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Supersymmetrization of the Chiral Extension of Standard Model

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Abstract

In this paper we investigate the supersymmetrization of the chiral extension of standard model. It is proven that such a theory maintains many important characteristics of the original one, but the generation number required for the breaking of asymptotic freedom is decreased from $> \frac{33}{8}$ to $> \frac{9}{4}$ which is compatible with the present experiments.

PACS 11.30 Pb-Supersymmetry.

PACS 12.50 Ch- Models of quarks and leptons.

I. Introduction

We have suggested a theory^[1], in which by starting from a free theory of fermions with masses to introduce gauge interactions by the usual method^[2]. The

new quarks, called l quarks, which appear during the course above and belong to the star representation with respect to the original quarks, called h quarks, were chosen to replace the leptons of the standard model. We also demonstrated that by an appropriate arrangement of spontaneous breaking the h quarks acquire small masses and the l quarks acquire large masses. If the generation number increases beyond $\frac{33}{8}$, the asymptotic freedom will cease to be effective. In this case we suggested that maybe it is possible to have some mechanisms which make the ground bound states of h quarks to be heavy and the ground bound states of l quarks to be light and also the other bound states to be more heavy.

However, for such a theory besides the problems of the naturalness and hierarchy, the requirement of large number of generations is very difficult to gain support from experiments. In order to overcome these difficulties, we will demonstrate here that a supersymmetric theory not only resolves the problems of the naturalness and hierarchy but also decreases the generation number dramatically from $\frac{33}{8}$ to $\frac{9}{4}$ which is compatible with the present experiments.

II. Lagrangian of the Supersymmetric Theory.

In this section we will discuss the formalism of the supersymmetrization of the chiral extension of standard model. The chiral superfield and anti-chiral superfield are expressed by Φ_L and Φ_R respectively

$$\Phi_L = A_L + 2\theta\psi_L - \theta^2 F_L, \quad \Phi_R = A_R + 2\bar{\theta}\bar{\psi}_R - \bar{\theta}^2 F_R, \quad A_L = A_{L1} + iA_{L2},$$

$$A_R = A_{R1} - iA_{R2}, \quad F_L = F_{L1} - iF_{L2}, \quad F_R = F_{R1} + iF_{R2}, \quad (1)$$

where $A_{L(R)1(2)}$, $F_{L(R)1(2)}$ are real fields. In Wess-Zumino gauge, the gauge superfield is expressed

$$V = -\theta\sigma^\mu\bar{\theta}A_\mu + i\bar{\theta}^2\theta\lambda - i\theta^2\bar{\theta}\bar{\lambda} - \frac{1}{2}\theta^2\bar{\theta}^2 D. \quad (2)$$

In the sequence of $U(1) \otimes SU(2) \otimes SU(3)$, we have superfields V_1, V_2, V_3 and the corresponding coupling constants g_1, g_2, g_3 . According to the representations of hypercharge Y , $SU(2)$ and $SU(3)$, there are two superfields Φ_L and Φ_R^c with representation $(\frac{1}{3}, 2, 3)$, two superfields Φ_{RU} and Φ_{LD}^c with representation $(\frac{1}{3}, 1, 3)$, two superfields Φ_{RD} and Φ_{LU}^c with representation $(-\frac{2}{3}, 1, 3)$, where c means the charge conjugation. The fields without c describe h quarks, otherwise they describe l quarks. U and D indicate the isotopic singlets corresponding to the up and down states of isotopic doublet. In addition we consider only two doublets of Higgs superfields Φ_{LH} and Φ_{RH} with representation $(1, 2, 1)$. For the convenience of discussion, we list all of the superfields used in this paper explicitly

