Electroweak Precision Measurements and the Minimal Supersymmetric
Standard Model(*)

Youichi Yamada(**), Kaoru Hagiwara and Seiji Matsumoto

Theory Group, KEK, 1-1 Oho, Tsukuba, Ibaraki 305, Japan

Abstract

We discuss supersymmetric contributions to the electroweak precision measurements in the Minimal Supersymmetric Standard Model for two cases: the quark-lepton universality violation $\delta_{q\ell}$ in charged currents and the ratio $R_b = \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons})$. The recent experimental data suggest deviations from the Standard Model for these observables, at the 1-$\sigma$ level for the former and at more than 3-$\sigma$ level for the latter. We analyze the non-oblique corrections from the SUSY particles to explain these discrepancies. The observed non-zero $\delta_{q\ell}$ may be explained by relatively light sleptons, charginos and neutralinos. Although the observed excess of $R_b$ can be explained by very light scalar top and chargino for low $\tan\beta$, this interpretation is severely constrained by the opening of exotic decay modes for the top quark.


(**) Present Address; Physics Department, University of Wisconsin, Madison, WI 53706, USA
§1. Introduction

Among many possible extensions of the Standard Model (SM), the Minimal Supersymmetric (SUSY) Standard Model (MSSM)\(^1\) has attracted much interest because it stabilizes the hierarchy between the grand unification scale and the electroweak scale, and at the same time it gives grand unification of the three gauge couplings consistent with the precision measurements. However, to solve the hierarchy problem, the MSSM has to contain many new particles, SUSY particles, near the present energy frontier. It is therefore expected that properties of these new particles may be probed by the present precise measurements of the electroweak processes, both at low energies and at the \(Z\) boson pole.

In general, loop corrections of the SUSY particles to the electroweak four-fermi processes are classified into two categories: the corrections to the vector boson propagators (oblique corrections) and the vertex and box corrections (non-oblique corrections). The oblique corrections are universal for various processes and have been extensively studied. In contrast, the non-oblique corrections are highly process-dependent. One therefore has to specify the process for the detailed study of these corrections.

In this paper we discuss two examples of the precision measurements where the non-oblique SUSY corrections play a crucial role. First, we study violation of the quark-lepton universality in charged current interactions. Second, we study deviation of the ratio \(R_b = \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons})\) from the SM prediction, the long-standing problem in the LEP/SLC precision measurements. These examples are also interesting because the present experimental data suggest the discrepancy with the SM, at the 1-\(\sigma\) level for the former and at more than 3-\(\sigma\) level for the latter.

This paper is organized as follows. In section 2, we discuss violation of the quark-lepton universality in charged current interactions. In section 3, we discuss a possible solution of the \(R_b\) problem in the MSSM. Section 4 is devoted for summary.

§2. Quark-lepton universality violation in charger currents

The tree-level universality of the charged current weak interactions is one of the important consequences of the SU(2)\(_L\) gauge symmetry of the fundamental theory. The universality between quarks and leptons is expressed as the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, for example,

\[
|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.
\]  

Present experimental data\(^2-4\) for these CKM matrix elements are extracted from the ratios of the amplitude of the semileptonic hadron decays to that of the muon decay, after subtracting
the known SM radiative corrections\cite{5,6}. Here we adopt the following values

\[ |V_{ud}| = 0.9745 \pm 0.0007[2,3], \quad |V_{us}| = 0.2205 \pm 0.0018[4], \quad |V_{ub}| = 0.003 \pm 0.001[4]. \quad (2.2) \]

Their squared sum is then

\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0017 \pm 0.0015. \quad (2.3) \]

The universality is violated at the 1-\(\sigma\) level.

In general, the universality (2.1) can be modified by process-dependent radiative corrections, due to the spontaneous breaking of the SU(2)\(_L\) symmetry. The data (2.2) lead to a small deviation from the quark-lepton universality (2.3), after the SM corrections are applied. This may suggest a signal for physics beyond the SM, although it is only at the 1-\(\sigma\) level.

Here we study the SUSY one-loop contribution to the quark-lepton universality violation in the low-energy charged currents\cite{7}. We refine previous works\cite{8,9} by extending the analysis to cover the whole parameter space of the MSSM and obtain constraints on SUSY particle masses from the 1-\(\sigma\) universality violation (2.3).

