Measuring stellar oscillations using equivalent widths of absorption lines

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ABSTRACT
Kjeldsen et al. (1995) have developed a new technique for measuring stellar oscillations and claimed a detection in the G subgiant η Boo. The technique involves monitoring temperature fluctuations in a star via their effect on the equivalent width of Balmer lines. In this paper we use synthetic stellar spectra to investigate the temperature dependence of the Balmer lines, Ca ii, Fe i, the Mg b feature and the G band. We present a list of target stars likely to show solar-like oscillations and estimate their expected amplitudes. We also show that centre-to-limb variations in Balmer-line profiles allow one to detect oscillation modes with ℓ ≤ 4, which accounts for the detection by Kjeldsen et al. of modes with degree ℓ = 3 in integrated sunlight.

Key words: stars: oscillations – Sun: oscillations – Sun: atmosphere – stars: individual: η Boo (HR 5235) – stars: individual: α Cen A (HR 5459) – δ Scuti.

1 INTRODUCTION
Measuring stellar oscillations is extremely difficult. The five-minute oscillations in the Sun, while rich in their information content, have tiny amplitudes. Observing similar oscillations in other stars therefore poses a great challenge (see ? for a recent review). Most attempts have sought to detect periodic Doppler shifts of spectral lines. A second method involves measuring the total stellar luminosity, which varies due to changes in temperature induced by acoustic waves in the stellar atmosphere. ?; hereafter referred to as KBVF) have proposed a new method in which they measure temperature fluctuations via their effect on the equivalent width of the Balmer lines. They presented strong evidence for oscillations in the G subgiant η Boo (see also ?; ?, with frequency splittings that were later found to agree with theoretical models (?; ?, Christensen-Dalsgaard, Bedding & Kjeldsen ?).

Here we examine the equivalent-width method in more detail. In Section 2 we use synthetic spectra to calculate the temperature sensitivity of the Balmer lines and other strong absorption features. In Section 3 we use these results to estimate oscillation amplitudes for a list of target stars. In Section 4 we discuss the centre-to-limb behaviour of absorption lines and show that observing Balmer-line equivalent widths allows one to detect non-radial oscillation modes with ℓ ≤ 4.

2 TEMPERATURE DEPENDENCE OF EQUIVALENT WIDTHS
The amplitude of an oscillation when measured in equivalent width (W) is related to the temperature fluctuations via

\[ \frac{\delta W}{W} = \frac{\partial \ln W}{\partial \ln T} \frac{\delta T}{T} \]  

(1)

Lines that are highly sensitive to temperature can be used to measure oscillations, while temperature-insensitive lines are useful as references. Also, detection sensitivity can be improved by combining observations of different lines, but only if the temperature dependence is known for each one.

To determine the function W(T), we shall assume that an oscillating star can be represented at all times by a model atmosphere in radiative equilibrium. A more sophisticated approach would require that the acoustic waves be treated explicitly (see ?; ?, ? and ?; ?, ?). With this assumption, we may calculate W(T) by considering a sequence of normal stellar atmospheres spanning a range of effective temperatures. This is best done using theoretical models, since databases of observed stellar spectra are inhomogeneous with respect to metallicity and surface gravity, and have uncertainties in Teff for each star. Note that in this paper we are not trying to estimate equivalent widths exactly. We are interested in relative changes as a function of effective temperature (and, in Section 4, of centre-to-limb position).
T. R. Bedding et al.

Figure 1. Synthetic spectra of the G band and the Mg b feature for a model atmosphere having $T_{\text{eff}} = 5800$ K. Equivalent widths were estimated by measuring the total flux in the regions enclosed by dashed lines.

We have used the theoretical model GRS88 (?) to calculate spectra of Hα, Hβ, Ca ii K, the Ca ii H + He blend and the 4383 Å Fe i line. Line profiles for the Balmer transitions were precalculated by K. Butler and T. Schöning (private communication) using VCS theory (?). We have also calculated spectra of the G band and the Mg b feature (see Figure 1) using the LTE code RAI11 by M. Spite, as extended for molecular lines by (?) — see (?) (? ?) for descriptions of the atomic and molecular data bases. The model atmospheres used in this case are from Gustafsson et al. (unpublished).