$$\begin{aligned} \Phi_L &= A_L + 2\theta\psi_L - \theta^2 F_L, & \Phi_R^c &= A_R^c + 2\bar{\psi}_R\bar{\theta} - \bar{\theta}^2 F_R^c, \\ \Phi_{RU} &= A_{RU} + 2\bar{\psi}_{RU}\bar{\theta} - \bar{\theta}^2 F_{RU}, & \Phi_{LU}^c &= A_{LU}^c + 2\theta\psi_{LU}^c - \theta^2 F_{LU}^c, \\ \Phi_{RD} &= A_{RD} + 2\bar{\psi}_{RD}\bar{\theta} - \bar{\theta}^2 F_{RD}, & \Phi_{LD}^c &= A_{LD}^c + 2\theta\psi_{LD}^c - \theta^2 F_{LD}^c, \\ \Phi_{LH} &= H_L + 2\theta h_L - \theta^2 f_L, & \Phi_{RH} &= H_R + 2\bar{h}_R\bar{\theta} - \bar{\theta}^2 f_R, \\ V_i &= -\theta\sigma^\mu\bar{\theta}A_{i\mu} + i\bar{\theta}^2\theta\lambda_i - i\theta^2\bar{\theta}\bar{\lambda}_i - \frac{1}{2}\theta^2\bar{\theta}^2 D_i, & i &= 1, 2, 3 \dots \dots \end{aligned} \quad (3)$$

We agree to expand V_2 and V_3 according to $\frac{\tau^1}{2}$ and $\frac{\lambda^a}{2}$ respectively, here τ^i are

the Pauli matrices, λ^a are the Gell-Mann matrices.

The total lagrangian includes supersymmetric invariant one and soft breaking one. The former comes from gauge fields, matter fields, Yukawa coupling terms and some quadratic terms; the latter includes all of the possible soft breaking terms [2]. It is easy to write down each of these terms, but our attention should be paid only to those which come from two kinds of quarks. Below we display only the part of Yukawa coupling term and the quadratic part which is supersymmetric invariant

$$\begin{aligned} L_Y = & - \{ \Phi_L^\dagger g_L \Phi_{LH}^c \Phi_{RU} + \Phi_R^{c\dagger} g_L \Phi_{LH} \Phi_{LU}^c + \Phi_L^\dagger g_R \Phi_{RH} \Phi_{RD} + \Phi_R^{c\dagger} g_R \Phi_{RH}^c \Phi_{LD}^c \\ & + \Phi_L^\dagger N \Phi_R^c + \Phi_{LD}^{c\dagger} N_u \Phi_{RU} + \Phi_{LU}^{c\dagger} N_d \Phi_{RD} + \mu \Phi_{RH}^\dagger \Phi_{LH} + h.c. \} F_1, \end{aligned} \quad (4)$$

where $g_{L(R)}$, $G_{L(R)}$ are the Yukawa coupling constants,

$$\Phi_{LH}^c = i\tau^2 \Phi_{LH}^\dagger, \quad \Phi_{RH}^c = i\tau^2 \Phi_{RH}^\dagger, \quad (5)$$

N , N_u and N_d , they come from the free theory with masses, are arbitrary and fixed matrices [1]. Index F_1 means to take the real part of its F term.

The expression of total lagrangian is rather tedious, so we need to introduce some notations, for which the details are given in appendix (A₁), (A₂), (A₃), (A₄)

By using these notations, we get the total lagrangian

$$L = L_1 + L_2 \quad (6)$$

$$\begin{aligned} L_1 = & -\frac{1}{4}F_{\mu\nu}^\dagger F^{\mu\nu} + \frac{1}{2}\bar{\lambda}_i \gamma^\mu D_\mu \lambda_i - [\bar{\lambda}_\mu (1+\gamma_5) \lambda + h.c.] + [\bar{\Psi}_L i \gamma^\mu D_\mu (1+\gamma_5) \Psi_L \\ & + (D^\mu A_L)^\dagger D_\mu A_L + \bar{\Psi}_L (1-\gamma_5) i \hat{\lambda} A_L - A_L^\dagger i \hat{\lambda} (1+\gamma_5) \Psi_L + (L \rightarrow R)] \\ & + [\bar{h}_L i \gamma^\mu D_\mu (1+\gamma_5) h_L + (D^\mu H_L)^\dagger D_\mu H_L + \bar{h}_L (1-\gamma_5) i \hat{\lambda} H_L \end{aligned}$$

$$\begin{aligned}
& -H_L^\dagger i\tilde{\lambda}(1+\gamma_5)h_L + (L \rightarrow R)] + [\mu\bar{h}_L(1-\gamma_5)h_R + mH_L^\dagger H_R + \text{h.c.}] \\
& \cdot \left\{ \frac{g_L^2}{8} [A_L^\dagger Y A_L + H_L^\dagger H_L - (L \rightarrow R)] + \frac{g_R^2}{8} [A_L^\dagger \tau^i A_L + H_L^\dagger \tau^i H_L - (L \rightarrow R)]^2 \right. \\
& \left. + \frac{g_L^2}{8} [A_L^\dagger \lambda^a A_L - A_R^\dagger \lambda^a A_R]^2 \right\}. \quad (7)
\end{aligned}$$