We study the decay \(f_2 \rightarrow f_1 \ell^- \bar{\nu}\) where \(f = (f_1, f_2)\) is a SU(2)\(_L\) fermion doublet, comparing the following two cases; muon decay \((f_1 = \nu_\mu, f_2 = \mu^-)\) and semileptonic hadron decays \((f_1 = u, f_2 = d, s, b)\). At the tree level, their decay amplitudes are identical up to the CKM matrix element, and are expressed in terms of the bare Fermi constant \(G_0 = g^2/4\sqrt{2}m_W^2\).

At the one-loop level, however, the decay amplitudes receive process-dependent non-oblique corrections as well as oblique ones. Following the formalism in Ref.\cite{10}, the corrected decay amplitudes of \(f_2 \rightarrow f_1 \ell^- \bar{\nu}\) are expressed as

\[ G_f = \frac{\bar{g}_W^2(0) + g^2\delta_{GF}}{4\sqrt{2}m_W^2}. \quad (2.4) \]

The effective coupling \(\bar{g}_W^2(0)\) contains the correction to the W-boson propagator and does not lead to the universality violation. The process-dependent term \(\delta_{GF}\), which represents the vertex and box corrections, gives the universality violation.

In the MSSM, \(\delta_{GF} = \delta_{GF}(\text{SM}) + \delta_{GF}(\text{SUSY})\), where \(\delta_{GF}(\text{SM})\) is the gauge vector loop contribution and \(\delta_{GF}(\text{SUSY})\) is the SUSY loop contribution. Since \(\delta_{GF}(\text{SM})\) has been subtracted in extracting the data (2.2), it is \(\delta_{GF}(\text{SUSY})\) which gives the universality violation shown in (2.3). \(\delta_{GF}(\text{SUSY})\) comes from the loops with left-handed sfermions \((\tilde{\nu}, \tilde{e}, \tilde{f}_1, \tilde{f}_2)_L\), charginos \(\tilde{C}_j (j = 1, 2)\), neutralinos \(\tilde{N}_i (i = 1 - 4)\) and a gluino \(\tilde{g}\). It is expressed as a sum \(\delta_f = \delta_f^{(v)} + \delta_f^{(e)} + \delta_f^{(b)}\), where we use the abbreviations \(\delta_f \equiv \delta_{GF}(\text{SUSY})\) etc. The explicit
forms of the correction $\delta_f^{(v)}$ to the $W^+ f_1 f_2$ vertex and the box correction $\delta_f^{(b)}$ are given in Ref.7. They are functions of the masses and mixing matrices of the above SUSY particles.

The quark-lepton universality violation (2.3) is now expressed as

$$\delta_{q\ell} \equiv \frac{\delta G_q}{G_q} - \frac{\delta G\mu}{G\mu} \equiv (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2)^{\frac{1}{2}} - 1$$

$$= \delta_q - \delta_\ell$$

$$= (\delta_q^{(v)} + \delta_\ell^{(v)} + \delta_q^{(b)}) - (2\delta_\ell^{(v)} + \delta_\ell^{(b)})$$

$$= (\delta_q^{(v)} + \delta_q^{(b)}) - (\delta_\ell^{(v)} + \delta_\ell^{(b)})$$

$$= -0.0009 \pm 0.0008. \quad (2.5)$$

We now give numerical estimates of $\delta_{q\ell}$. We assume generation independence of the sfermion masses, and impose the following mass relations suggested by the minimal supergravity model with grand unification

$$M_1 = \frac{5}{3} M_2 \tan^2 \theta_W, \quad M_\tilde{g} = \frac{g_s^2}{g^2} M_2, \quad M_Q^2 = M_{L}^2 + 9 M_2^2, \quad (2.6)$$

to reduce the number of independent parameters.

In Fig.1, the quark-lepton universality violation $\delta_{q\ell}$ is shown in the $(M_2, \mu)$ plane for $\tan \beta = 2$ and $m_{\tilde{\nu}} = (50, 100)$ GeV. The solid lines are contours for constant $\delta_{q\ell}$'s. The regions below the thick solid lines ($\delta_{q\ell} = -0.0001$) are consistent with the 1-$\sigma$ universality violation (2.3). The regions below the thick dashed lines are excluded by LEP-I experiments because SUSY particles have not been observed in Z decays. Therefore the regions under the thick solid lines and above the thick dashed lines are favored by the present data. As seen in the figure, the SUSY parameters which satisfy the universality violation (2.3) and the LEP-I bound tend to lie in the $M_2 \ll |\mu|$ region, where the lighter chargino and neutralinos are gaugino-like. On the other hand, when $M_2 \gg |\mu|$, $\delta_{q\ell}$ tends to be positive and disfavors the negative deviation (2.3). We also find that the $\tan \beta$ dependence is not significant and that the gluino contribution to $\delta_{q\ell}$ is completely negligible, less than $O(10^{-6})$ in magnitude.

