Collective Mechanism of Cooling—Damping Resistance in 2 D Pairs and Plasmons of Crystal Absorbers for Muon Beams

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The conception of cooling resistance R_c and damping resistance R_d are introduced. The explicit formulations of energy spread $\langle \Delta E^2 \rangle_m$ and emittance ϵ_0 of muon beam are deduced as a function of elementary excitation in absorbers. Both are the monotone increasing functions in the range of optimum cooling and damping (0.5—5 GeV) where pairs and plasmons are the dominate excitations in crystal. The excitation energy of plasmons is above pairs which distribute in $0 \rightarrow \hbar\omega_{\pi}^{max}$. Thus, plasmons are the main ressitans of cooling and damping.

So decreasing the quantum energy of plasmon can decrease energy spread and emittance of muon beam. A conception of multi-layer 2D plasmon absorber is suggested to strengthening the effects of cooling and damping.

I. Introduction

A. N. Skrinsky's early proposal of muon collider using G. Budker's idea of ionization cooling and damping [1] has been relivened up since that the complete scenario of 0.2—2 TeV particle factory was presented by D. Neuffer and R. Palmer in 1994. [2, 3]

Muon storage rings have been researching and developing for muon $g-2$ experiments by AGS at BNL. [4] Ionization cooling and damping of muons is often mentioned in connection with muon storage rings, because muon beams are produced from the decay of pions and have the large phase—space area.

The optimum cooling and damping region obtained by Neuffer [2] and simulated by Van Ginneken is 0.5—5 GeV. [5] In this energy region the electric field of muon beam flattens and extends, so that the distant—collision contribution to energy loss increase as $\ln \beta\gamma$ by relativistic effect. Hence, the collective plasmon excitation in crystal by distant—collision is significant. The high quantum energy of plasmon makes energy loss and its rate of muon energy decrease. It is a resistance of cooling and damping. Hence, it should be decreased for strengthening cooling and damping.

II. Collective Effect of Energy Loss in Muon Cooling and Damping

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In Neuffer's original work, the elementary excitation of energy loss is considered as an average energy of ionization I . The energy loss is considered as [2]

$$-\frac{dE}{dz} = K \frac{Z}{A} \frac{1}{\beta^2} \left[\ln \left(\frac{2m_e c^2 \beta^2 \gamma^2}{I} \right) - \beta^2 \right] \quad (2.1)$$

$$K = 4\pi N_A r_e^2 m_e c^2 = 0.307075 \text{ [MeV g}^{-1} \text{cm}^2] \quad (2.2)$$

$$\frac{dE}{dz}: \text{ [MeV g}^{-1} \text{cm}^2], \quad A: \text{ [g mol}^{-1}],$$

where I is simply the average ionization energy in the range of $0.5\text{GeV} < E_\mu < 5\text{GeV}$ which is the optimum muon energy range of cooling and damping. It is true, because in very low energy excitation of phonon and very high energy excitation of radiation are outside this optimum muon energy range, the ionization is a local effect, but in crystal the collective effects of pairs and plasmons will be actually happen. The quasi particle representation of elementary excitation is resulted from the fact that the valence electrons in crystal are mobile and interacting each other on the compensated charge background of ion lattice.

These kinds of electron system certainly have density fluctuations, and the interaction between electrons is long range Coulomb interaction. Hence, even if any micro-fluctuation of local density is excited by a penetrating muon, it will surely influence the distribution of whole electron system. Therefore, the elementary excitation of density vibration resulted by muon appears as a collective effect. Thus, (2.1) should be replaced by a new formula including the density effects.

Therefore the general formula of energy loss is

$$-\frac{dE}{dz} = K \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \left(\frac{2m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} \right) - \beta^2 - \frac{\delta}{2} \right] \quad (2.3)$$

where δ denotes the density effect correction

$$\frac{\delta}{2} \rightarrow \ln \frac{\hbar \omega_p}{I} + \ln \beta \gamma - \frac{1}{2} \quad (E_\mu > 0.5\text{GeV}) \quad (2.4)$$

$T_{\max}$ denotes the maximum kinetic energy which can be imparted to a free electron in a single collision

$$T_{\max} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma \frac{m_e}{m_\mu} + \left(\frac{m_e}{m_\mu}\right)^2} \quad (2.5)$$

Hence, for $E_\mu > 0.5 \text{ GeV}$

$$-\frac{dE}{dz} = K \frac{Z}{A} \frac{\gamma^2}{\gamma^2 - 1} \left[\frac{1}{2} \ln \frac{(2m_e c^2)^2 (\gamma^2 - 1)}{[1 + 2\gamma \frac{m_e}{m_\mu} + (\frac{m_e}{m_\mu})^2] (I)^2} + \frac{1}{2} \right] \quad (2.6)$$

where I is the energy of pair (lower I) and plasmon (higher I).

