AL

MPI-PhT/95-61

July 1995

Longitudinal W 's in Top Quark Hadronic Decays

Bodo Lampe

*Max-Planck-Institut für Physik, Werner-Heisenberg-Institut
Föhringer Ring 6, D-80805 München*

3w 9551

ABSTRACT

The hadronic decay of the top quark $t \rightarrow bW^+ \rightarrow bq_1\bar{q}_2(g)$ is analyzed in the rest system of the W . The angular distribution between the heavy quarks and the thrust direction gives information about the amount of longitudinal and transverse W 's in the decay. Higher order QCD corrections are calculated and their structure is elucidated in some detail. It is argued that this observable will be an ideal tool to study top quark properties at Tevatron and LHC.

1. Introduction Recently, evidence has been found for the production of top quarks at the Fermilab Tevatron collider at $\sqrt{s} = 1.8 \text{ TeV}$ ¹. Under the hypothesis that the events are due to top quark production, the top quark mass was determined to be $m_t = 176 \pm 13 \text{ GeV}$ by CDF and $m_t = 199 \pm 20 \text{ GeV}$ by D0. These values were obtained by measuring the invariant mass of the top quark decay products on the basis of the leading standard model processes for top quark production and decay. In combination with the LEP1 analysis there is now conclusive evidence for the existence of a top quark. In the next years CDF and D0 will presumably be able to reduce the error to $< 10 \text{ GeV}$. At the LHC² a really precise measurement of m_t with an expected error of about 1 GeV will be possible.

Once the mass of the top quark is established, the important question arises whether the top quark properties are precisely as predicted by the standard model. In the standard model the couplings of the top quark to the vector bosons are completely fixed by the family repetition structure. For example, the tbW coupling which induces almost all top quark decays should be a pure V-A interaction. It fixes both the total hadronic decay rate Γ_{had} and the amount of longitudinal relative to transverse W 's in top decay. In leading order these quantities are given by

$$\Gamma_{\text{had}} = \Gamma_L + \Gamma_T = \frac{2N_c G_F m_t^3}{8\sqrt{2}\pi} (1-f)^2 (1+2f) \quad (1)$$

$$R_L = \frac{\Gamma_L}{\Gamma_T} = \frac{1}{2f} \quad (2)$$

where $f = \frac{m_W^2}{m_t^2}$. In my opinion the V-A coupling should be very carefully tested because it involves two heavy particles, t and W , and it is conceivable that deviations from the standard model show up first in this coupling.³ One should note that due to the large top quark mass the ratio R_L is quite large (≈ 2 for $m_t = 160 \text{ GeV}$, ≈ 3.1 for $m_t = 200 \text{ GeV}$), in contrast to processes in which the W couples to light fermions, where longitudinal cross sections usually vanish in leading order. Γ_{had} is probably difficult to measure⁴ but the ratio Γ_L/Γ_T can be inferred by studying distributions of top quark decay products from the decay $t \rightarrow bW^+ \rightarrow bf_1\bar{f}_2$.

In fact in the rest system of the W one has the following expansion

$$\frac{d\Gamma}{d\cos\theta} = \frac{3}{8}\Gamma_T(1+\cos^2\theta) + \frac{3}{4}\Gamma_L\sin^2\theta + \frac{3}{4}\Gamma_P\cos\theta \quad (3)$$

where θ is the angle between the heavy quark direction and the direction of the light fermion f_1 or f_2 . The main aim of this letter is to report on the oneloop QCD

corrections to eq. 2 which will turn out to be of the order of -5% . Other corrections, like electroweak⁵, b-mass and finite W -width corrections are smaller, $\leq O(1\%)$.

