Phenomenological constraints on a scale-dependent gravitational coupling

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We investigate the astrophysical and cosmological implications of the recently proposed idea of a running gravitational coupling on macroscopic scales. We find that when applied to the rotation curves of galaxies, their flatness requires still the presence of dark matter. Bounds on the variation of the gravitational coupling from primordial nucleosynthesis, change of the period of binary pulsars, gravitational lensing and the cosmic virial theorem are analysed.

1. INTRODUCTION

The flatness of the rotation curves of galaxies and the large structure of the Universe indicate that either the Universe is predominantly made up of dark matter of exotic nature, i.e. non-baryonic, and/or that on large scales gravity is distinctively different from that on solar system scales, where Newtonian and post-Newtonian approximations are valid. The former possibility has been thoroughly investigated (see Ref. [1] for a review) and is an active subject of research in astroparticle physics. The second possibility, however relevant, has drawn less attention. This is essentially due to the fact that until recently no consistent and appealing modification of Newtonian and post-Newtonian dynamics has been put forward. Many of these attempts [2], although consistent with observations, were most often unsatisfactory from the theoretical point of view. Actually, it has been recently shown that under certain fairly general conditions it is unlikely that relativistic gravity theories can explain the flatness of the rotation curves of galaxies [3]. These conditions however do not exclude the class of generalizations of General Relativity that involve higher-derivatives. Quantum versions of these theories were shown to exhibit asymptotic freedom in the gravitational coupling [4] and one would expect this property to manifest itself on large scales. This possibility would surprisingly imply that quantum effects could mimic the presence of dark matter [5], as well as induce other cosmological phenomena [6, 7]. One striking implication of these ideas is the prediction [6, 7] that the power spectrum on large scales would have more power than the one predicted by the $\Omega = 1$ Cold Dark Matter (CDM) Model, in agreement with what is observed by IRAS [8]. Furthermore, due to the increase in the gravitational constant on large scales one finds that the energy density fluctuations grow quicker than in usual matter dominated Friedmann-Robertson-Walker models [6, 7]. Moreover, one can explain with a scale-dependent $G$ the discrepancy between determinations of the Hubble’s parameter made at different scales, as suggested in [6], and studied in [9].

Nevertheless, independently of the possible running of the gravitational constant in a higher derivative theory of gravity, it is worthwhile analysing the constraints on the scale-dependence of $G$ from astrophysical and cosmological phenomena, where such an effect would be dominant. On the other hand, in the last few years there has been a revival of Brans-Dicke like theories, with variable gravitational coupling, that has led to a number of constraints on possible time variations of $G$. Of course, some of the constraints on $\dot{G}$ can be written as constraints on $\Delta G$ over scales in which a graviton took a time $\Delta t$ to propagate. For instance, during nucleosynthesis the largest distance that a graviton could have tra-

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versed is the horizon distance at that time, i.e. a few light-seconds to a few light-minutes, approximately the Earth-Moon distance. Such a distance is too small for quantum effects to become appreciable, as we discuss below. However, those effects become important at kiloparsec (kpc) distances and therefore could be relevant for discussing the rotation curves of galaxies. We shall actually show, for a particular theory [5, 6], that the rotation curves of spiral galaxies cannot be entirely explained by the running of G, so some amount of dark matter is required, which is still consistent with the upper bound on baryonic matter coming from primordial nucleosynthesis. On the other hand, we could impose bounds on a possible variation of G from a plethora of cosmological and astrophysical phenomena at large scales, although the lack of precise observations at those scales make the bounds rather weak [10]. Of course, a difficulty in examining constraints on the variation of G is that in all gravitational phenomena the gravitational coupling appears in the factor GM, and hence we cannot distinguish a variation in G from the existence of some type of dark matter.

2. ASYMPTOTIC FREEDOM OF THE GRAVITATIONAL COUPLING

The main idea behind the results of Refs. [5, 6] is the scale dependence of the gravitational coupling. The inspiration for this comes from the property of asymptotic freedom exhibited by 1-loop higher-derivative quantum gravity models [4]. Since there exists no screening mechanism for gravity, asymptotic freedom may imply that quantum gravitational effects act on macroscopic and even on cosmological scales, a fact which has of course some bearing on the dark matter problem [5] and on the large scale structure of the Universe [6, 7]. It is in this framework that a power spectrum which is consistent with the observations of IRAS [8] and COBE [11] can be obtained [6, 7].