For all models we have adopted solar surface gravity and metallicity; calculations of the Hα line by (?) indicate that changing these parameters has little effect on equivalent widths. The microturbulent velocity was assumed to be 1 km/s (note this value is not critical for the strong lines we are considering) and the damping constants were obtained by fitting to the solar spectra (and also the Arcturus spectrum, for the G band and Mg b). The models do not include the chromospheric temperature inversion, but chromospheric effects should have negligible influence on the equivalent widths of the metal lines considered here because only the innermost line cores are formed at $\log \tau_{5000} < -3.5$. For the Balmer line, equivalent widths near the limb are significantly influenced by non-LTE effects in the line core, as discussed in Section 4.1 below.

The upper panel in Figure 2 shows $W(T_{\text{eff}})$ as measured from our synthetic spectra. The lower panel shows the slope $(\partial \ln W) / (\partial \ln T_{\text{eff}})$, which we obtained simply by measuring the differences between adjacent points in the upper panel. The slope for the Balmer lines is 5–7, consistent with the value of 6 adopted by KBVF, which was based on published models (?) (?). We see that the Mg b feature is especially promising for oscillation measurements. The Fe i lines are also useful since, although they are individually weaker, they are numerous and very sensitive to temperature.

### 3 OSCILLATIONS IN TARGET STARS

In the Sun, the amplitudes of individual oscillation modes can vary considerably over timescales of several days. However, different modes vary independently and the maximum amplitude, taken over all modes, stays roughly constant. When the solar oscillations are measured in bolometric luminosity, this amplitude is $(\delta L/L)_{\text{bol}} = 4.1$ ppm (parts-per-million) — see (?) (KB, hereafter referred to as KB) and references therein.

From KB, equations 5 and 7, amplitudes of solar-like oscillations in other stars should scale roughly as

\[
(\delta L/L)_{\text{bol}} = \frac{L/L_\odot}{(M/M_\odot)(T_{\text{eff}}/5777\, \text{K})} 4.1\, \text{ppm.} \tag{2}
\]

This luminosity variation is due almost entirely to changes in temperature (the change in radius is negligible). Therefore, using $L \propto R^2 T^4$ and Equation 1 we can write

\[
\delta W = \frac{\partial \ln W}{\partial \ln T_{\text{eff}}} \frac{L/L_\odot}{(M/M_\odot)(T_{\text{eff}}/5777\, \text{K})} 1.0\, \text{ppm.} \tag{3}
\]

Equation 3 gives an estimate for the oscillation signal expected from equivalent-width measurements of a spectral line. The photon noise, on the other hand, scales as the
Hence, assuming photon noise to be the dominant noise,
square root of the number of photons detected in the line. Hence, assuming photon noise to be the dominant noise source, the signal-to-noise ratio for a fixed observing time will scale as:

\[ S/N \propto \frac{\partial \ln W}{\partial \ln \text{Teff}} \frac{L \sqrt{W}}{M \text{Teff}^{0.2m}}, \]  

(4)

where \( m \) is the stellar magnitude at the relevant wavelength.

We have applied Equations 3 and 4 to a sample of bright stars that are likely to undergo solar-like oscillations. Table 1 shows fundamental parameters and expected oscillation characteristics for the stars, which were selected from The Bright Star Catalogue (\( ? \)) to have \( V < 4.2 \) and \( 0.35 < B - V < 1.2 \). A further restriction to include only main-sequence stars and subgiants was made on the basis of density, as described below. We also omitted known variables (e.g., \( \delta \) Scuti stars) and close binaries having nearly-
equal components. For the remaining binaries we give the parameters for the primary only (except \( \alpha \) Cen, whose components are listed separately).

For the Sun, \( \alpha \) Cen A and B, Procyon (\( \alpha \) CMi), \( \eta \) Boo, \( \beta \) Hyi and \( \varepsilon \) Eri we adopted the same fundamental parameters as used in KB and KBVF. For the other stars, we estimated effective temperatures using \( \text{Teff} \approx 11000 \text{K} / (B - V + 1.24) \), used parallaxes from The Bright Star Catalogue and took bolometric corrections from \( ? \) \( ? \). We made no reddening corrections except for HR 1543, which has \( E(B - V) = 0.03 \). Masses were estimated using stellar models in Fig. 1 of \( ? \), which assumes solar metallicity, OPAL opacities and convective overshoot.