It has been shown by CDF and $D0$ ¹ that for many events the momenta of the top quark decay products can be reconstructed, if use is made of the condition $(f_1 + \bar{f}_2) = m_W^2$. Therefore it will be possible to determine Γ_L/Γ_T precisely via eq. 3, in particular at LHC, where more than 100 000 top quarks per year will be produced. Some nice studies will be possible in the case of the semileptonic top decay $f_1 = \nu_e, \bar{f}_2 = \ell^+$. For example, the lepton energy distribution⁶ (in the top rest system) or the forward backward asymmetry of the leptons with respect to the heavy quark axis⁷ (in the W rest system) can be examined. As compared to these, the ratio $\frac{\Gamma_L}{\Gamma_T}$ has the advantage that it can be obtained both in semileptonic and hadronic top decays and therefore with a much higher statistics. To be specific, our discussion will be carried out for hadronic decays. In this case QCD effects from the W decay, $W \rightarrow q\bar{q}g$, have to be included and θ should be defined as the angle between the heavy quarks and the thrust direction defined by the jets from the light quarks (fig. 1). QCD effects from W decay on R_L will be discussed in section 5. In the narrow width approximation for the W they are completely separate from QCD corrections to the tbW vertex.

2. Leading Order Analysis Consider the hadronic decay of a top quark in the rest system of the W . In this system the 4-momenta of t and b are identical and q_1 and q_2 are produced back to back. The particle momenta can be parametrized as follows

$$\begin{aligned} t &= \frac{m_t^2}{2m_W} (f_+, 0, 0, f_-) \\ b &= \frac{m_t^2}{2m_W} f_- (1, 0, 0, 1) \\ q_1 &= \frac{m_W}{2} (1, 0, \sin \theta, \cos \theta) \\ q_2 &= \frac{m_W}{2} (1, 0, -\sin \theta, -\cos \theta) \end{aligned} \tag{4}$$

where $f_{\pm} = 1 \pm f$. In writing down this parametrization we have used the narrow width approximation $W^2 = m_W^2$, and $m_b = 0$. These approximations are good to $\leq 1\%$, even in leading order⁷. In the narrow width approximation there is only one free kinematical parameter, the angle θ .

Theoretical analysis of top quark decay is usually carried out in the t -rest system. This system provides the most natural notion of longitudinal W 's because

the W polarisations can be defined with reference to the W direction, e.g., $\varepsilon_L = (f_-, 0, 0, f_+)/2\sqrt{f}$. The W -rest system, on the other hand, is more convenient from the experimental point of view because R_L can be obtained directly from the angular distribution (3). In the top rest system no simple analogue of this equation exists. As is clear from fig. 2 the meaning of “longitudinal” and “transversal” is the same in both systems, provided one takes reference to the heavy quark direction. The lowest order distribution $\frac{d\Gamma_0}{d\cos\theta}$ is given by⁷

$$d\Gamma_0(t \rightarrow bq_1\bar{q}_2) = 2N_c \frac{3G_F m_t^3}{32\sqrt{2}\pi} f_-^2 d\cos\theta \left[f_+ + 2f\cos\theta - f_- \cos^2\theta \right] \quad (5)$$

From (5) the results eq. 1 and 2 can be read off. The term linear in $\cos\theta$ cannot be examined in hadronic top decays because quark (q_1) and antiquark ($\bar{q}_2$) jets cannot be discriminated. It is responsible for the forward backward asymmetry in semileptonic top quark decays discussed in⁷.

3. Virtual Gluon Exchange at the tbW Vertex Coming now to the QCD corrections we first examine the effect of virtual gluon exchange fig. 3 on R_L . The tbW vertex including virtual gluon exchange is given by⁸

$$\Gamma_\mu(t \rightarrow bW^+) = \frac{-ig}{\sqrt{2}} \left[H\gamma_\mu \frac{1}{2}(1 - \gamma_5) - C_F \frac{\alpha_s}{2\pi} H_- \frac{i\sigma_{\mu\nu}W^\nu}{2m_t} \frac{1}{2}(1 + \gamma_5) \right] \quad (6)$$

