NEUTRINO MASSES AND MIXING*

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The investigation of neutrino properties is a very important problem of today’s neutrino physics. The key problem is the problem of neutrino masses. If neutrinos are massive, they can be mixed. If the total lepton number is conserved, massive neutrinos are Dirac particles. If the neutrino masses are generated by an interaction that does not conserve the total lepton number, neutrinos with definite mass are Majorana particles. All this possibilities correspond to different gauge theories and it is very important that they can be distinguished experimentally. There are more than 60 different experiments that are going on at present on the search for effects of the masses, nature and mixing of neutrinos, and there is a general belief that investigation of neutrino properties could allow us to reach new physics.

In this reports I will discuss:
1. possibilities of neutrino mixing.
2. neutrino oscillations in vacuum and in matter.
3. experimental indications in favor of neutrino masses and mixing.

From all existing data it follows that flavor neutrinos $\nu_e, \nu_\mu$ and $\nu_\tau$ interact with matter via standard $CC$ and $NC$ interactions

$$\mathcal{L}^{CC} = -\frac{g}{2\sqrt{2}}j^{CC}_\alpha W^\alpha + h.c., \quad \mathcal{L}^{NC} = -\frac{g}{2\cos\theta_W}j^{NC}_\alpha Z^\alpha,$$

(1)

where

$$j^{CC}_\alpha = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_l \gamma_\alpha \nu_l + \ldots, \quad j^{NC}_\alpha = \sum_{l=e,\mu,\tau} \bar{\nu}_l \gamma_\alpha \nu_l + \ldots$$

(2)

Let us notice that the CC interaction determines the concept of flavor neutrinos. For example, we call $\nu_\mu$ the particle that is produced together with $\mu^+$ in $\pi^+ \rightarrow \mu^+ \nu_\mu$ decay, and so on. According to the neutrino mixing hypothesis, a flavor neutrino field $\nu_{lL}$ is a unitary superposition of left-handed components of fields of neutrinos with definite masses:

$$\nu_{lL} = \sum_i U_{li} \nu_{iL}, \quad l = e, \mu, \tau$$

(3)

where $\nu_i$ is the field of the neutrino with mass $m_i$ and $U$ is a unitary matrix.

The type of neutrino mixing is determined by the type of neutrino mass term. There are three possible neutrino mass terms [1]. In the case of the Dirac mass term

$$\mathcal{L}^D = -\sum_{l', l} \bar{\nu}_{l'} M_{l' l} \nu_l + h.c.$$  \hspace{1cm} (4)

three neutrinos with definite masses are Dirac particles. The Dirac mass term can be generated by the standard Higgs mechanism. If this mass term enters in the Lagrangian, the total lepton number

$$L = L_e + L_\mu + L_\tau$$  \hspace{1cm} (5)

is conserved.

There are two possible neutrino mass terms that do not conserve L:

1. The left-handed Majorana mass term

$$\mathcal{L}^M_L = -\frac{1}{2} \sum_{l', l} (\nu_{l'I})^c M_{l'I} \nu_{lL} + h.c.$$ \hspace{1cm} (6)

where \((\nu_{l'I})^c = C\bar{\nu}_{lL}^T\) is the charge conjugated spinor. This mass term can be generated in models with a Higgs triplet. Let us notice that theory with \(\mathcal{L}^M_L\) is the most economical theory of massive neutrinos (only left-handed fields enter in the Lagrangian). From Eq.(6), for the flavor neutrino field we have

$$\nu_{lL} = \sum_{i=1}^{3} U_{li} \chi_i L$$ \hspace{1cm} (7)

where \(\chi_i = \chi_i^c \equiv C\chi_i^T\) is the field of a Majorana neutrino with mass \(m_i\).