$$\begin{aligned}
L_2 = & -[\bar{\Psi}_L H(1-\gamma_5)\Psi_R + A_L^\dagger \bar{h}_{LR}(1-\gamma_5)\Psi_R + A_R^\dagger \bar{h}_{RL}(1+\gamma_5)\bar{\Psi}_L + \text{h.c.}] \\
& - (H A_R)^\dagger H A_R - (H^\dagger A_L)^\dagger H^\dagger A_L - (A_R^\dagger G_L A_L^\dagger - A_R^\dagger G_L A_L^\dagger + \mu^* H_R^\dagger)(\text{h.c.}) \\
& - (A_L^\dagger g_R A_{RD} - A_L^\dagger G_R A_{RD}^\dagger + \mu H_L^\dagger)(\text{h.c.}) - A_L^\dagger M_L^2 A_L - A_R^\dagger M_R^2 A_R \\
& - [A_L^\dagger M_{LR} A_R + \text{h.c.}], \quad (8)
\end{aligned}$$

where m, μ, M_L^2, M_R^2 and M_{LR} come from the soft breaking ^[3], and

$$A_L^\dagger = i\tau^2 A_L^*, \quad A_R^\dagger = i\tau^2 A_R^*, \quad (9)$$

$$\begin{aligned}
\bar{h}_{LR} &= \begin{pmatrix} 0 & g_L \bar{h}_L^\dagger & g_R \bar{h}_R^\dagger \\ G_R^\dagger \bar{h}_R^\dagger & 0 & 0 \\ G_L^\dagger \bar{h}_L^\dagger & 0 & 0 \end{pmatrix}, \quad \bar{h}_{RL} = \begin{pmatrix} 0 & G_R \bar{h}_L^\dagger & G_L \bar{h}_L^\dagger \\ g_L^\dagger \bar{h}_L^\dagger & 0 & 0 \\ g_R^\dagger \bar{h}_R^\dagger & 0 & 0 \end{pmatrix}, \\
H &= \begin{pmatrix} N & g_L H_L^\dagger & g_R H_R^\dagger \\ G_R^\dagger H_R^\dagger & N_u & 0 \\ G_L^\dagger H_L^\dagger & 0 & N_d \end{pmatrix}. \quad (10)
\end{aligned}$$

The transposition in (10) acts only on the isotopic components.

The gauge field sector and Higgs sector of (6) are the same as one met in the supersymmetric version of the standard model; Haber, Kane and Gunion

have already investigated them thoroughly^[4]. Therefore we will emphasize on the sectors of quarks and their partners only, especially their mass spectra and wave mixings. One can see from (6),(7),(8) that in the generation space, L_1 is a scalar expressed in unit matrix; L_2 , in general, involves non-diagonal matrices. In the next section we will devote to deal with L_2 and discuss how to determine the mass spectra and wave mixings.

III. Mass Spectra and Wave Mixings

For any given g_L, g_R, G_L and G_R , one can make them to be positive diagonal by unitary transformation or wave function redefinition. It is easy to see that under unitary transformation, the formalism of L_1 keeps unchanged. However $N, N_u, N_d, M_L^2, M_R^2$ and M_{LR} will experience some changes; nevertheless, their arbitrary and fixed properties are still preserved. These mean that equivalently $N, N_u, N_d, M_L^2, M_R^2$ and M_{LR} can also be treated as invariants. The L_2 now becomes

$$\begin{aligned}
L_2 = & -\{[\bar{\Psi}_L H(1-\gamma_5)\Psi_R + A_L^\dagger \bar{h}_{LR}(1-\gamma_5)\Psi_R + A_R^\dagger \bar{h}_{RL}(1+\gamma_5)\bar{\Psi}_L + \text{h.c.}] \\
& + (H A_R)^\dagger H A_R + (H^\dagger A_L)^\dagger H^\dagger A_L + (A_R^\dagger G_L A_L^\dagger - A_R^\dagger G_L A_L^\dagger + \mu^* H_R^\dagger)(\text{h.c.}) \\
& + (A_L^\dagger k g_R A_{RD} - A_L^\dagger G_R K A_L^\dagger + \mu H_L^\dagger)(\text{h.c.}) + A_L^\dagger M_L^2 A_L + A_R^\dagger M_R^2 A_R \\
& + [A_L^\dagger M_{LR} A_R + \text{h.c.}], \quad (11)
\end{aligned}$$