$H = 1 + O(\alpha_s)$ is the leading order term rescaled by QCD corrections. H_- is given by

$$H_- = -\frac{\ln f_-}{f} \quad (7)$$

The corresponding matrix element for $t \rightarrow bq_1\bar{q}_2$ is

$$\left| M_{\text{Born}} + M_{\text{virtual}} \right|^2 = H(tq_2)(bq_1) - C_F \frac{\alpha_s}{2\pi} H_-(bq_2)(bq_1). \quad (8)$$

It can now easily be seen that the effect of the $\sigma_{\mu\nu}$ -term is a longitudinal piece $\sim \sin^2\theta$ in the matrix element and therefore a contribution only to the numerator of R_L not to the denominator

$$R_L^{\text{virtual}} = \frac{1}{2f} \left(1 + C_F \frac{\alpha_s}{2\pi} \frac{f_- \ln f_-}{f} \right) \quad (9)$$

This will be added to the real gluon emission contribution presented in the next section. It should be noted that the infrared and mass singularities of the virtual

corrections are solely in the $O(\alpha_s)$ term of H and that R_L gets no contribution from the $O(\alpha_s)$ part of H because this term cancels between the numerator and denominator of R_L . This proves (via the Lee Nauenberg theorem) that the real gluon emission diagrams treated in section 4 give a finite correction to R_L , too.

4. Real Gluon Emission from the Heavy Quarks Real gluon emission from the heavy quarks in the W rest system implies that the 3-momenta of t - and b -quark are not parallel any more (whereas f_1 and f_2 are still back to back). Therefore, two (!) angular distributions $\frac{d\Gamma}{d\cos\theta_t}$ and $\frac{d\Gamma}{d\cos\theta_b}$ may be defined, one with respect to the t direction and one with respect to the b direction. In lowest order these two distributions are identical but they get different QCD corrections. Correspondingly, two ratios $R_L = \frac{\Gamma_L}{\Gamma_T}$, of the form

$$R_t = \frac{1}{2f} \left(1 - C_F \frac{\alpha_s}{2\pi} D_t \right) \quad (10)$$

and

$$R_b = \frac{1}{2f} \left(1 - C_F \frac{\alpha_s}{2\pi} D_b \right) \quad (11)$$

exist and can be determined experimentally. D_b and D_t are to be calculated in the following. Among the two quantities, R_t is the more interesting one, because it is equal to the ratio $\frac{\#L}{\#T}$ of longitudinal over transverse W 's in the top quark rest system with the W chosen as the reference direction. This follows qualitatively from fig. 2 and will be shown quantitatively later.

In addition to the θ -dependence an azimuthal dependence arises in the matrix element for $t \rightarrow bgq_1\bar{q}_2$ (fig. 4). This can be traced to W polarization contributions different from the diagonal ones, $\Gamma_T = (++) + (--)$, $\Gamma_L = (00)$, and $\Gamma_P = (++) - (--)$, and may be of interest in future studies of top quark decay. In the present calculation the azimuthal dependence was integrated out. To

uncover the azimuthal dependence, let us parametrize the momenta as follows:

$$\begin{aligned}
W &= (m_W, 0, 0, 0) \\
q_1 &= \frac{m_W}{2} (1, \sin \theta \sin \varphi, \sin \theta \cos \varphi, \cos \theta) \\
q_2 &= W - q_1 \\
b &= \frac{m_t^2}{2m_W} f_- x_b (1, 0, \sin \alpha, \cos \alpha) \\
t &= \frac{m_t^2}{2m_W} f_+ x_t (1, 0, 0, \beta) \\
g &= t - W - b
\end{aligned} \tag{12}$$

Here x_b and x_t are the rescaled quark energies,

$$\begin{aligned}
x_b &= 1 - \frac{2tg}{m_t^2 f_-} \\
x_t &= 1 - \frac{2bg}{m_t^2 f_+}
\end{aligned} \tag{13}$$