III. Cooling Resistance and Elementary Excitations

Let I denote the elementary excitation of absorber, and $\langle \Delta E_\mu^2 \rangle_{\text{rms}}$ denotes the equilibrium energy spread of cooling muon. We define the resistance of cooling

$$R_c \equiv \frac{d}{dI} \langle \Delta E_\mu^2 \rangle_{\text{rms}} \quad (3.1)$$

If we can prove that $R_c > 0$ in the optimum energy range of cooling, then a more powerful cooling can be obtained by decreasing cooling I .

The equilibrium energy spread in (3.1) in ionization cooling accomplished with random fluctuation heating has been obtained by Neuffer based on the average ionization energy I [2]

$$\langle \Delta E^2 \rangle_{\text{rms}} = \frac{1}{2} E_{\text{cool}} \quad (3.2)$$

$$E_{\text{cool}} = n_c \left(-\frac{dE}{dz} \right) \delta(0) \quad (3.3)$$

$$n_c = \frac{1}{\left[\frac{\partial}{\partial E_\mu} \left(-\frac{dE}{dz} \right) \delta(0) + \frac{\eta}{E_0} \left(-\frac{dE}{dz} \right) \frac{d\delta}{dx} \right]} \quad (3.4)$$

where E_0 is the central energy of muon beam, E_{cool} is the total acceleration energy to obtain e^{-1} cooling. n_c is the cooling cycles. $\delta(0)$ is the minimum penetrating thickness (width) of crystal absorber. $\frac{d\delta(x)}{dx}$ is the gradient of absorber width in x direction which is also the direction of bunch width σ_x . y is the direction of bunch height σ_y , and z is the direction of bunch length and beam. η is the value of Courant-Snyder dispersion function of absorber. Now substituting (2.6) into (3.3) and (3.4) and furtherly substituting this result into (3.2). in the cases of

$$\left(\frac{m_e}{m_\mu} \right)^2 \ll 1, \quad \gamma^4 \equiv \left(\frac{E_\mu}{m_e c^2} \right)^4 \gg 1 \quad (3.5)$$

(3.2) is approximately

$$\langle \Delta E^2 \rangle_{\text{rms}} = \frac{l m_\mu c^2}{B} \quad (3.6)$$

where

$$B \equiv 2\gamma + \frac{\gamma}{A} + \frac{\eta}{E_0} \frac{1}{\delta(0)} \frac{d\delta(x)}{dx} \quad (3.7)$$

$$A \equiv \ln \frac{2m_\mu c^2}{I} + \frac{1}{2} \ln \frac{(\gamma^2 - 1)}{1 + 2\gamma \frac{m_e}{m_\mu}} \quad (3.8)$$

If $0.5 \text{ GeV} < E_\mu < 5 \text{ GeV}$, $l < 1 \text{ KeV}$, substituting (3.6), (3.7), (3.8) into (3.1), then

$$R_e = \frac{d \langle \Delta E_\mu^2 \rangle_{\text{rms}}}{dI} = \frac{m_\mu c^2}{B} \left[1 - \frac{\gamma}{A^2 B} \right] \approx \frac{M_\mu c^2}{B} > 0 \quad (3.9)$$

(3.9) proves that the energy spread of cooled muon beam is a monotone increasing function of elementary excitation.

Thus decreasing the elementary excitation of absorbers l can decreasing the equilibrium energy spread $\langle \Delta E_\mu^2 \rangle_{\text{rms}}$ in muon cooling.

IV. Transvers Damping Resistance and Elementary Excitations

The unnormalized emittance of equilibrium damping bunch.

$$\varepsilon_0 = \frac{\beta}{2} \frac{14^2}{E_\mu \left(-\frac{dE}{dz} L_R \right)} \quad E_\mu [\text{MeV}] \quad (4.1)$$

where

$$L_R = \frac{716.4 \text{ gcm}^{-2} \text{A}}{Z(Z+1) \ln(287/\sqrt{A})} \quad (4.2)$$

Define transverse damping resistance R_e ,

$$R_e \equiv \frac{d\varepsilon_0}{dI} \quad (4.3)$$

Substituting (4.1) into (4.3),

$$R_1 = \frac{\beta}{2} \frac{14^2}{E_s L_R} \left(-\frac{dE}{dz} \right)^{-2} K \frac{Z}{A} \frac{\gamma^2}{\gamma^2 - 1} \frac{1}{I} > 0 \quad (4.4)$$

(4.4) proves that the emittance of damped muon beam is a monotone increasing function of elementary excitation. Hence, decreasing elementary excitation I of absorbers can decrease emittance ϵ_0 .