where g_L, g_R, G_L, G_R , are positive diagonal matrices, k and K are two indepen-

dent CKM matrices, and also

$$\begin{aligned} \bar{h}_{LR} &= \begin{pmatrix} 0 & g_L \bar{h}_L^c & k g_R \bar{h}_R^c \\ G_R K h_R^c & 0 & 0 \\ G_L \bar{h}_L & 0 & 0 \end{pmatrix}, \quad \bar{h}_{RL} = \begin{pmatrix} 0 & K' G_R \bar{h}_R^c & G_L \bar{h}_L \\ g_L \bar{h}_L^c & 0 & 0 \\ g_R k \bar{h}_R^c & 0 & 0 \end{pmatrix}, \\ H &= \begin{pmatrix} N & g_L H_L^c & k g_R H_R^c \\ G_R K H_R^c & N_u & 0 \\ G_L H_L^c & 0 & N_d \end{pmatrix}. \end{aligned} \quad (12)$$

After spontaneous breaking and substituting H_L and H_R by their vacuum expectation values into L_2 ,

$$\langle H_L \rangle_0 = \begin{pmatrix} 0 \\ \frac{v_L}{\sqrt{2}} \end{pmatrix}, \quad \langle H_R \rangle_0 = \begin{pmatrix} 0 \\ \frac{v_R}{\sqrt{2}} \end{pmatrix}. \quad (13)$$

We obtain the quark mass term L_{mass}^q

$$\begin{aligned} L_{mass}^q &= - \begin{pmatrix} u_{hL} \\ d_{hL}^c \end{pmatrix} \begin{pmatrix} \frac{g_L v_L}{\sqrt{2}} & N K^\dagger \\ N_u & \frac{G_R v_R}{\sqrt{2}} \end{pmatrix} \frac{1-\gamma_5}{2} \begin{pmatrix} u_{hR} \\ K d_{hR}^c \end{pmatrix} \\ &- \begin{pmatrix} k^\dagger d_{hL} \\ u_{hL}^c \end{pmatrix} \begin{pmatrix} \frac{g_R v_R}{\sqrt{2}} & k^\dagger N \\ N_d & \frac{G_L v_L}{\sqrt{2}} \end{pmatrix} \frac{1-\gamma_5}{2} \begin{pmatrix} d_{hR} \\ -u_{hR}^c \end{pmatrix} + h.c. \end{aligned} \quad (14)$$

According to the definition of $\psi_{L(R)}^{(c)}$ of (1), there are relations between $\psi_{L(R)}^{(c)}$ and the wave function $(u_{L(R)}^{(c)}, d_{L(R)}^{(c)})$ of physical fermion [1].

$$\begin{aligned} \psi_L &= \frac{1}{\sqrt{2}} \begin{pmatrix} u_{hL} \\ d_{hL} \end{pmatrix}, \quad \psi_{RU} = \frac{1}{\sqrt{2}} u_{hR}, \quad \psi_{RD} = \frac{1}{\sqrt{2}} d_{hR}; \\ \psi_R^c &= \frac{1}{\sqrt{2}} \begin{pmatrix} d_{hR}^c \\ -u_{hR}^c \end{pmatrix}, \quad \psi_{LD}^c = \frac{1}{\sqrt{2}} d_{hL}^c, \quad \psi_{LU}^c = \frac{1}{\sqrt{2}} u_{hL}^c. \end{aligned} \quad (15)$$

One notes that in (14) $(u_{hL(R)}, d_{hL(R)}^c)$ decouples with $(d_{hL(R)}, u_{hL(R)}^c)$, this is because of charge conservation, and the decoupling between left-handed component and right-handed component is because of the chiral property of spinor.