The quantities $1 - x_b$ and $1 - x_t$ are the Dalitz variables which will appear in the phase space. By eq. 13 they are related to the propagator poles tg and bg . A priori, the matrix element for real gluon emission from a heavy quark line has singularities for $tg \rightarrow 0$ and $bg \rightarrow 0$ but these drop out when the contribution to R_L is formed. This will be explicitly shown later. $f_{\pm} = 1 \pm f$ and $f = \frac{m_W^2}{m_t^2}$ are defined as before. $\beta = \sqrt{1 - \frac{4f}{f_+^2 x_t^2}}$ follows from the mass shell condition $t^2 = m_t^2$. The angle α is also completely fixed for given x_b and x_t . This follows from $g^2 = (t - b - W)^2 = 0$:

$$1 - \beta \cos \alpha = 2f \frac{f_+(1 - x_t) + f_- x_b}{f_+ f_- x_t x_b} \tag{14}$$

The angular orientation of the event is defined by the polar angle θ and the azimuthal angle φ . In the parametrization given above θ is the angle between t and q_1 and we are going to discuss R_t first. The other ratio, R_b , is more conveniently discussed in the system where b is put along the z -direction. This will be done later.

The phase space can be given in terms of the four variables x_t, x_b, θ and φ :

$$dR_4(t; b, g, q_1, q_2) = m_t^2 \int_0^{m_t^2} dW^2 \frac{\pi^2}{32} f_+ f_- \int_0^1 d(1 - x_b) \int_0^U d(1 - x_t) \int_{-1}^1 d \cos \theta \int_0^{2\pi} d\varphi \tag{15}$$

where the upper limit for $1 - x_t$ is

$$U = \frac{f_-^2 x_b (1 - x_b)}{f_+ (1 - f_- (1 - x_b))} \tag{16}$$

The matrix element fig. 4 can be expanded as^{7,9}

$$\begin{aligned}
|M|^2 = & -C_F \frac{\alpha_s}{2\pi} \left\{ \frac{3}{8} Z_T (1 + \cos^2 \theta) + \frac{3}{4} Z_L \sin^2 \theta \right. \\
& + \frac{3}{4} Z_P \cos \theta + \frac{3}{\sqrt{2}} Z_N \sin \theta \cos \varphi - \frac{3}{4} Z_V \sin^2 \theta (2 \cos^2 \varphi - 1) \\
& \left. + \frac{3}{\sqrt{2}} Z_F \sin \theta \cos \theta \cos \varphi \right\}
\end{aligned} \tag{17}$$

where the coefficients Z depend only on x_t, x_b and f . Integrating over φ one recovers the form of eq. 3. The correction D_t can be obtained from the combination $2fZ_L - Z_T = (1 + 2f)Z_L - (Z_T + Z_L)$

$$D_t(\text{real}) = \frac{3f_+}{ff_-} \int_0^1 d(1 - x_b) \int_0^U d(1 - x_t) (2fZ_L - Z_T)$$

This is because, generically,

$$\frac{\Gamma_L}{\Gamma_T} = \frac{L_0 + \alpha_s L_1}{T_0 + \alpha_s T_1} = \frac{L_0}{T_0} \left\{ 1 + \alpha_s \frac{L_1 T_0 - T_1 L_0}{L_0 T_0} \right\} \tag{18}$$

where $L_0 \sim 1, T_0 \sim 2f, L_1 \sim Z_L, T_1 \sim Z_T$ in appropriate units. The combination eq. 18 is free of infrared and collinear singularities.