For each star we used Equation 3 to calculate the oscil-
amplitude observing techniques (luminosity and velocity) and can be for stars oscillating at low frequencies (i.e., sub-giants). Also, we have neglected $1/f$ noise sources, such as instrumental drift, which will be important for stars oscillating at low frequencies (i.e., sub-giants).

The ranking of targets in Table 1 may differ for the other observing techniques (luminosity and velocity) and can be calculated using the formulae in KB. The Table also includes rough estimates for the frequency of maximum oscillation amplitude $\nu_{\text{max}}$ and the large frequency separation $\Delta \nu_0$, computed by scaling from the Sun (see KB). These frequencies will be important when choosing targets and designing observing programs. The large separation depends on the average density of the star and the table only includes stars for which $\Delta \nu_0 > 1/(\text{day}) = 11.57 \mu$Hz, in order to restrict the sample to main-sequence stars and subgiants.

The table also shows the quantity

$$\nu_{\text{rot}} \sin i = v_\sin i/(2\pi R),$$

where $\nu_{\text{rot}}$ is rotation frequency of the star. Components of an oscillation mode that is split by stellar rotation will be separated by approximately $\nu_{\text{rot}}$ (see below).

Finally, it is worth mentioning the double-lined spectroscopic binary $\alpha$ Equ (HR 8131), whose components (G5 III and A5 V) are slightly too faint to be included in Table 1. The masses and luminosities of both components have been derived from interferometric and spectroscopic observations by ? (7), making this system particularly valuable for testing stellar models.

4 MODE SENSITIVITY

4.1 Calculations

From equivalent-width observations of the daytime sky, KBVF detected solar oscillations with degrees $\ell = 0, 1, 2$ and 3. The detection of $\ell = 3$ was puzzling, given that the observations did not resolve the solar disk. The answer is straightforward: the Balmer lines are much weaker at the edge of the solar disk than at the centre (see Figure 3), so their integrated profiles are strongly weighted towards the disk centre. This has lead us to investigate in detail the sensitivity of the equivalent-width method to modes with different values of $\ell$.

The calculation involves integrating the oscillations over the stellar surface, including the effects of limb darkening (?; ?; ?). The oscillations of a star can be described in terms of spherical harmonics $Y^m_\ell(\theta, \phi).$ If the star is rotating slowly, there is no preferred direction and we are free to choose a coordinate system with the polar axis pointed towards the observer. In this case, all integrations of spherical harmonics will be zero unless $m = 0$. Even for stars in which rotation is important, the following analysis is still useful, as discussed at the end of this section.

For $m = 0$, the spatial response function for each $\ell$ (the ratio of the observed to the actual amplitude) is

$$S_\ell = 2\sqrt{2\ell+1} \int_0^1 P_\ell(\mu) I(\mu) F(\mu) \mu d\mu.$$  

Here $\mu = \cos \theta$, $P_\ell(\mu)$ is the Legendre polynomial of degree $\ell$, $I(\mu)$ is the centre-to-limb variation of continuum intensity (classical limb darkening) and $F(\mu)$ is the centre-to-limb variation of the oscillation signal.

The function $I(\mu)$ for the Sun is well approximated by

$$I(\mu) = 1 - u_2(1 - \mu) - v_3(1 - \mu^2),$$

where the coefficients vary with wavelength (?). The function $F(\mu)$ depends on how the observations are made. If the oscillations are observed via the Doppler shift of a spectral line then $F(\mu) = \mu$, corresponding to the projection of a nearly-radial pulsation along the line of sight. For observations in intensity there is no projection effect (to a good approximation) and $F(\mu) = 1$. If the oscillations are ob-

![Figure 3](image-url): Observed Hα spectra of the Sun at the centre and edge of the disk, taken from ? (7). The wings of the Hα line are much weaker in the lower diagram, whereas the metal lines and the core of Hα show little variation.
Our general conclusion is that the Balmer lines are strongly limb darkened, while the other lines have almost constant strength from centre to limb. For our purposes, it is sufficient to approximate $F(\mu)$ for each spectral line using a linear function:

$$F(\mu) = 1 - c(1 - \mu).$$

By fitting a straight line to each set of points in Figure 4 (giving more weight to the central portion of the disk, since this contributes the greatest area), we obtained the following values for $c$: 0.95 for Hα, 1.0 for Hβ (which we also adopt for Hγ and Hδ), −0.2 for Ca II K, 0.1 for the Ca II H + Hδ blend, −0.2 for Fe I, 0.05 for the G band and 0.15 for the Mg b feature.