In Fig.2, the 1-$\sigma$ allowed region of masses of the sneutrino $\tilde{\nu}$ and the lighter chargino $\tilde{C}_1$ from the quark-lepton universality violation (2.3) is shown. The 1-$\sigma$ (67% C.L.) upper bounds are roughly $m_{\tilde{\nu}} < 220$ GeV and $m_{\tilde{C}_1} < 600$ GeV, respectively. Therefore, the 1-$\sigma$ deviation (2.3) from the quark-lepton universality tends to favor light sleptons and relatively light chargino and neutralinos with significant gaugino components. It is interesting that while the upper bound of $m_{\tilde{\nu}}$ generally decreases with increasing $m_{\tilde{C}_1}$, it increases between $m_{\tilde{C}_1} \approx 50$ GeV and $m_{\tilde{C}_1} \approx 100$ GeV, similar to the case of $\delta_\ell$ that has been studied in Ref.12.
Fig. 1. The SUSY contribution to the quark-lepton universality violation parameter $\delta_{q\ell}$ in the $(M_2, \mu)$ plane for $\tan \beta = 2$ and $m_{\tilde{\nu}}$(GeV) = 50(a), 100(b). The SUSY contribution explains the universality violation (2.3) at the 1-$\sigma$ level in regions below the thick solid lines. The regions below the thick dashed lines are excluded by LEP-I experiments.

Finally, we comment on the sign of $\delta_{q\ell}$. As seen in Fig.1, $\delta_{q\ell}$ takes both signs, contrary to the observation of Ref.8 where only cases with very light sfermions ($M_{\tilde{\nu}} < m_Z/2$, $M_{\tilde{\nu}} < m_Z$) were studied and only negative $\delta_{q\ell}$ was found. This sign change is caused by the cancellation between the vertex and the box corrections in (2.5).

§3. $R_b$ problem

The present precision measurements of the electroweak processes at the Z pole show an excellent agreement with the Standard Model for the observed top quark mass,

$$m_t(\text{GeV}) = \begin{cases} 
176 \pm 8 \pm 10[13], \\
199 \pm 19_{21} \pm 22[14], 
\end{cases}$$ (3.1)
except for the ratios $R_b = \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons})$ and $R_c = \Gamma(Z \to c\bar{c})/\Gamma(Z \to \text{hadrons})$. The discrepancy in these ratios, sometimes called “$R_b$ crises”, is a long-standing problem in the precision test of the SM. The most recent preliminary data\textsuperscript{15,16} for the 1995 summer conferences show

$$R_b(1995) = 0.2219 \pm 0.0017, \quad R_c(1995) = 0.1543 \pm 0.0074,$$

(3.2)

for the fit with unconstrained $R_c$ and

$$R_b(1995) = 0.2206 \pm 0.0016,$$

(3.3)

for the fit with fixed $R_c = 0.171$ (the SM prediction). The discrepancies of $R_b$ in (3.2, 3.3) with the SM prediction\textsuperscript{10} are larger than the 3-$\sigma$ level. This discrepancy of $R_b$ between experiments and the SM has become more serious than it was a year ago, since the deviation (3.3) is larger than that of the previous data\textsuperscript{17}, $R_b(1994) = 0.2202 \pm 0.0020$ and also because we can no more decrease the discrepancy by requiring a light top quark ($m_t < 150$ GeV) after the direct measurements of $m_t$\textsuperscript{13,14}. It is therefore natural to consider the possibility that physics beyond SM manifests itself in the process $Z \to b\bar{b}$.