The integral eq. 18 can be done. Details can be taken from the appendix. Adding the virtual contributions one obtains

$$\begin{aligned}
D_t = & -\frac{f_- \ln f_-}{f} + \frac{1}{f_-^2} \left\{ 2f_-(f^2 - 12f - 6) - (5 + 15f + 14f^2) \ln f \right. \\
& + 2 \left[\frac{\pi^2}{2} - L_2(1 + \sqrt{f}) - L_2(1 - \sqrt{f}) \right] (7 + 15f + 2f^2) \\
& \left. + \frac{L_2(1 - \sqrt{f}) - L_2(1 + \sqrt{f})}{\sqrt{f}} (5 + 22f + 23f^2 - 2f^3) \right\}
\end{aligned} \tag{19}$$

Numerically, this gives $D_t = 1.3720$ for $m_t = 160 \text{ GeV}$ and $D_t = 1.5347$ for $m_t = 200 \text{ GeV}$ corresponding to a correction of about -3.5% to $\frac{\Gamma_L}{\Gamma_T}$. The complete m_t dependence of the correction is shown in fig. 5.

Eq. 19 is the main result of this article. I have checked that the same result is obtained if one works in the t -rest system and calculates the correction to $\frac{\#L}{\#T}$. I have used the following parametrization of momenta

$$\begin{aligned}
t &= (m_t, 0, 0, 0) \\
W &= \frac{m_t}{2} f_+ x_W (1, 0, 0, \beta_W) \\
g &= \frac{m_t}{2} f_- x_g (1, 0, \sin \alpha_g, \cos \alpha_g)
\end{aligned} \tag{20}$$

where $\beta_W = \sqrt{1 - \frac{4f}{f_+^2 x_W^2}}$ and $\cos \alpha_g$ follows from $(t - W - g)^2 = 0$. I have obtained the combination $(1 + 2f)L - (T + L)$ by contracting the heavy quark tensor with $(1 + 2f)\varepsilon_L^\mu \varepsilon_L^\nu - (-g^{\mu\nu} + \frac{W^\mu W^\nu}{m_W^2})$ where $\varepsilon_L = \frac{f+x_W}{2\sqrt{f}}(\beta_W, 0, 0, 1)$. The resulting expression has to be integrated with the measure

$$dR_3(t; b, W, g) = \frac{m_t^2 \pi^2}{4} f_+ f_- \int_0^1 dx_g \int_0^{\frac{f_-^2 x_g (1-x_g)}{f_+ (1-f_- x_g)}} d(1-x_W) \quad (21)$$

given in terms of quantities defined in the t -rest system and the result of this integration completely agrees with eq. 19.

The corrections to R_b can be obtained by similar methods. One should work in the W rest system and put the b -quark along the z -direction. Eventually one obtains

$$D_b = -\frac{2}{f_-^2} \left\{ -f_-^2 f_- + \left(\frac{f_-}{2} - f_-^2 \right) \ln f_- \right\} - \frac{f_-}{f} \ln f_- \quad (22)$$

Numerically, this gives $D_b = 2.57$ for $m_t = 160\text{GeV}$, corresponding to a correction of about -6% to $\frac{\Gamma_L}{\Gamma_T}$. D_b is represented graphically in fig. 5 as a function of m_t .

5. Effects from $W \rightarrow q_1 \bar{q}_2(g)$ R_L gets corrections from gluons emitted from the light quark legs, fig. 6. Of course, there is no longitudinal component in the decay of W 's into massless fermions but there is an effect on the angular distribution eq. 3. To calculate it, I have obtained an expansion of $|M(\text{fig.6})|^2$ a la eq. 17 based on the following parametrization of momenta

$$\begin{aligned} W &= (m_W, 0, 0, 0) \\ b &= \frac{m_t^2}{2m_W} f_- (1, \sin \theta \sin \varphi, \sin \theta \cos \varphi, \cos \theta) \\ t &= W + b \\ q_1 &= \frac{m_W}{2} x_1 (1, 0, 0, 1) \\ q_2 &= \frac{m_W}{2} x_2 (1, 0, \sin \theta_2, \cos \theta_2) \end{aligned} \quad (23)$$

with $\cos \theta_2 = 1 + \frac{2}{x_1 x_2} - \frac{2}{x_1} - \frac{2}{x_2}$ from $(W - q_1 - q_2)^2 = 0$.