Our Balmer-line models indicate that $c$ varies quite slowly with effective temperature ($c \propto T_{\text{eff}}^{-0.5}$), so the results given here apply to a broad class of stars.

Having obtained approximations for $I(\mu)$ and $F(\mu)$, we can now integrate Equation 7. Results for the five lowest-degree modes ($\ell = 0, \ldots, 4$) are:

$$\begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{2}{\sqrt{3}} & \frac{2}{\sqrt{5}} & \frac{2}{\sqrt{5}} & \frac{2}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{4}} & \frac{2}{\sqrt{5}} & \frac{2}{\sqrt{5}} & \frac{2}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{6}} & \frac{2}{\sqrt{7}} & \frac{2}{\sqrt{8}} & \frac{2}{\sqrt{8}} & \frac{2}{\sqrt{8}} \\ \frac{2}{\sqrt{8}} & \frac{2}{\sqrt{9}} & \frac{2}{\sqrt{10}} & \frac{2}{\sqrt{10}} & \frac{2}{\sqrt{10}} \end{pmatrix} \times \begin{pmatrix} 1 \\ c-1 \\ c-1 \\ c-1 \\ c \end{pmatrix} \begin{pmatrix} \frac{1}{u_2} \\ \frac{1}{v_2} \end{pmatrix}.$$ (10)

This matrix product, which should be evaluated from right to left, can be used to estimate the mode sensitivity of each observing method as a function of wavelength. Note that Equation 10 can be used for velocity measurements by setting $c = 1$ and for intensity measurements by setting $c = 0$.

### 4.2 Results and discussion

Figure 5 shows the sensitivities as calculated from Equation 10, expressed relative to $\ell = 0$. In the case of velocity and intensity observations (solid and dotted curves), there are published solar oscillation data with which to compare the calculations:

(i) Observations in velocity from the South pole using the Na i line (589 nm) by ? (?) give $S_1/S_0 = 1.5$, $S_2/S_0 = 1.2$, $S_3/S_0 = 0.6$ and $S_4/S_0 = 0.2$ (the scatter is about $\pm 0.2$).

(ii) Observations in bolometric intensity from the Solar Maximum Mission by ? (?) give $S_1/S_0 = 1.25$ and $S_2/S_0 = 0.83$ (scatter about $\pm 0.06$).

(iii) Observations in luminosity with the IPhIR green channel (500 nm) by ? (?) give $S_1/S_0 = 1.09$ and $S_2/S_0 = 0.79$ (scatter about $\pm 0.07$).

In each case, there is good agreement with our results.

Turning to equivalent-width measurements, we see from Figure 5 that the Balmer lines give a response similar to that of velocity measurements. This follows from their strong centre-to-limb variation and is consistent with the solar observations by KBVF — in particular, we can now explain...
Finally, we consider a rotating star. A mode with a particular \((n, \ell)\) will split into a multiplet having \(m = -\ell, \ldots, \ell - 1, \ell\), with the components being separated by about \(\nu_{\text{rot}}\) (the oscillation power summed over these peaks is conserved). If the star is seen pole-on, only modes with \(m = 0\) are observable and the analysis of this section holds exactly: we would not notice that the star is rotating. At the other extreme, in a star seen from near the equator (e.g., the Sun) the modes with \(|\ell - m|\) odd will have zero amplitude because the Legendre function is antisymmetric around the equator. For \(\ell = 1\) we would therefore only see two modes \((m = \pm 1)\). The total power will be the same as for a non-rotating star, so the amplitudes of these two modes will each be multiplied by \(1/\sqrt{2}\) relative to the non-rotating case. For \(\ell = 2\) we would see three modes \((m = -2, 0, 2)\) with relative amplitudes of \(\sqrt{3/2}, 1/2\), and \(\sqrt{3/2}\). Other cases can readily be calculated using the formulae given by ? (?).

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