In the MSSM, the radiative corrections to the $Zb\bar{b}$ vertex by SUSY particles and/or the extra Higgs scalars have a possibility to explain the discrepancy in $R_b$ by increasing $\Gamma(Z \to b\bar{b})$ over the other partial decay widths into light quarks. There have recently appeared a lot of works on this subject\textsuperscript{18-29}. Here we discuss the $R_b$ problem in the MSSM and show the constraints on the SUSY particles to explain the $R_b$ data in (3.3)\textsuperscript{*} for a very simple case. We also discuss the difficulty in the MSSM explanation of the $R_b$ problem due to exotic decays of the top quark.

The MSSM contributions to the $Zb\bar{b}$ vertex come from the loops with (i) $t - H^+$, (ii) $\tilde{t} - \tilde{C}$, (iii) $b - (h^0, H^0, P^0)$ and (iv) $\tilde{b} - \tilde{N}$, where $(\tilde{t}, \tilde{b})$, $H^+$, $(h^0, H^0)$ and $P^0$ denote the top and bottom squarks, the charged Higgs scalar, the light and heavy neutral Higgs scalars and the Higgs pseudoscalar, respectively. The magnitudes and the signs of these loop corrections strongly depend\textsuperscript{18,20,22} on the value of $\tan \beta$. When $\tan \beta$ is small, typically $\tan \beta < 30$, the loops (i,ii) are dominant, due to the large top-quark Yukawa-coupling. When $\tan \beta$ is sufficiently large, $\tan \beta > 30$, however, the loops (iii,iv) become important because of the

\textsuperscript{*} In this paper, we use the data (3.3) obtained by using $R_c = 0.171$. We assume here that the discrepancy of $R_c$ in (3.2) is caused\textsuperscript{10} by systematic uncertainty in flavor tagging, which is less serious for $R_b$.  

6
large bottom quark Yukawa coupling $y_b \sim m_b \tan \beta$. In both cases, to give a correction comparable to the present discrepancy between (3.3) and (3.4), the masses of new particles in the loops should be very light, typically lighter than $m_Z$.

Here we discuss only the case with small $\tan \beta$, for brevity. In this case, the contributions (iii, iv) above are negligible. The sign of the correction to $R_b$ by the loop (i) with the charged Higgs scalar $H^+$ is always negative and it worsens$^{18,20}$ the discrepancy. On the other hand, the loop contribution (ii) with charginos and the scalar top $\tilde{t}$ can give positive correction$^{18,20}$ to $R_b$. The correction by the $\tilde{t} - \tilde{C}$ loop can be comparable to the discrepancy $R_b(1995) - R_b(\text{SM})$ only if all the following conditions are satisfied:

- the mass of the lighter top squark $\tilde{t}_1$ and that of the lighter chargino $\tilde{C}_1$ are small, at most $\tilde{t}_1, \tilde{C}_1 < m_Z$,
- $\tilde{C}_1$ contains a significant higgsino component,
- $\tilde{t}_1$ is nearly right-handed, $\tilde{t}_1 \sim \tilde{t}_R$,
- $\tan \beta$ is very small, $\tan \beta \sim 1$.

The last three conditions are necessary$^{20}$ to obtain a large $\tilde{t}_1 - \tilde{C}_1 - b$ coupling.

Here we show typical results of our numerical calculation of $R_b$ in the MSSM for very simple cases. Fig. 3 shows $R_b(\text{MSSM})$ in the $(M_2, \mu)$-plane, under the following conditions

$$\tan \beta = 1, \ m(\tilde{t}_1 = \tilde{t}_R) = 46 \text{ GeV}, \ m_{H^+} \gg m_Z,$$

$$m(\text{other SUSY particles}) \gg m_Z.$$ (3.5)

The conditions (3.5) are chosen for maximizing the SUSY contribution to $R_b$. $m_t = 175 \text{ GeV}$, $M_1 = \frac{5}{3} M_2 \tan^2 \theta_W$ and the LEP-I constraints on charginos and neutralinos are also imposed. We can see from Fig. 3 that the allowed region of $(M_2, \mu)$ to give $R_b(\text{MSSM}) > 0.2190$ (consistent with the data (3.3) at 1-σ level) is very narrow and it can be explored by the coming first upgrade of LEP I (“LEP 1.5”). The allowed region of $(M_2, \mu)$ with $R_b(\text{MSSM}) > 0.2175$ (consistent with the data at 95% C.L.) is wider, but it is completely covered by experiments at LEP 200. Therefore, if the SUSY contribution explains the excess of $R_b$, we can produce both $\tilde{C}_1$ and $\tilde{t}_1$ in the LEP 1.5 or LEP 200. In the case with large $\tan \beta$, a very light $P^0$ is necessary$^{18,20,22}$ to give large $R_b(\text{MSSM})$.