V. Collective Mechanism of Elementary Excitation of 3D Pairs and Plasmon

In the picture of elementary excitation, the I in section II, III, and IV is composed of quantum energy sets of pairs $\{\hbar\omega_{k,q}\}$ and plasmons $\{\hbar\omega_q\}$, which can be calculated by Kubo's linear response approximation of Heisenberg quantum mechanic equation of electron systems with collective effect excited by an external impulse of muon. [6]

The results are:

$$\hbar\omega_{k,q}^{\text{pair}} < \{I = \hbar\omega_{k,q}\} < \hbar\omega_{k,q}^{\text{pair}} \quad (5.1)$$

where

$$\hbar\omega_{k,q}^{\text{pair}} = \begin{cases} 0 & (q < 2k_F) \\ \frac{1}{2m}(q^2 - 2k_F q) & (q \geq 2k_F) \end{cases} \quad (5.2)$$

$$\hbar\omega_{k,q}^{\text{pair}} = \frac{1}{2m}(q^2 + 2k_F q) \quad (5.3)$$

where q is wave vector of quasi-particles of pair. k_F is Fermi wave vector.

$$I = \hbar\omega_q^{3D} = \hbar\omega_p^{3D} + \hbar \frac{3V_F}{10\omega_p^{3D}} q^2 > \hbar\omega_{k,q}^{\text{pair}} \quad (5.4)$$

is plasmon energy in ordinary 3 dimensional absorbers, where V_F is Fermi velocity,

$$\hbar\omega_p^{3D} = \hbar \sqrt{\frac{4\pi N_{3D} e^2}{m}} = 28.816 \sqrt{\rho \frac{Z}{A}} \quad (5.5)$$

is the first order of plasmon energy, where N_{3D} is the 3D density of valence electrons, ρ is the density and the second term in (5.4) is the correction of fermi surface charactor of crystals.

The metal absorber parameters about 3D plasmons are following:

Valence	1	2	3	4	
Metal	Cu	Be	Al	Pb	
$N_{3D}(\text{cm}^{-3})$	8.45	24.2	18.06	13.20	$\times 10^{22}$
$r_s(a_H)$	2.67	1.88	2.07	2.30	
K_F Fermi Vector(cm^{-1})	1.36	1.93	1.75	1.57	$\times 10^8$
V_F Fermi Velocity(cm s^{-1})	1.57	2.23	2.02	1.82	$\times 10^8$
E_F Fermi Energy(eV)	7.00	14.14	11.63	9.37	
T_F Fermi Temperature ($^{\circ}\text{K}$)	8.12	16.41	13.49	10.87	$\times 10^4$

VI. 2D Plasmon Excitation

The problem is that the elementary excitation, $I = \hbar\omega_i^{3D}$ of (5.4) has an energy gap of $\hbar\omega_i^{3D}$ (5.5) which is higher than $\hbar\omega_{i,q}^{\text{max}}$. It makes the energy of elementary excitation increase, and the energy loss of muon decrease in the range of $E_\mu > 0.5 \text{ GeV}$. [7, 8]

The 2D plasmon devices can be carried out by multi-layer sandwich of GaAs/GaAlAs. The area is about 2 inches, The thickness is $1 \mu\text{m}$ which composed of 50 layers of 10 nm GaAs layer and 10 nm GaAlAs layer. Every layer of GaAs + GaAlAs is a 2D plasmon layer. They have the much lower elementary excitation energy. It can be used to cool and damp a flat muon beam of $1 \mu\text{m}$ height which is 10^3 times of the NLC beam height. [9] Hence, it is easier to focus.

In 2D plasma there is no energy gap. And the equivalent $\hbar\omega_i^{3D}$ is

$$\hbar\omega_i^{3D} = \hbar\omega_i^{2D} \sqrt{q} = \hbar \sqrt{\frac{4\pi N_{2D} q e^2}{m_e}} \quad (6.1)$$

where N_{2D} is 2 dimension density.

$$N_{2D} q \approx 10^{12} \text{cm}^{-2} \times 8 \text{cm}^{-1}$$

hence

$$I \equiv \hbar\omega_i^{2D} = \hbar\omega_i^{2D} \sqrt{q} \approx \frac{1}{10} \hbar\omega_i^{3D} \quad (6.2)$$

Now we obtain an interesting result that the quantum energy of elementary excitation $I \equiv \hbar\omega_i^{2D}$ in 2D plasmon absorbers (6.2) is proved as much lower than that $I \equiv \hbar\omega_i^{3D}$ of 3D plasmon absorber (5.4).

Hence, according to the positive resistance of cooling and damping (3.9) and (4.4). The

cooling and damping effects of 2D plasmon absorbers are much better than that of ordinary 3D plasmon absorbers.

Thus, this quantum effect is an advantageous for muon cooling and damping.

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