2. The Dirac and Majorana mass term

$$\mathcal{L}^{D+M} = \mathcal{L}^M_L + \mathcal{L}^D + \mathcal{L}^M_R$$  \hspace{1cm} (8)

This is the most general neutrino mass term that does not conserve \(L\) and include both \(\nu_{lL}\) and \(\nu_{lR}\). Dirac and Majorana mass term is typical for GUT models. For the mixing we have

$$\nu_{lL} = \sum_{i=1}^{6} U_{li} \chi_i L \hspace{1cm} (\nu_{lR})^c = \sum_{i=1}^{6} U_{li} \chi_i L \hspace{1cm} (9)$$

where \(\chi_i \ (i = 1, ..., 6)\) is the field of a Majorana particle with mass \(m_i\) and \(U\) is a 6 \times 6 mixing matrix.

In the framework of the models with a Dirac and Majorana mass term, exists the most popular mechanism of neutrino mass generation, the so called see-saw mechanism [2]. Assume that \(\mathcal{L}^M_L = 0\), \(\mathcal{L}^D\) is characterized by parameters that are of the order of the fermion masses and \(\mathcal{L}^M_R\) is characterized by parameters that are of the order of \(M_{GUT}\).

In this case, in the spectrum of the six Majorana particles there are three neutrinos with small masses

$$m_i \simeq \frac{(m_{l'})^2}{M_i} \hspace{1cm} (i = 1, 2, 3)$$ \hspace{1cm} (10)
and three particles with very heavy masses $M_i \simeq M_{GUT}$. Here $m_i^p$ is the mass of the up-quark or charged lepton in the corresponding generation. The see-saw mechanism provides a natural explanation of the smallness of neutrino masses.

If all the masses of Majorana particles in (9) are small, transitions of flavor neutrino into sterile states $\nu_l \to \bar{\nu}_{l'}$ become possible ($\bar{\nu}_{L}$ is a left-handed antineutrino, a quantum of the right-handed field $\nu_{lR}$).

After this short review of possible schemes of neutrino mixing, let us turn to the neutrino oscillations, a phenomenon that was first considered by B. Pontecorvo [3]. If there is neutrino mixing and neutrino masses are small, the state of a flavor neutrino $\nu_l$ with momentum $|\vec{p}| \gg m_i$ is a coherent superposition of the states of neutrinos with definite mass and negative helicities:

$$|\nu_l> = \sum_i |i><i|\nu_l>$$

where $|i>$ is an eigenstate of the free Hamiltonian,

$$H|i> = E_i|i>, \quad E_i \simeq p + \frac{m_i^2}{2p} \quad \text{and} \quad <\nu_l|i> = U_{li}. \quad (12)$$

If the beam of neutrinos at $t = 0$ is described by the state $|\nu_l>$, at time $t$ for the state vector of the beam we have

$$|\nu_l>_t = e^{-iHt}|\nu_l>$$

The flavor content of the neutrino beam is analyzed with the help of CC weak interactions. For the amplitude of the transition $\nu_l \to \nu_{l'}$ at the time $t$, from Eqs.(11) and (13) we have

$$A_{\nu_l \to \nu_{l'}}(t) = <\nu_{l'}|e^{-iHt}|\nu_l> = \sum_i U_{l'i} e^{-iE_it} U_{li}^* \quad (14)$$

Analogously, for the transition amplitude between antineutrino flavor states we have

$$A(\bar{\nu}_l \to \bar{\nu}_{l'}) = \sum_i U_{l'i}^* e^{-iE_it} U_{li} \quad (15)$$

Comparing the expressions (14) and (15), we have the following relation for the transition probabilities:

$$P(\nu_l \to \nu_{l'}) = P(\bar{\nu}_{l'} \to \bar{\nu}_l) \quad (16)$$

where $P(\nu_l \to \nu_{l'}) = |A(\nu_l \to \nu_{l'})|^2$. It is obvious that this relation is a consequence of CPT invariance.