Even before the LEP upgrades, we have to check the effects of the new particles with masses $\gtrsim m_Z/2$ to other experiments, which are all consistent with the SM. In some cases, these particles cause no harm. For example, the effect of light $\tilde{t}_R$ and $\tilde{C}_1$ to the oblique corrections $(S, T)$ can be sufficiently small since the SUSY contributions to $S$ and $T$ are dominated by the corrections from left-handed sfermions$^{12}$. In fact, the SUSY contribution to the $Zb\bar{b}$ vertex may lower$^{22-26}$ the fitted value of the strong coupling constant $\alpha_s(m_Z)$,
Fig. 3. $R_b$(MSSM) in the $(M_2,\mu)$ plane for $\tan\beta = 1$, $m(\tilde t_i ) = \tilde t_R$(GeV) = 46(a), 90(b) and $m_t = 175$ GeV. The regions below the upper dashed lines can be explored by LEP 200. The regions below the lower dashed lines are excluded by LEP-I.

to make it consistent with the low-energy measurements of $\alpha_s$. However, other phenomena like the rare $b$ decay\cite{28}, $\text{Br}(b \rightarrow s\gamma)$, are significantly affected by these light new particles.

The most serious consequence of the explanation of the $R_b$ problem by light SUSY particles is the exotic decays of the top quark\cite{25,28,29}, where exotic decay widths become comparable to the standard mode $t \rightarrow bW^+$ which has been observed\cite{13,14}. In the case with small $\tan\beta$, the decay mode $t \rightarrow \tilde t \tilde N_i$ opens. Its branching ratio is generally very large\cite{25,28}. In Fig.4, we show $\text{Br}(t \rightarrow \tilde t \tilde N)$ for the same conditions (3.5) as in Fig.3. As can be seen in the figure, in most of the parameter region that gives $R_b$(SUSY) > 0.2175, $\text{Br}(t \rightarrow \tilde t \tilde N)$ is larger than 0.2. In the case with large $\tan\beta$, we instead obtain large $\text{Br}(t \rightarrow bH^+)$, which is also exotic\cite{29}. Although some articles\cite{23,30} consider the possibility of detecting these exotic decays, the measurements of the top quark production\cite{13,14}, which suggest $\sigma_{\text{exp}}(p\bar p \rightarrow t \rightarrow bW^+) \geq \sigma_{\text{SM}}(p\bar p \rightarrow t)$ at present, put severe upper limit on the exotic $t$-decay branching fraction and disfavor the explanation of the $R_b$ problem by light SUSY particles.

§4. Summary

We have discussed the SUSY contributions to the quark-lepton universality violation in charged currents and the anomaly in $R_b$ at the $Z$ pole. In both cases, the non-oblique radiative corrections play crucial role. We have studied the constraints on the SUSY particles to explain recent measurements of above two observables, which suggest physics beyond the Standard Model. The observed 1-$\sigma$ violation of the quark-lepton universality, if it turns out to be real, may be explained by relatively light sleptons and chargino. In contract, although it is possible to explain the 3-$\sigma$ excess of $R_b$ by very light scalar top and chargino, this possibility is severely restricted by the upper limit for exotic decays of the top quark.
Fig. 4. The branching ratio $\text{Br}(t \to \tilde{t}\tilde{N})$ in the $(M_2, \mu)$ plane. The experimental bounds by LEP-I (lower dashed lines) and LEP 200 (upper dashed lines), and the region with $R_b(MSSM) > 0.2190, 0.2175$ (thick solid lines) are also shown. Parameters are the same as in Fig.3.

Note added. After the seminar, we received a paper\textsuperscript{31} in which the analysis using the recent data (3.3) was given. Ref.31 claims that the SUSY explanation with high $\tan\beta$ and light $P^0$ is ruled out by constraints from decays $Z \to b\bar{b}P^0$ and $b \to c\tau\bar{\nu}_\tau$.

Acknowledgements

We thank V. Barger for careful reading of the manuscript. The work of Y. Y. is supported in part by the JSPS Fellowships and the Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture of Japan No. 07-1923.

References

16) K. Hagiwara, talk in this Seminar.