In this parametrization the angle θ is the angle between the heavy quark direction and the direction of q_1 . The matrix element $|M(\text{fig.6})|^2$ has poles in $\frac{2q_1 g}{m_W^2} = 1 - x_2$ and $\frac{2q_2 g}{m_W^2} = 1 - x_1$. However, these poles drop out in the combination $2fZ_L - Z_T$

which determines the correction to R_L . In fact one obtains

$$\begin{aligned} R_L^{q_1} &= \frac{1}{2f} \left[1 - C_F \frac{\alpha_s}{2\pi} \frac{f_-}{f} (1 + 2f) \int_0^1 d(1 - x_1) \int_0^{x_1} d(1 - x_2) 2 \frac{x_1 + x_2 - 1}{x_1^2} \right] \\ &= \frac{1}{2f} [1 - C_F \frac{\alpha_s}{2\pi} \frac{f_-}{f} (1 + 2f)] \end{aligned} \quad (24)$$

Note that there is no contribution from virtual gluon exchange. The result eq. 24 is not very satisfactory because experimentally the jet q_1 cannot be distinguished from q_2 or g . The z axis to choose is not q_1 but the thrust axis, the axis of the most energetic of the jets q_1, q_2 and g . Therefore the integration in eq. 24 should be carried out with the conditions $x_1 > x_2$ and $x_1 > x_3 = 2 - x_1 - x_2$. In the other two cases $x_2 = \max(x_1, x_2, x_3)$ resp. $x_3 = \max(x_1, x_2, x_3)$ one should integrate

$$R_L^{q_2} = \frac{1}{2f} [1 - C_F \frac{\alpha_s}{2\pi} \frac{f_-}{f} (1 + 2f) \int_0^1 d(1 - x_1) \int_0^{x_1} d(1 - x_2) 2 \frac{x_1 + x_2 - 1}{x_2^2}] \quad (25)$$

resp.

$$R_L^g = \frac{1}{2f} [1 - C_F \frac{\alpha_s}{2\pi} \frac{f_-}{f} (1 + 2f) \int_0^1 d(1 - x_1) \int_0^{x_1} d(1 - x_2) 4 \frac{x_1 + x_2 - 1}{(2 - x_1 - x_2)^2}] \quad (26)$$

obtained in the frames where q_2 resp. g define the z direction.

Adding up the contributions from the three regions one obtains

$$\begin{aligned} R_L &= \frac{1}{2f} [1 - C_F \frac{\alpha_s}{2\pi} \frac{f_-}{f} (1 + 2f) \int_{\frac{2}{3}}^1 dT (-\frac{8}{T^2} + \frac{16}{T} - 6)] \\ &= \frac{1}{2f} [1 + C_F \frac{\alpha_s}{2\pi} \frac{f_-}{f} (1 + 2f) (16 \ln \frac{2}{3} + 6)] \end{aligned} \quad (27)$$

This is our main result for the correction from the light quark side. It corresponds to a correction of about -5% for $m_t = 160 \text{ GeV}$ and about -8% for $m_t = 200 \text{ GeV}$.

The corresponding relative correction $D = -(16 \ln \frac{2}{3} + 6) \frac{f_-}{f} (1 + 2f)$ (in units of $-C_F \frac{\alpha_s}{2\pi}$) has to be added to the corrections D_t resp. D_b from the heavy quark side.

6. Conclusions In this article the complete oneloop QCD corrections to $\frac{\Gamma_L}{\Gamma_T}$ have been calculated. Actually, beyond the leading order two such ratios, one with respect to the top, the other one with respect to the bottom direction can be defined. In the W rest system they are easily determined, to the extend that the 4-momenta of $b(t)$ and the thrust axis of the jets from the W can be reconstructed. We have also included the effect of gluon emission from the light quark lines which give an interesting modification if the thrust direction is given by the gluon. In this

connection one may ask the question, whether it is possible to discriminate a hard gluon emitted from q_1 or q_2 (fig. 6) from a hard gluon emitted from the heavy quark lines t, b (fig. 4). The answer is yes, if the event is fully reconstructed and the W -mass shell condition $(q_1 + q_2 + g)^2 = m_W^2$ resp. $(q_1 + q_2)^2 = m_W^2$ is used.