If CP invariance in the lepton sector takes place, we have

$$U_{li} = U_{li}^* \quad \text{for Dirac neutrinos} \quad \text{and} \quad U_{li} \eta_i = U_{li}^* \quad \text{for Majorana neutrinos.} \quad (17)$$

Here $\eta_i$ is the CP parity of the Majorana neutrino with mass $m_i(\eta_i = \pm i)$. From Eqs.(14), (15) and (17) it follows that in the case of CP invariance in the lepton sector

$$P(\nu_l \to \nu_{l'}) = P(\bar{\nu}_l \to \bar{\nu}_{l'}) \quad (18)$$
Let us enumerate the neutrino masses in the following way:

\[ m_1 < m_2 < m_3 \ldots \]  

The expression for the transition amplitude can be written in the form

\[ A(\nu_l \rightarrow \nu_{l'} ) = e^{-iE_1 t} \left[ \sum_{i=2,3} U_{li} \left( e^{-i\frac{\Delta m^2_{ii} R}{2p}} - 1 \right) U^*_{l'i} + \delta_{l'l} \right] \]  

where \( \Delta m^2_{ii} = m_i^2 - m_1^2 \), and \( R \simeq t \) is the distance between the neutrino source and the detector. From Eq.(20) it is obvious that transitions between different flavor neutrino states require both mixing (non-diagonality of \( U \)) and \( \Delta m^2_{ii} \neq 0 \). For neutrino oscillations to be observable, it is necessary that at least one \( \Delta m^2 \) satisfy the following inequality:

\[ \Delta m^2 \gtrsim \frac{p}{R} (\text{MeV}) \]  

(21)

Let us notice that from this inequality it follows that solar neutrino experiments have an enormous sensitivity to the parameter \( \Delta m^2 \) (\( \Delta m^2 \geq 10^{-10} \text{ eV}^2 \)).

The data on the search for neutrino oscillations are usually analyzed under the simplest assumption that only two flavor neutrino fields are mixed. In this case

\[ U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \]  

(22)

where \( \theta \) is the mixing angle. From Eq.(15), for the transition probabilities we have

\[ P(\nu_l \rightarrow \nu_{l'} ) = \frac{1}{2} \sin^2 2\theta \left( 1 - \cos \frac{\Delta m^2 R}{2p} \right) \]  

(23)

\[ P(\nu_l \rightarrow \nu_l ) = 1 - P(\nu_l \rightarrow \nu_{l'}) \]  

(24)

The more realistic case is that with three generation mixing and a neutrino mass hierarchy [4]. Let us assume that \( \Delta m^2_{21} \) is so small that in any experiment with terrestrial or atmospheric neutrinos

\[ \frac{\Delta m^2_{21} R}{p} \ll 1. \]  

(25)

The mixing between the first and the second generation with small \( \Delta m^2_{21} \) can be responsible for the suppression of the flux of solar \( \nu_e \)'s. For the probability of \( \nu_l \rightarrow \nu_{l'} \) (\( l' \neq l \)) transitions, from Eq.(20) we have

\[ P(\nu_l \rightarrow \nu_{l'}) = \frac{1}{2} A_{\nu_l;\nu_{l'}} \left( 1 - \cos \frac{\Delta m^2}{2p} \right) \]  

(26)

Here

\[ A_{\nu_{l'};\nu_l} = 4|U_{l'3}|^2|U_{3l}|^2 \]  

(27)
is the amplitude of $\nu_l \leftrightarrow \nu_{l'}$ oscillations and $\Delta m^2 = m_3^2 - m_1^2$. The expression for the survival probability can be obtained from the conservation of the total probability. We have

$$P(\nu_l \rightarrow \nu_l) = 1 - \sum_{l'} P(\nu_l \rightarrow \nu_{l'}) = 1 - \frac{1}{2} B_{\nu_l;\nu_l} \left(1 - \cos \frac{\Delta m^2 R}{2p}\right)$$

where

$$B_{\nu_l;\nu_l} = \sum_{l' \neq l} A_{\nu_l;\nu_l}$$

Using the unitarity of the mixing matrix, from Eqs.(27) and (29) we find

$$B_{\nu_l;\nu_l} = 4 |U_{l3}|^2 \left(1 - |U_{l3}|^2\right)$$

Thus, if the inequality (25) is satisfied for oscillations of terrestrial and/or atmospheric neutrinos:

1. All the oscillation channels $\nu_\mu \leftrightarrow \nu_\tau$, $\nu_\mu \leftrightarrow \nu_e$, $\nu_e \leftrightarrow \nu_\tau$ are characterized by the same $\Delta m^2$.
2. The amplitudes of exclusive and inclusive channels are connected by the relation (29).
3. The relation (18) is satisfied even if CP is violated in the lepton sector.
4. The oscillations in all channels are characterized by the three parameters $\Delta m^2$, $|U_{e3}|^2$ and $|U_{\mu3}|^2$ ($|U_{\tau3}|^2 = 1 - |U_{e3}|^2 - |U_{\mu3}|^2$).