We briefly outline this proposal. Removing the infinities generated by quantum fluctuations and ensuring renormalizability of a quantum field theory requires a scale-dependent redefinition of the physical parameters. Furthermore, the removal of those infinities still leave the physical parameters with some dependence on finite quantities whose particular values are arbitrary. These can be assigned by specifying the value of the physical parameters at some momentum or length scale; once this is performed, variations on scale are accounted for by appropriate changes in the values of the physical parameters via the renormalization group equations (RGEs). Thus, the equations of motion in the quantum field theory of gravity should be similar to the ones of the classical theory, but with their parameters replaced by the corresponding ‘improved’ values, that are solutions of the corresponding RGEs. However, since gravity couples coherently to matter and exhibits no screening mechanism, quantum fluctuations of the gravitational degrees of freedom contribute on all scales. One must therefore include the effect of these quantum corrections into the gravitational coupling, G, promoting it into a scale-dependent quantity. One-loop quantum gravity models indicate that the coupling G(μ²/μ²) is asymptotically free, where μ is a reference momentum, meaning that G grows with scale [4]. A typical solution for G(μ²/μ²) was obtained in Ref. [5] setting the β-functions of matter to vanish and integrating the remaining RGEs:

\[ G(\mu^2/\mu^2) = G_{lab} \delta(r, r_{lab}) , \]  

where G_{lab} is the value of G measured in the laboratory at a length scale r_{lab}, and \δ(r, r_{lab}) is a growing function of r. In order for the asymptotic freedom of G(μ²/μ²) to have an effect on for instance the dynamics of galaxies and their rotation curves, the function \δ(r, r_{lab}) should be close to one for r < 1 kpc, growing significantly only for r ≥ 1 kpc. A convenient parametrization for \δ(r, r_{lab}) from the fit of Ref. [5] in the kpc range is the following:

\[ \delta(r, r_{lab}) = 1.485 \left[ 1 + \beta \left( \frac{r}{r_0} \right)^\gamma \ln \left( \frac{r}{r_0} \right) \right] , \]  

where \beta ≃ 1/30, \gamma ≃ 1/10 and r_0 = 10 kpc.

We mention that a scale dependence of the gravitational constant also arises from completely
different reasons in the so-called stochastic inflation formalism [12] and that the scaling behaviour and screening of the cosmological constant was also discussed in the context of the quantum theory of the conformal factor in four dimensions [13].

In what follows we shall use the fit (1),(2) in our analysis of the rotation curves of galaxies, and extract a prediction for the distribution of dark matter. However, before we pursue this discussion let us present some of the ideas developed in Refs. [5-7]. As discussed above, the classical equations have to be ‘improved’ by introducing the scale dependence of the gravitational coupling. This method suggests that the presence of cosmological dark matter could be replaced by an asymptotically free gravitational coupling. Assuming that the Friedmann equation describing the evolution of a flat Universe is the improved one, then:

\[ H^2(\ell) = \frac{8\pi}{3} G \left( \frac{a^2_0 \ell^2}{a^2 \ell^2} \right) \rho_m, \]  

where \( a = a(t) \) is the scale factor, \( H = \dot{a}/a \) is the Hubble parameter, \( \rho_m \) is the density of matter, \( \ell \) is the comoving distance and \( \ell_* \) is some convenient length scale.

From Eq. (2) one sees that the present physical density parameter, \( \Omega_{\text{phys}}^0 \), is by construction equal to one. However, the quantity which is usually referred to as density parameter is actually:

\[ \Omega_0 = \frac{8\pi}{3} G \rho_{m0}, \]  

where \( H_0 \) is the present Hubble parameter for a given large scale distance, \( r = r_* \). This leads to \( \Omega_0 \) as a growing function of scale, which is in agreement with observations for a constant \( \rho_{m0} \).

Furthermore, from Eq. (3) one can clearly see the scale dependence of the Hubble parameter [6, 7, 9]. Moreover, as shown in Refs. [6] and [7], the power spectrum resulting from these considerations is similar to that of a low density Cold Dark Matter model with a large cosmological constant [14].

### 3. Rotation Curves of Galaxies

Let us now turn to the discussion of the implications of the fit (2) for the rotation curves of galaxies. It is a quite well established observational fact that the rotation curves of spiral galaxies flatten after about 10 to 20 kpc from their centre, which of course is a strong dynamical evidence for the presence of dark matter and/or of non-Newtonian physics. The rotation velocity of the galaxy is given by the non-relativistic relation,

\[ v^2 = \frac{G(r)M(r)}{r}, \]  

which approaches a constant value some distance from the centre, e.g. \( v_0^2 = 220 \text{ km/s} \) for the Milky Way. Assuming that the gravitational coupling is precisely Newton’s constant \( G_N \) and imposing that the rotation velocity is constant, using the Virial Theorem at \( r = R \equiv 500 \text{ kpc} \), one finds the standard expression for the mass distribution of dark matter:

\[ M_N(r) = M_N(R) \frac{r}{R}. \]  