In the same way, one obtains the mass term $L_{mass}^{\tilde{q}}$ of squarks. In this case, the expression of $L_{mass}^{\tilde{q}}$ becomes more complicated owing to the existence of couplings between left-handed components and right-handed components. We leave the complete expression in the appendix (A5) (A6) and consider only a special case in which the terms proportional to μ and M_{LR} are neglected, then

$L_{mass}^{\tilde{q}}$ becomes

$$\begin{aligned} L_{mass}^{\tilde{q}} &= - \begin{pmatrix} A_{LU} \\ A_{LD}^c \end{pmatrix}^\dagger W_u \begin{pmatrix} A_{LU} \\ A_{LD}^c \end{pmatrix} - \begin{pmatrix} A_{RU} \\ A_{RD}^c \end{pmatrix}^\dagger T_u \begin{pmatrix} A_{RU} \\ A_{RD}^c \end{pmatrix} \\ &- \begin{pmatrix} A_{LD} \\ A_{LU}^c \end{pmatrix}^\dagger W_d \begin{pmatrix} A_{LD} \\ A_{LU}^c \end{pmatrix} - \begin{pmatrix} A_{RD} \\ A_{RU}^c \end{pmatrix}^\dagger T_d \begin{pmatrix} A_{RD} \\ A_{RU}^c \end{pmatrix}, \end{aligned} \quad (16)$$

where W_u, W_d, T_u, T_d are given by (A6). In principle, the spectra and wave mixings are determined by the matrices of (14) and (16), but obviously it is very difficult to obtain an analytical expression. In order to have a qualitative impression of this theory, we make a further simplification by assuming $k=K=1$ and $N, N_u, N_d, M_L^2, M_R^2$ to be real and diagonal with the corresponding diagonal

elements $N_i, N_u, N_d, M_{Li}^2, M_{Ri}^2$. Below we will give the analytical expressions of mass spectra and wave mixings.

Under the assumptions mentioned above, the matrices of (14) and (16) are reduced to the type of 2×2 matrix for each generation. For an example, let us consider the i 's generation of the first matrix M_i of (14), then by the Cartan decomposition [9] we should solve the unknown hermitian matrix H_i defined by

$$M_i = \begin{pmatrix} \frac{g_{Li} v_L}{\sqrt{2}} & N_i \\ N_u & \frac{g_{Ri} v_R}{\sqrt{2}} \end{pmatrix} = U H_i, \quad (17)$$

where U is an unitary matrix needed to be determined also. H_i satisfies the following equation

$$M_i^\dagger M_i = H_i^2. \quad (18)$$

According to (17), H_i should be of the form

$$H_i = \begin{pmatrix} h_{i11} & h_{i12} \\ h_{i21} & h_{i22} \end{pmatrix}, \quad (19)$$

where all of the elements are real. By solving (18) we get

$$h_{i11} = \{[(\frac{G_{Li}^2 v_L^2}{2} + N_i^2 - \frac{g_{Li}^2 v_L^2}{2} - N_u^2)^2 + 4(N_i \frac{g_{Li} v_L}{\sqrt{2}} + N_u \frac{g_{Ri} v_R}{\sqrt{2}})^2]^{-1} \\ \{(\frac{g_{Li}^2 v_L^2}{2} + N_u^2)(\frac{G_{Li}^2 v_L^2}{2} + N_i^2 - \frac{g_{Li}^2 v_L^2}{2} - N_u^2)^2 + 2(N_i \frac{g_{Li} v_L}{\sqrt{2}} + N_u \frac{g_{Ri} v_R}{\sqrt{2}})^2 \\ (\frac{g_{Li}^2 v_L^2}{2} + N_u^2 + g_{Li} g_{Ri} \frac{v_L v_R}{2} - N_i N_u) - (N_i \frac{g_{Li} v_L}{\sqrt{2}} + N_u \frac{g_{Ri} v_R}{\sqrt{2}})^2 \\ (\frac{G_{Li}^2 v_L^2}{2} + N_i^2 - \frac{g_{Li}^2 v_L^2}{2} - N_u^2)\} \}^{\frac{1}{2}}, \quad (20)$$

$$h_{i12} = [\frac{g_{Li}^2 v_L^2}{2} + N_u^2 - h_{i11}^2]^{\frac{1}{2}}, \quad h_{i22} = -h_{i11} + \frac{1}{h_{i12}} (N_u \frac{g_{Li} v_L}{\sqrt{2}} + N_u \frac{g_{Ri} v_R}{\sqrt{2}}).$$