Instead of R_L it is sometimes more convenient to use the ratios $\frac{\Gamma_L}{\Gamma_T + \Gamma_L} = \frac{R_L}{1 + R_L}$, $\frac{\Gamma_T}{\Gamma_T + \Gamma_L} = \frac{1}{1 + R_L}$ or $\frac{\Gamma_L - \Gamma_T}{\Gamma_L + \Gamma_T} = \frac{R_L - 1}{R_L + 1}$. For example, $\frac{\Gamma_L - \Gamma_T}{\Gamma_L + \Gamma_T}$ measures the asymmetry between the longitudinal and transverse contributions to the total rate. The QCD expression for these ratios can easily be obtained from D (=the correction to R_L in units of $-C_F \frac{\alpha_s}{2\pi}$), namely

$$\begin{aligned}\frac{\Gamma_L}{\Gamma_T + \Gamma_L} &= \frac{1}{1 + 2f} \left(1 - C_F \frac{\alpha_s}{2\pi} \frac{2fD}{1 + 2f} \right) \\ \frac{\Gamma_T}{\Gamma_T + \Gamma_L} &= \frac{2f}{1 + 2f} \left(1 + C_F \frac{\alpha_s}{2\pi} \frac{D}{1 + 2f} \right) \\ \frac{\Gamma_L - \Gamma_T}{\Gamma_T + \Gamma_L} &= \frac{1 - 2f}{1 + 2f} \left(1 - C_F \frac{\alpha_s}{2\pi} \frac{4fD}{1 - 4f^2} \right)\end{aligned}\tag{28}$$

Appendix In this appendix two integral representations of D_t , eq. 19, are given. The first one is given in terms of x_g and x_W and can be derived in the top quark rest system. The second one is given in terms of x_t and x_b and derived in the W rest system. Both results may be of experimental interest in case that cuts on x_g, x_W, x_t or x_b are to be introduced so that not the “true” ratio $\frac{\Gamma_L}{\Gamma_T}$ but some cut-dependent approximation to it is determined.

$$\begin{aligned}D_t = & -\frac{f_- \ln f_-}{f} + \frac{1}{2f_-^3} \int_0^1 \frac{dx_g}{x_g^2} \int_0^{\frac{f_- x_g(1-x_g)}{4(1-f_-x_g)}} \frac{d(1-x_W)}{f_+^2 x_W^2 - 4f} \left\{ 8(1-x_W)^2(2f^4 + 7f^3 + 9f^2 + 5f + 1) \right. \\ & + 4x_g^3(-2f^6 + 5f^5 - f^4 - 6f^3 + 4f^2 + f - 1) \\ & + 4x_g^2(1-x_W)^2(f^5 + f^4 - 2f^3 - 2f^2 + f + 1) \\ & + 4x_g^2(1-x_W)(-2f^6 + 3f^5 + 5f^4 - 6f^3 - 4f^2 + 3f + 1) \\ & + 4x_g^2(2f^6 - 4f^5 - 2f^4 + 8f^3 - 2f^2 - 4f + 2) \\ & + 4x_g(1-x_W)^2(4f^5 + 10f^4 + 4f^3 - 8f^2 - 8f - 2) \\ & \left. + 4x_g(1-x_W)(-6f^5 - 3f^4 + 12f^3 + 6f^2 - 6f - 3) \right\}\end{aligned}$$