Assuming instead a running gravitational coupling (2), the condition that the rotation velocity is constant yields:

\[ M(r) = \frac{0.673}{1 + \beta\left( \frac{r}{r_0} \right)^\gamma \ln \left( \frac{r}{r_0} \right)} M_N(r). \]  

Equation (7) reveals after simple computation that the running of the gravitational coupling reduces the amount of dark matter required to explain the flatness of the rotation curves of galaxies by about 44%, assuming that galaxies stretch up to a distance of about 500 kpc. This result [10] (see also Ref. [15]) is a clear prediction of the dependence of the gravitational coupling with scale and, in particular, of the fit (2). Furthermore, since the possibility that the Galactic halo is entirely made up of baryonic dark matter is barely consistent with the nucleosynthesis bounds on the amount of baryons [16], the running of \( G \) is quite welcome since it reduces the required amount of baryonic dark matter in the halo. An entertaining hypothesis could be that precisely this effect is responsible for the reduction in the microlensing event rates across the halo in the direction of
the Large Magellanic Cloud [17, 18] with respect to those along the bulge of our galaxy, as reported by [19].

4. BOUNDS ON THE VARIATION OF $G$ WITH SCALE

In this section we constrain the variation of the gravitational coupling given by the fit (2) with bounds from primordial nucleosynthesis, binary pulsars and gravitational lensing and also discuss the effect that a scale-dependent $G$ has on the peculiar velocity field [10].

4.1. Primordial nucleosynthesis

As mentioned in the introduction, one could obtain bounds on the variation of the gravitational coupling from observations of the light elements' abundances in the Universe. Such observations are in agreement with the standard primordial nucleosynthesis scenario, but there is still some room for variations in the effective number of neutrinos, the baryon fraction of the universe and also in the value of the gravitational constant. For instance, the predicted mass fraction of primordial $^4$He can be parametrised, in theories with a variable gravitational coupling, in the following way [20],

$$Y_p = 0.228 + 0.010 \ln \eta_{10} + 0.327 \log \xi,$$

where $\eta_{10}$ is the baryon to photon ratio in units of $10^{-10}$ and $\xi$ is the ratio of the Hubble parameter at nucleosynthesis and its present value, itself proportional to the square root of the corresponding gravitational constant. In the fit (8) it is assumed that the effective number of light neutrinos is $N_\nu = 3$ and that the neutron lifetime is $\tau_n = 887$ seconds.

By running the nucleosynthesis codes for different values of $G$, it was shown in Ref. [21] that a variation of $\Delta G/G = 0.2$ on the values of the gravitational coupling was compatible with the observations of the primordial $D$, $^3He$, $^4He$ and $^7Li$ abundances at 1σ level.

This result will now be used to constrain the running of $G$ in an asymptotically free theory of gravity. In a theory with a scale-dependent gravitational constant, the maximum value of $G$ at a given time is the one that corresponds to the physical horizon distance at that time. During primordial nucleosynthesis, the horizon distance grows from a few light-seconds to a few light-minutes, i.e. less than a few milliparsecs. At that scale we find $\Delta G/G = 0.07$, see Eq. (2), which is much less than the allowed variation of $G$ given in [21]. Therefore, primordial nucleosynthesis does not rule out the possibility of an asymptotically free gravitational coupling. Of course, a light-second is about the distance to the Moon, and there are similar constraints on a variation of $G$ at this scale coming from lunar laser ranging, $\Delta G/G < 0.6$ [22].

4.2. Binary pulsars

The precise timing of the orbital period of binary pulsars and, in particular, of the pulsar PSR 1913+16, provides another way of obtaining a model-independent bound on the variation of the gravitational coupling [23]. Since the semimajor axis of that system is just about a few light-seconds, the resulting limits on the variation of $G$ can be readily compared with the ones arising from nucleosynthesis. The observational limits on the rate of change of the orbital period, mainly due to gravitational radiation damping, together with the knowledge of the relevant Keplerian and post-Keplerian orbiting parameters, allows one to obtain the following limit [23]:

$$\sigma \equiv \frac{\Delta G}{G} < 0.08 \, h^{-1},$$

where $h$ is the value of the Hubble parameter in units of 100 km/s/Mpc. For $h = 0.8$, [24] one obtains $\sigma = 0.1$ which is more stringent than the nucleosynthesis bound, but is still compatible with the fit (2).