In the papers [5] the existing data were analyzed in the framework of the model with a neutrino mass hierarchy. We shall describe the results of these analyses later.

Now we turn to the discussion of the transitions of flavor neutrinos in matter [6, 7]. Let us consider a beam of neutrinos with momentum $\vec{p}$. For the wave function in the flavor representation $a_{\nu_l}(t) = \langle \nu_l | \psi(t) \rangle$ the following evolution equation holds:

$$i \frac{\partial a_{\nu_l}}{\partial t} = Ha_{\nu_l} \quad \text{with} \quad H = H_0 + H_I$$

The free Hamiltonian $H_0$ is given by

$$\langle \nu_{l'} | H_0 | \nu_l \rangle = (UEU^\dagger)_{l'l} = p \delta_{l'l} + \left(U \frac{m^2}{2p} U^\dagger\right)_{l'l}$$

The second term of $H$ is the effective Hamiltonian of the coherent interactions of the neutrino with matter. The neutral current interaction is $\nu_e - \nu_{\mu} - \nu_{\tau}$ symmetric. This interaction cannot change the flavor content of the beam. The contribution to $H_I$ comes from the CC part of the $\nu_e - e$ interaction:

$$H_I(t) = 2 \frac{G_F}{\sqrt{2}} \int \bar{\nu}_{eL} \gamma^\alpha \nu_{eL} \bar{e}\gamma_\alpha (1 + \gamma_5) e \, d^3x$$

Taking into account that

$$\langle \phi | \int \bar{e}\gamma_\alpha (1 + \gamma_5) e \, d^3x | \phi \rangle = \delta_{\alpha 0} \rho_e(t)$$

$$\langle \phi | \int \bar{e}\gamma_\alpha (1 + \gamma_5) e \, d^3x | \phi \rangle = \delta_{\alpha 0} \rho_e(t)$$
where $\rho_e$ is the density of electrons and $|\phi> \equiv \psi$ is the state vector of matter, we have

$$(H_I(t))_{\nu_e;\nu_e} = \sqrt{2} G_F \rho_e(t) \delta_{\nu_e \nu_e} \tag{35}$$

All the other matrix elements of $H_I(t)$ are equal to zero. It is important that the Hamiltonian of the effective interactions of neutrino with matter depends on time. It was shown in Ref.[7] that due to this dependence resonance transitions of flavor neutrinos in matter become possible.

Let us consider the simplest case of two neutrino flavors. Omitting the term $\frac{1}{2} \text{Tr} H$ that is proportional to the unit matrix and does not change the flavor content of the beam, we obtain:

$$H(t) = \frac{1}{4p} \begin{pmatrix} -X(t) & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & X(t) \end{pmatrix} \tag{36}$$

Here

$$\Delta m^2 = m_2^2 - m_1^2 \quad \text{and} \quad X(t) = \Delta m^2 \cos 2\theta - 2\sqrt{2} G_F \rho_e(t). \tag{37}$$

The Hamiltonian $H(t)$ can be easily diagonalized. We have

$$H(t) = U(t) E(t) U^+(t) \quad \text{with} \quad U(t) = \begin{pmatrix} \cos \theta(t) & \sin \theta(t) \\ -\sin \theta(t) & \cos \theta(t) \end{pmatrix}. \tag{38}$$

Here $\theta(t)$ is the mixing angle in matter and $E_{ik}(t) = E_i(t) \delta_{ik}, \ E_{1,2}(t)$ being the energies of neutrinos in matter (up to a constant). We have

$$\sin 2\theta(t) = \frac{\Delta m^2 \sin 2\theta}{\sqrt{X^2(t) + \Delta m^4 \sin^2 2\theta}} \tag{39}$$

$$\cos 2\theta(t) = \frac{X(t)}{\sqrt{X^2(t) + \Delta m^4 \sin^2 2\theta}} \tag{40}$$

$$E_{1,2} = \mp \frac{1}{4p} \sqrt{X^2(t) + \Delta m^4 \sin^2 2\theta} \tag{41}$$

