frac{M^4}{f^2} |\phi|^2 + \frac{1}{3} \frac{M^4}{f^4} |\phi|^4.$$ \hspace{1cm} (15)

Then the constraints on the model are as follows:

1. The inflaton field $\psi$ must dominate the energy density. This means that $M^4 < m_W^2 |\psi|^2$. So, for $\psi \sim m_{pl}$ during inflation, we need $M \leq 10^{11}$ GeV.

2. Strings can form when $H^2 \sim \frac{M^4}{f^4}$, where $H \sim (m_W/m_{pl}) |\psi|$ during inflation. Since $f \sim 10^{15}$ GeV is fixed whereas $\psi$ continually decreases we obtain $M \leq 10^9$ GeV. Such intermediate mass scales responsible for dynamical supersymmetry breaking are possible.

The string parameters are similar to the previous cases. The mass per unit length of the cosmic strings is given by $\mu \sim (10^{16}$ GeV$)^2$, as required. The thickness of the cosmic strings is $\delta_\phi \sim m_{GUT}^{-1} \sim f/M^2 \sim 10^{13} m_{GUT}^{-1}$, i.e., $10^{13}$ times as large as the usual strings. Note that the coefficient in front of the $\phi^4$ term is $\frac{1}{3} \frac{M^4}{f^4} \sim 10^{-25}$, approximately the same value that was required in the example studied in Section IIA.

**III. Discussion and Conclusion**

Inflation using soft terms with a single moduli field by itself is unsatisfactory because inadequate density fluctuations are produced. We have proposed that cosmic defects may be formed at the end of an inflationary epoch and provide the large scale structure. We have focused on cosmic strings as an example. The cosmic strings that can be produced during moduli inflation are ‘fat’ compared to usual strings, with thickness ranging from $10^{10}$ to $10^{13}$ times the usual values. Thus the thickness ranges from $(10^{10} - 10^{13}) m_{GUT}^{-1} \sim (10^{-20} - 10^{-17})$ cm. However, the earliest observable effects from the strings would be produced at nucleosynthesis, by which time even these fat strings would be ‘thin’ relative to the horizon size, roughly $10^{10}$ cm. [At that time the production of gravitational waves by the strings might serve to constrain them very weakly]. Certainly the most likely observable effects would be produced subsequent to the time of recombination at $T \sim$ eV, by which time the initial fatness of the strings would be completely irrelevant. The horizon size at recombination is roughly $10^{20}$ cm. Thus these strings follow the usual evolution [22].

If the potential for the string field is minimized at $\sim 10^{16}$ GeV, then the required value of mass per unit length of the cosmic strings is obtained. We examined three different string field potentials: 1) $V_\phi = \lambda (\phi^4 - \eta^2)^2$ required $\lambda \sim 10^{-24}$, which is not
very well motivated; 2) \( V_\phi = V_0 - m_W^2 |\phi|^2 + b \frac{|\phi|^{n+4}}{m_{pl}^2} \) needed \( n \approx 6 \) for \( b \sim 1 \). Smaller values of \( b \) may be obtained from string theory by imposing discrete symmetries, which would allow more reasonable values of \( n \); 3) \( V_\phi = M^4 \cos^2 \frac{|\phi|}{f} \) required \( f \sim 10^{15} \) GeV, not a well-motivated value. While none of these potentials is perfect, we hope that the examples presented are illustrative.

We would also like to point out that there are other ways to produce cosmic strings during inflation. First, Basu, Guth and Vilenkin [23] have studied the production of cosmic defects that arise out of fluctuations of the vacuum during inflation. Second, Kofman, Linde, and Starobinsky [24] have proposed that cosmic defects may be able to arise due to parametric resonance giving rise to temperature effects that induce a phase transition during reheating after inflation. If either of these two mechanisms is active, these would be alternative ways to generate cosmic defects, and thereafter large scale structure, in a model of single moduli inflation.

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