4.3. Gravitational lensing

Gravitational lensing of distant quasars by intervening galaxies may provide, under certain assumptions, yet another method of constraining, on large scales, the variability of the gravitational coupling. The four observable parameters associated with lensing, namely, image splittings, time delays, relative amplification and optical depth do depend on $G$, more precisely on the product $GM$, where $M$ is the mass of the lensing object. This
dependence might suggest that limits on the variability of $G$ could not be obtained before an independent determination of the mass of the lensing object. However, as the actual bending angle is not observed directly, the relevant quantities are the distance of the lensing galaxy and of the quasar. Since these quantities are inferred from the redshift of those objects, they depend on their hand on $G$, on the Hubble constant, $H_0$, and on the density parameter, $\Omega_0$. However, as we have previously seen, a scale-dependent gravitational coupling implies also a dependence on scale of $H_0$ and $\Omega_0$, see Eqs. (3) and (4). This involved dependence on scale makes it difficult to proceed as in Ref. [25], where gravitational lensing in a flat, homogeneous and isotropic cosmological model, in the context of a Brans-Dicke theory of gravity, was used to provide a limit on the variation of $G$:

$$\frac{\Delta G}{G} = 0.2 . \quad (10)$$

Since for this limit $\Omega_0 = 1$ was assumed, while in a scale-dependent model it is achieved via the running of the gravitational coupling, the bound (10) constrains only residual variations of $G$ that have not been already taken into account when considering the dependence on scale of $H_0$ and $\Omega_0$. Of course, for models where the cosmological parameters are independent of scale, the bound (10) can be readily used to constrain the variability of $G$ on intermediate cosmological scales. It is worth stressing that this method, besides being one of the few available where this variability is directly constrained at intermediate cosmological times between the present epoch and the nucleosynthesis era, it is probably the only one which can realistically provide in the near future even more stringent bounds on even larger scales by observing the lensing of light from far away quasars caused by objects at redshifts of order $z \geq 1$.

4.4. Peculiar velocity field

Since we expect the effects of a running $G$ to become important at very large scales, one could try to explore distances of hundreds of Mpc, where the gravitational coupling is significantly different from that of our local scales. That is the realm of physical cosmology where of particular importance is the study of the peculiar velocity field. A possible signature of the running of $G$ would be a mismatch between the velocity fields and the actual mass distribution, such that at large scales the same mass would pull more strongly. To be more specific, in an expanding universe there is a relation between the kinetic and gravitational potential energy of density perturbations known as the Layzer-Irvine equation (see eg. Ref. [26]) that can be written as a relation between the mass-weighted mean square velocity $\bar{v}^2$ and the mass autocorrelation function $\xi(r)$,

$$\bar{v}^2(r) = 2\pi G \rho_0 J_2(r) , \quad (11)$$

where $\rho_0$ is the mean local mass density and $J_2(r) = \int_0^r r dr \xi(r)$. The galaxy-galaxy correlation function can be parametrized by $\xi(r) \sim (r/r_0)^{-1.8}$ with $r_0 = 5h^{-1}$ Mpc, while the cluster-cluster correlation function has the same expression with $r_0 = 20h^{-1}$ Mpc. This means that the velocity field (11) should be proportional to $(r/r_0)^{0.2}$, unless the gravitational constant has some scale dependence. So far the relation seems to be satisfied, under rather large observational errors (see Ref. [27] for a review). Unfortunately, the errors are so large that it would be premature to infer from this a scale dependence of $G$. Even worse, phenomenologically there is a proportionality constant between the galaxy-galaxy correlation function and the actual mass correlation function, the so-called biasing factor, which is supposed to be scale dependent and could mimic a variable gravitational constant. However, future sky surveys might be able to constrain more strongly the relation (11) by measuring peculiar velocities with better accuracy at larger distances and it might then be possible to extract the scale-dependence of $G$.

5. CONCLUSIONS

We have seen that the running of the gravitational coupling is compatible with the observational fact that the rotation curves of galaxies are constant provided some amount of baryonic dark matter is allowed, actually about 44% less than what is required for a constant $G$. This
could also explain why we see less microlensing events towards the halo than in the direction of the bulge of our galaxy. Failure in reproducing the predicted distribution of baryonic dark matter would signal either that the approach adopted here is unsuitable or that the fit (2) is inadequate. We have looked for possible bounds on variations of \( G \) with scale from primordial nucleosynthesis, variations in the period of binary pulsars, macroscopic gravitational lensing and deviations in the peculiar velocity flows. Unfortunately, as observational errors tend to increase with the scale probed, we cannot yet seriously constrain an increase of \( G \) with scale, as proposed by the asymptotically free theories of gravity.

REFERENCES