As it is clear from Eqs.(39) and (40), the mixing angle in matter depends on the density of electrons. Assume that at some point $x_R = t_R$ the condition

$$\Delta m^2 \cos 2\theta = 2\sqrt{2} G_F \rho_e(t_R) p \tag{42}$$

is satisfied. At this point the diagonal elements of the Hamiltonian $H(t)$ are equal to zero and, as it follows from Eq.(41), for any value of $\theta \neq 0$ the mixing is maximal: $\theta(t_R) = \pi/4$.

The condition (42) is called resonance condition. Let us notice that at the point $t = t_R$ the distance between the energy levels of neutrinos in matter is minimal:

$$E_2(t_R) - E_1(t_R) = \frac{\Delta m^2 \sin 2\theta}{2p} \tag{43}$$
The resonance condition (42) can be written in the form

$$\Delta m^2 \cos 2\theta \simeq 0.7 \times 10^{-7} \rho E \text{eV}^2$$

(44)

where $$\rho$$ is the density of matter in g/cm$$^3$$. In the center of the sun $$\rho \simeq 10^2$$ g/cm$$^3$$ and the energy of solar neutrinos is $$\simeq 1$$ MeV. Thus, for solar neutrinos the resonance condition (44) is satisfied at $$\Delta m^2 \simeq 10^5$$ eV$$^2$$. The solution of the evolution equation shows that in a wide region of the values of the parameters $$\Delta m^2$$ and $$\sin^2 2\theta$$ the probability of solar neutrinos to survive depends on neutrino energy and can be significantly less than one.

Now we turn to a short discussion of the results of the experiments aimed to reveal the effects of neutrino masses and mixing [8]. There are three groups of experiments of this type:

1. The experiments on the search for effects of neutrino mass through precise measurement of the high energy part of the $$\beta$$-spectrum. The classical process is $$\beta$$-decay of $$^3H$$:

$$^3H \rightarrow ^3He + e^- + \bar{\nu}_e$$

(45)

The spectrum of the electrons in this decay is determined by the phase space

$$\frac{dN}{dT} = C pE(Q-T)\sqrt{(Q-T)^2 - m_{\nu}^2} F(E)$$

(46)

Here $$p$$ and $$E$$ are electron momentum and total energy, $$T = E - m_e$$, $$m_{\nu}$$ is the mass of the electron neutrino, $$F(E)$$ is the Fermi-function that describes the electromagnetic interaction of the final particles, and $$Q \simeq 18.6$$ keV is the released energy. There is no indications in favor of non-zero $$m_{\nu}$$ from experiments of this type. In the latest experiments the following upper bounds were obtained:

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$$m_{\nu}$$ (eV)</th>
<th>$$m_{\nu}$$ (eV$$^2$$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mainz</td>
<td>&lt; 7.2</td>
<td>$$-39 \pm 34 \pm 15$$</td>
</tr>
<tr>
<td>Troizk</td>
<td>&lt; 4.35</td>
<td>$$-4.1 \pm 10.3$$</td>
</tr>
</tbody>
</table>

Let us notice that $$m_{\nu_e} < 160$$ keV and $$m_{\nu_e} < 24$$ MeV.

2. The experiments on the search for neutrinoless double $$\beta$$-decay

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^-$$

(47)

These decays are allowed only if neutrinos are massive and Majorana particles (the total lepton number $$L$$ is not conserved). The matrix element of the ($$\beta\beta$$)$_{0\nu}$ decay is proportional to

$$\langle m \rangle = \sum_i U_{ei}^2 m_i$$

(48)

where the factor $$U_{ei}^2$$ is due to two $$e-\nu_i$$ vertices and $$m_i$$ is due to the neutrino propagator. Neutrinoless double $$\beta$$-decay was not observed in experiment. The best lower limit on the life time was reached in the Heidelberg-Moscow experiment for $^{76}Ge$: $$T_{1/2}(^{76}Ge) \geq 7.2 \times 10^{24}$$ y. From this data it follows that $$|\langle m \rangle| \lesssim 1$$ eV.
Table 1