From (20), the mass of the up quark of the i 's generation m_{ui} and the mass

of the down quark of the i 's generation M_{di} are determined

$$m_{ui} = |\lambda_i^{(-)}|, \quad M_{di} = \lambda_i^{(+)}, \quad (21)$$

$$\lambda_i^{(\pm)} = \frac{1}{2} \{ h_{i11} + h_{i22} \pm [(h_{i11} + h_{i22})^2 + 4h_{i12}^2]^{\frac{1}{2}} \}. \quad (22)$$

At the same time, one can obtain U , and M_i

$$U = M_i H_i^{-1}, \quad M_i = U_i^\dagger \begin{pmatrix} m_{ui} & 0 \\ 0 & M_{di} \end{pmatrix} V_i, \\ V_i = \begin{pmatrix} a_i & b_i \\ b_i & -a_i \end{pmatrix}, \quad U_i = \begin{pmatrix} \frac{1}{m_{ui}} & 0 \\ 0 & \frac{1}{M_{di}} \end{pmatrix} V_i M_i^\dagger, \\ a_i = (h_{i22} - \lambda_i^{(-)}) \left[(h_{i22} - \lambda_i^{(-)})^2 + h_{i12}^2 \right]^{-\frac{1}{2}}, \\ b_i = -h_{i12} \left[(h_{i22} - \lambda_i^{(-)})^2 + h_{i12}^2 \right]^{-\frac{1}{2}}, \\ h_{i11} - \lambda_i^{(+)} = - (h_{i22} - \lambda_i^{(-)}). \quad (23)$$

Obviously, V_i is the mixing matrix which determines the wave mixing between u_{iR} and d_{iR}^c , U_i is the mixing matrix which determines the wave mixing between u_{iL} and d_{iL}^c . It is easy to show that in the case of where $G_{RVR}, G_{LVL} \gg g_{LVL}, g_{RVR}, N, N_u, N_d$, then we have $m_{ui} \ll M_{di}$ which is needed by us to ensure

the possibility that the leptons are the bound states of l quarks and also U_i, V_i are nearly unit matrices to ensure that B violation where B-L is conserved is very weak ^[1].

With the replacements of $G_{LiVL} \rightarrow G_{RiVR}$, $G_{RiVR} \rightarrow G_{LiVL}$ and $N_{di} \rightarrow N_{di}$, the quantities corresponding to down h quark and up l quark can be obtained. In addition, by using the same method the mass spectra and wave mixings of squarks can be determined. Then again, it is easy to see that without soft breaking, each quark and its partner have the same mass, the wave mixing between quarks is the same as that between their partners. But if all of the soft breakings come in, four squarks are mixed.

III. Discussion

As it has been mentioned above that the original theory ^[1] has the difficulty related with a requirement of large number of generations, here, we will show that by the supersymmetrization the situation changes dramatically. For the usual supersymmetric theory of the standard model, at one-loop approximation the β_3 function have been already calculated^[6]

$$\beta_3 = (2N_g - 9) \frac{g^3}{16\pi^2}, \quad (24)$$

N_g is the number of generations. For the supersymmetric theory of the chiral extension of the standard model, the number of quarks and also squarks is doubled, so (24) should be replaced by

$$\beta_3 = (4N_g - 9) \frac{g^3}{16\pi^2}. \quad (25)$$

⁽²⁵⁾ means that if the number of generations is larger than $\frac{9}{4}$, the asymptotic freedom ceases to be effective. This is an interesting result. On the one hand, it tells us that this theory does not contradict with the fact that the experiments have confirmed that there are three generations. On the other hand, if the physical reality had chosen the number of generations to be one or two, we would not have had the opportunity to construct our theory. So, the conclusion is that three generations may not be an occasional coincidence.

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Appendix

In the text, we have used some notations, here are their definitions. For gauge fields, they are

$$A_\mu = \begin{pmatrix} A_{1\mu} \\ A_{2\mu}^i \\ A_{3\mu}^a \end{pmatrix}, \lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2^i \\ \lambda_3^a \end{pmatrix}, D = \begin{pmatrix} D_1 \\ D_2^i \\ D_3^a \end{pmatrix},$$

$$F_{\mu\nu} = \begin{pmatrix} F_{1\mu\nu} \\ F_{2\mu\nu}^i \\ F_{3\mu\nu}^a \end{pmatrix} = \begin{pmatrix} \partial_\mu A_{1\nu} - \partial_\nu A_{1\mu} \\ \partial_\mu A_{2\nu}^i - \partial_\nu A_{2\mu}^i - ig_2 T^{ijk} A_{2\mu}^j A_{2\nu}^k \\ \partial_\mu A_{3\nu}^a - \partial_\nu A_{3\mu}^a - ig_3 T^{abc} A_{3\mu}^b A_{3\nu}^c \end{pmatrix},$$