$$\begin{aligned}
D_t = & -\frac{f_- \ln f_-}{f} + \frac{1}{4ff_-} \int_0^1 \frac{d(1-x_b)}{x_b^2(1-x_b)^2} \int_0^{\frac{f_-x_b(1-x_b)}{f_+(1-f_-(1-x_b))}} \frac{d(1-x_t)}{1-x_t} \\
& \left\{ \cos^2 \alpha \left[(1-x_t)(4+12f+8f^2) + (1-x_t)(1-x_b)(-16-48f-32f^2) \right. \right. \\
& \quad + (1-x_t)(1-x_b)^2(24+72f+48f^2) + (1-x_t)(1-x_b)^3(-16-48f-32f^2) \\
& \quad + (1-x_t)(1-x_b)^4(4+12f+8f^2) + (1-x_b)(-2+6f^2-4f^3) \\
& \quad + (1-x_b)^2(8-24f^2+16f^3) + (1-x_b)^3(-12+36f^2-24f^3) \\
& \quad \left. \left. + (1-x_b)^4(8-24f^2+16f^3) + (1-x_b)^5(-2+6f^2-4f^3) \right] \right. \\
& \quad + (1-x_t)(-4-12f-8f^2) + (1-x_t)(1-x_b)(16+48f+32f^2) \\
& \quad + (1-x_t)(1-x_b)^2(-24-64f-40f^2) + (1-x_t)(1-x_b)^3(16+32f+16f^2) \\
& \quad + (1-x_t)(1-x_b)^4(-4-4f) + (1-x_b)(2-6f^2+4f^3) \\
& \quad + (1-x_b)^2(-8+24f^2-16f^3) + (1-x_b)^3(12-36f^2+24f^3) \\
& \quad \left. \left. + (1-x_b)^4(-8+24f^2-16f^3) + (1-x_b)^5(2-6f^2+4f^3) \right\}
\end{aligned}$$

Acknowledgements I would like to thank G. Kramer for some very helpful ideas. Discussions with W.J. Marciano and A. Soni are gratefully acknowledged.

REFERENCES

1. F. Abe et al., CDF Collaboration, *Phys. Rev. D* **50** (1994) 2966;
F. Abe et al., CDF Collaboration, Fermilab-Pub-95/022-E;
S. Abachi et al., D0 Collaboration, Fermilab-Pub-95/028-E.
2. E. Reya, P.M. Zerwas, Top Physics at LHC, in. Proc. of the LHC Workshop, Aachen 1990.
3. B. Lampe, *Phys. Lett. B* **291** (1992) 166; *Phys. Lett. B* **302** (1993) 120;
M. Jezabek, J.H. Kühn, *Phys. Lett. B* **329** (1994) 317.
4. M. Jezabek, J.H. Kühn, Karlsruhe preprint TTP-03-4 (1993);
V.A. Khoze, W.J. Stirling, L.H. Orr, *Nucl. Phys. B* **378** (1992) 413.
5. T. Kuruma, *Z. Phys. C* **57** (1993) 551.
6. M. Jezabek, J.H. Kühn, *Nucl. Phys. B* **320** (1989) 20.
7. B. Lampe, MPI-PhT/95-29 (1995).
8. M. Jezabek, J.H. Kühn, *Nucl. Phys. B* **314** (1989) 1;
A. Czarnecki *Phys. Lett. B* **252** (1990) 467;
C.S. Li, R.J. Oakes, T.C. Yuan, *Phys. Rev. D* **43** (1991) 3759.
9. G. Köpp, G. Kramer, W.F. Palmer, G.A. Schuler, *Z. Phys. C* **48** (1990) 327.
J.G. Körner, G.A. Schuler, *Z. Phys. C* **46** (1990) 93.

Table
Relative Magnitude of QCD Corrections to R_t and R_b
in units of $-C_F \frac{\alpha_s}{2\pi}$

m_t	D_t	D_b
100	0.778	1.269
120	1.071	1.868
140	1.249	2.268
160	1.372	2.570
180	1.463	2.816
200	1.535	3.025
220	1.592	3.209
240	1.638	3.373

thrust direction

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5 Relative magnitude of QCD corrections to Γ_{total} , R^b and R^t in units of $-C_F \frac{\alpha_s}{2\pi}$

Fig. 6