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Neutrino energy (MeV)</th>
<th>Expected flux (cm$^{-2}$sec$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p + p \rightarrow d + e^+ + \nu_e$</td>
<td>$\leq 0.42$</td>
<td>$6.0 \times 10^{10}$</td>
</tr>
<tr>
<td>$^7Be + e^- \rightarrow ^7Li + \nu_e$</td>
<td>0.86</td>
<td>$4.9 \times 10^9$</td>
</tr>
<tr>
<td>$^8B \rightarrow ^8Be + e^+\nu_e$</td>
<td>$\leq 14$</td>
<td>$5.7 \times 10^6$</td>
</tr>
</tbody>
</table>

3. The experiments on the search for neutrino oscillations. The search for neutrino oscillations is the most sensitive method to reveal neutrino masses and mixing. We shall considered first the solar neutrino experiments.

The most important reactions of the solar $pp$ cycle in which neutrinos are produced presented in Table 1. The expected fluxes are result of calculations in the framework of the standard solar model [9] (SSM). There is one model independent constraint on the fluxes of solar neutrinos. The energy of the sun is produced in the transition

$$2e^- + 4p \rightarrow ^4He + 2\nu_e$$

Thus the production of energy in the sun is accompanied by the emission of neutrinos. If we assume that the sun is in a stable state, we have

$$\frac{1}{2} Q \sum_{i=pp,...} \left( 1 - 2 \frac{\bar{E}_i}{Q} \right) \Phi_i = \frac{L_\odot}{4\pi R^2}$$

Here

$$Q = 4m_p + 2m_e - m_{^4He} \simeq 26.7 MeV ,$$

$L_\odot$ is the luminosity of the sun, $R$ is the distance between the sun and the earth, $\Phi_i$ is the total flux of neutrinos from the source $i$ ($i = pp, ^7Be,...$) and $\bar{E}_i$ is the average energy of neutrinos from the source $i$. The results of the four solar neutrino experiments are presented in Table 2. These results are presented in SNU (1 SNU = $10^{-36}$ events/(atom·sec)). As it seen from Table 2, the event rates in all solar neutrino experiments are significantly less than the predicted event rates. If we accept the neutrino fluxes predicted by the SSM, the existing data can be described under the simplest assumption of transitions between two neutrino types. If matter effects are important, the following values were found for the mixing parameters:

1. $\sin^2 2\theta \simeq 8 \times 10^{-3}$ , $\Delta m^2 \simeq 5 \times 10^{-6} eV^2$ ,
2. $\sin^2 2\theta \simeq 0.8$ , $\Delta m^2 \simeq 10^{-5} eV^2$ .

The existing data can also be described by vacuum oscillations. In this case, for the parameters $\sin^2 2\theta$ and $\Delta m^2$ the following values were found: $\sin^2 2\theta \simeq 0.8, \Delta m^2 \simeq 8 \times 10^{-11} eV^2$. 

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Table 2. The result of solar neutrino experiments.

<table>
<thead>
<tr>
<th>Experiment, reaction, threshold</th>
<th>Data</th>
<th>Prediction of SSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homestake $\nu_\text{e}^{37}\text{Cl} \rightarrow e^{-37}\text{Ar}$, $E_{\text{th}} = 0.81\text{MeV}$</td>
<td>$2.55 \pm 0.35 \text{SNU}$</td>
<td>$9.3 \pm 1.4 \text{SNU}$</td>
</tr>
<tr>
<td>Gallex $\nu_\text{e}^{71}\text{Ga} \rightarrow e^{-71}\text{Ge}$, $E_{\text{th}} = 0.23\text{MeV}$</td>
<td>$77 \pm 8.5 \pm 5 \text{SNU}$</td>
<td>$131.5 \pm 6 \text{SNU}$</td>
</tr>
<tr>
<td>Sage $\nu_\text{e} \rightarrow \nu_\text{e}$, $E_{\text{th}} \simeq 7\text{MeV}$</td>
<td>$69 \pm 11 \pm 6 \text{SNU}$</td>
<td>$131.5 \pm 6 \text{SNU}$</td>
</tr>
<tr>
<td>Kamiokande data/SSM = $\nu_\text{e}$</td>
<td>$0.51 \pm 0.04 \pm 0.06$</td>
<td>$\nu_\text{e}$</td>
</tr>
</tbody>
</table>