$$D_\mu \lambda = \begin{pmatrix} \partial_\mu \lambda_1 \\ \partial_\mu \lambda_2^i - ig_2 T^{ijk} A_{2\mu}^j \lambda_2^k \\ \partial_\mu \lambda_3^a - ig_3 T^{abc} A_{3\mu}^b \lambda_3^c \end{pmatrix}, \quad (A1)$$

where i, a are the SU(2) and SU(3) index respectively; T^i, T^a are the adjoint representation of SU(2) and SU(3) respectively; $A_{1\mu}, A_{2\mu}^i$ and $A_{3\mu}^a$ are the gauge fields of U(1), SU(2) and SU(3). λ_1, λ_2^i and λ_3^a are the partner fields of $A_{1\mu}, A_{2\mu}^i$ and $A_{3\mu}^a$. D_1, D_2^i, D_3^a are auxiliary fields of U(1), SU(2) and SU(3).

For quarks and squarks, the following notations are used

$$\begin{aligned} A_L &= \begin{pmatrix} A_L \\ A_{LD}^c \\ A_{LU}^c \end{pmatrix}, \quad \Psi_L = \begin{pmatrix} \psi_L \\ \psi_{LD}^c \\ \psi_{LU}^c \end{pmatrix}, \quad F_L = \begin{pmatrix} F_L \\ F_{LD}^c \\ F_{LU}^c \end{pmatrix}, \\ A_R &= \begin{pmatrix} A_R^c \\ A_{RU} \\ A_{RD} \end{pmatrix}, \quad \Psi_R = \begin{pmatrix} \psi_R^c \\ \psi_{RU} \\ \psi_{RD} \end{pmatrix}, \quad F_R = \begin{pmatrix} F_R^c \\ F_{RU} \\ F_{RD} \end{pmatrix}; \\ \Lambda &= \begin{pmatrix} \Lambda_1 \\ \Lambda_2^i \\ \Lambda_3^a \end{pmatrix}, \quad \Lambda_1 = \frac{g_1}{2} Y = \frac{g_1}{2} \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{2}{3} \end{pmatrix}, \\ \Lambda_2^i &= \frac{g_2}{2} \begin{pmatrix} \tau^i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Lambda_3^a = \frac{g_3}{2} \begin{pmatrix} \lambda^a & 0 & 0 \\ 0 & \lambda^a & 0 \\ 0 & 0 & \lambda^a \end{pmatrix}, \\ \widehat{A}_\mu &= \Lambda_1 A_{1\mu} + \Lambda_2^i A_{2\mu}^i + \Lambda_3^a A_{3\mu}^a, \quad \widehat{D} = \Lambda_1 D_1 + \Lambda_2^i D_2^i + \Lambda_3^a D_3^a, \\ \widehat{\lambda} &= \Lambda_1 \lambda_1 + \Lambda_2^i \lambda_2^i + \Lambda_3^a \lambda_3^a, \quad D_\mu = \partial_\mu + i \widehat{A}_\mu. \end{aligned} \quad (A2)$$

For Higgs fields, we define

$$\Lambda = \begin{pmatrix} \Lambda_1 \\ \Lambda_2^i \end{pmatrix}, \quad \Lambda_1 = \frac{g_1}{2}, \quad \Lambda_2^i = \frac{g_2}{2} \tau^i, \quad \widehat{A}_\mu = \Lambda_1 A_{1\mu} + \Lambda_2^i A_{2\mu}^i, \\ \widehat{\lambda} = \Lambda_1 \lambda_1 + \Lambda_2^i \lambda_2^i, \quad \widehat{D} = \Lambda_1 D_1 + \Lambda_2^i D_2^i, \quad D_\mu = \partial_\mu + i \widehat{A}_\mu. \quad (A3)$$