The lower bound of the event rate in gallium experiments $Q_{\text{Ga}}$ can be found from the luminosity constraint (50). In fact, assuming that $P(\nu_\text{e} \rightarrow \nu_\text{e}) = 1$, we have

$$Q_{\text{Ga}} = \sum_i \bar{\sigma}_i \Phi_i \geq \bar{\sigma}_{pp} \sum_i \Phi_i \simeq 80 \text{SNU} \quad (54)$$

This lower bound does not contradict the values of $Q_{\text{Ga}}$ measured in the Gallex and Sage experiments. Let us compare, however, the data of different experiments under the assumption that nothing happens with solar $\nu_\text{e}$’s on their way from the sun to the earth. Let us compare, for example, the data from the Homestake and Kamiokande experiments. In the Kamiokande experiment only $^8\text{B}$ neutrinos are detected (the threshold is $\simeq 7\text{MeV}$). The flux $\Phi_B$ can be determined from the data of this experiment. Using this flux it is possible to obtain the contribution of $^7\text{Be}$ and other neutrinos to the event rate of the Homestake experiment. In Ref.[10] it was found that

$$Q_{\text{Cl}}(\nu_{\text{Be}}, \ldots) = -0.66 \pm 0.52 \text{SNU} \quad (55)$$

From this value it follows that

$$Q_{\text{Cl}}(\nu_{\text{Be}}, \ldots) < 0.46 \text{SNU} \quad (95\% \text{C.L.}) \quad (56)$$

On the other hand, all standard solar models give

$$Q_{\text{Cl}}(\nu_{\text{Be}}, \ldots) = 1.1 \pm 0.1 \text{SNU} \quad (57)$$

Thus, rather strong indications in favor of neutrino mixing follow from the analysis of the data of different solar neutrino experiments.

If the mass of the heaviest neutrino is in the eV region, neutrinos can solve the problem (or part of the problem) of dark matter. Two new experiments at CERN, CHORUS [11] and NOMAD [12], are searching for $\nu_\mu \leftrightarrow \nu_\tau$ oscillations. For $\Delta m^2 \geq 10 \text{eV}^2$ these experiments are sensitive to $A_{\nu_\mu,\nu_\tau} \geq 6 \times 10^{-4}$. If there is a hierarchy of neutrino masses and $\Delta m^2_{21} = \Delta m^2_{2} - m^2_{1}$ is relevant for the suppression of the flux of solar $\nu_\text{e}$’s, the probability
of $\nu_l \to \nu_{l'}$ ($l' \neq l$) transitions is given by Eq. (26). As it is well known, there is a hierarchy of couplings between generations in the quark sector. Let us assume that the hierarchy of couplings is a general phenomenon valid also for the lepton sector. In this case

$$|U_{e3}|^2 \ll |U_{\mu 3}|^2 \ll 1 \quad \text{and} \quad |U_{\tau 3}|^2 \simeq 1.$$  

(58)

For the oscillation amplitudes, from Eqs. (27) and (58) we obtain

$$A_{\nu_e;\nu_e} \ll A_{\nu_\mu;\nu_e} \ll 1$$  

(59)

$$A_{\nu_\mu;\nu_e} \simeq \frac{1}{4} A_{\nu_e;\nu_e} A_{\nu_\mu;\nu_\mu}$$  

(60)

Thus, if there is a hierarchy of couplings in the lepton sector, $\nu_\mu \to \nu_\tau$ is the dominant transition.

In conclusion, we shall discuss some indications in favor of neutrino mixing that follow from the data on beam-stop neutrino experiment and atmospheric neutrino experiments. Recently the LSND collaboration published [13] the results of the experiment on search for $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations. The sources of neutrinos in this experiment are $\pi^+ \to \mu^+ \nu_\mu$ and $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$ decays at rest. In the experiment $\bar{\nu}_e$’s were searched for via the observation of the process $\bar{\nu}_e p \to e^+ n$. Nine candidate events were found with an estimated background of $2.1 \pm 0.3$ events. A possible interpretation of this result (which requires confirmation) are $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations with an amplitude $10^{-3} \leq A_{\bar{\nu}_\mu;\nu_e} \leq 10^{-2}$ and $2 \times 10^{-4} \text{eV}^2 \leq \Delta m^2 \leq 5 \text{eV}^2$. We have analyzed the LSND result together with the results of the other experiments in the framework of the model with mixing of three massive neutrino fields and a neutrino mass hierarchy [5]. If the elements $|U_{e3}|$ and $|U_{\mu 3}|$ are small (as in the case of a hierarchy of couplings in the lepton sector), the LSND positive signal contradicts the negative results from the other experiments. The LSND result is compatible with the results from other experiments only in the case of a rather unusual mixing in the lepton sector with large $|U_{\mu 3}|$ and small $|U_{e3}|$ and $|U_{\tau 3}|$ (in this case $\nu_\mu$ is the heaviest neutrino).

Atmospheric neutrinos are produced in the decays

$$\pi (K) \to \mu \nu_\mu, \quad \mu \to e \nu_e \nu_\mu$$  

(61)

Thus, in the atmospheric neutrino flux $N_{\nu_\mu}/N_{\nu_e} \simeq 2$. For the ratio

$$R = \left( \frac{N_{\mu}}{N_e} \right)_{\text{obs}} / \left( \frac{N_{\mu}}{N_e} \right)_{\text{MC}}$$  

(62)

the following value was obtained in the Kamiokande experiment:

$$R = 0.60^{+0.06}_{-0.05} \pm 0.05$$  

(63)

Here $N_\mu(N_e)$ is the number of muon and electron events and $(N_\mu/N_e)_{\text{MC}}$ is the predicted ratio. The atmospheric neutrino anomaly was observed also in the IMB and Soudan experiments:

$$R = 0.54 \pm 0.05 \pm 0.12 \quad \text{(IMB)}$$  

(64)

$$R = 0.64 \pm 0.17 \pm 0.09 \quad \text{(Soudan)}$$  

(65)
On the other hand, in the Frejus experiment $R = 0.99 \pm 0.13 \pm 0.08$. The Kamiokande data can be described under the assumption of $\nu_\mu \leftrightarrow \nu_\tau$ or $\nu_\mu \leftrightarrow \nu_e$ oscillations. For the oscillation parameters the following values were obtained:

\[
5 \times 10^{-3} \leq \Delta m^2 \leq 3 \times 10^{-2} \text{eV}^2, \quad 0.7 \leq \sin^2 2\theta \leq 1 \quad (\nu_\mu \leftrightarrow \nu_\tau) \quad (66)
\]

\[
7 \times 10^{-3} \leq \Delta m^2 \leq 8 \times 10^{-2} \text{eV}^2, \quad 0.6 \leq \sin^2 2\theta \leq 1 \quad (\nu_\mu \leftrightarrow \nu_e) \quad (67)
\]

New experiments in search for neutrino oscillations are now under preparation. I have in mind the so called long baseline neutrino experiments:

- KEK–Super-Kamiokande (250 Km)
- Fermilab–Soudan (730 Km)
- CERN–Gran Sasso (730 Km)

The appearance ($\nu_\mu \rightarrow \nu_\tau$, $\nu_\mu \rightarrow \nu_e$) and disappearance ($\nu_\mu \rightarrow \nu_\mu$) channels will be investigated and the indications in favor of neutrino mixing that come from the atmospheric neutrino experiments will be checked.

In conclusion, we would like to stress that the problem of neutrino masses and mixing is the central problem of today’s neutrino physics. The investigations of this problem could allow us to reach physics beyond the standard model. At present different indications exist that neutrinos are massive and mixed. New experiments may appear crucial for the problem.

References