For soft breaking, we define

$$\begin{aligned} M_L^2 &= \begin{pmatrix} m_L^2 & 0 & 0 & 0 \\ 0 & m_L^2 & 0 & 0 \\ 0 & 0 & m_{LD}^2 & 0 \\ 0 & 0 & 0 & m_{LU}^2 \end{pmatrix}, \quad M_R^2 = \begin{pmatrix} m_R^2 & 0 & 0 & 0 \\ 0 & m_R^2 & 0 & 0 \\ 0 & 0 & m_{RU}^2 & 0 \\ 0 & 0 & 0 & m_{RD}^2 \end{pmatrix}, \\ M_{LR} &= \begin{pmatrix} m_{LR} & 0 & 0 & 0 \\ 0 & m_{LR} & 0 & 0 \\ 0 & 0 & m_{LRU} & 0 \\ 0 & 0 & 0 & m_{LRD} \end{pmatrix}, \quad \mu' = \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2^i & 0 \\ 0 & 0 & \mu_3^a \end{pmatrix}. \end{aligned} \quad (A4)$$

Below is the expression of the mass term of squarks in the presence of soft breaking

$$L_{\text{mass}}^{\tilde{q}} = - \begin{pmatrix} A_{LU} \\ A_{LD}^c \\ A_{RU} \\ A_{RD}^c \end{pmatrix}^\dagger \begin{pmatrix} W_u & S_u \\ S_u^\dagger & T_u \end{pmatrix} \begin{pmatrix} A_{LU} \\ A_{LD}^c \\ A_{RU} \\ A_{RD}^c \end{pmatrix}$$

$$\begin{pmatrix} A_{LD} \\ A_{LU} \\ A_{RD} \\ A_{RU} \end{pmatrix} \begin{pmatrix} W_d & S_d \\ S_d^\dagger & T_d \end{pmatrix} \begin{pmatrix} A_{LD} \\ A_{LU} \\ A_{RD} \\ A_{RU} \end{pmatrix} \quad (A5)$$

$$\begin{aligned} W_u &= \begin{pmatrix} m_L^2 + NN^\dagger + \frac{g_L^2 v^2}{2} & NK^\dagger G_R \frac{v}{\sqrt{2}} + g_L N_u^\dagger \frac{v}{\sqrt{2}} \\ G_R KN^\dagger \frac{v}{\sqrt{2}} + N_u g_L \frac{v}{\sqrt{2}} & m_D^2 + N_u N_u^\dagger + \frac{G_L^2 v^2}{2} \end{pmatrix}, \\ W_d &= \begin{pmatrix} m_L^2 + NN^\dagger + k g_R^2 k^\dagger \frac{v^2}{2} & N G_L \frac{v}{\sqrt{2}} + k g_R N_d^\dagger \frac{v}{\sqrt{2}} \\ G_L N^\dagger \frac{v}{\sqrt{2}} + N_d g_R k^\dagger \frac{v}{\sqrt{2}} & m_U^2 + N_d N_d^\dagger + \frac{G_L^2 v^2}{2} \end{pmatrix}, \\ T_u &= \begin{pmatrix} m_{RU}^2 + N_u^\dagger N_u + \frac{g_L^2 v^2}{2} & g_L N \frac{v}{\sqrt{2}} + N_u^\dagger G_R K \frac{v}{\sqrt{2}} \\ N^\dagger g_L \frac{v}{\sqrt{2}} + K^\dagger G_R N_u \frac{v}{\sqrt{2}} & m_R^2 + N^\dagger N + K^\dagger G_R^2 K \frac{v^2}{2} \end{pmatrix}, \\ T_d &= \begin{pmatrix} m_{RD}^2 + N_d^\dagger N_d + \frac{g_R^2 v^2}{2} & -g_R k^\dagger N \frac{v}{\sqrt{2}} - N_d^\dagger G_L \frac{v}{\sqrt{2}} \\ -N^\dagger k g_R \frac{v}{\sqrt{2}} - G_L N_d \frac{v}{\sqrt{2}} & m_R^2 + N^\dagger N + \frac{G_L^2 v^2}{2} \end{pmatrix}, \\ S_u &= \begin{pmatrix} \mu^* \frac{g_L v}{\sqrt{2}} & m_{LR} \\ m_{LRU} & \mu^* G_R K \frac{v}{\sqrt{2}} \end{pmatrix}, \quad S_d = \begin{pmatrix} \mu^* k g_R \frac{v}{\sqrt{2}} & -m_{LR} \\ m_{LRD} & -\mu^* G_R \frac{v}{\sqrt{2}} \end{pmatrix}. \quad (A6) \end{aligned}$$

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