Higher Covariant Derivative Pauli-Villars Regularization for
Gauge Theories in the Batalin-Vilkovisky Formalism.

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Abstract

The combined method of Higher Covariant Derivatives and Pauli-Villars regularization to regularize pure Yang-Mills theories is formulated in the framework of Batalin and Vilkovisky. However, BRS invariance is broken by this method and suitable counterterms should be added to restore it. The 1-loop counterterm is presented. Contrary to the scheme of Slavnov, this method is regularizing and leads to consistent renormalization group functions, which are the same as those found by other regularization schemes.

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1 Introduction

In order to regularize gauge theories, viz. Yang-Mills theories, the most successful and practical regularization method has become dimensional regularization [1]. Though for theories, whose properties explicitly depend on the dimension of spacetime —like e.g. chiral gauge theories or supersymmetric gauge theories— dimensional regularization requires careful treatment to say the least. Moreover one can even not define the path integral in dimensional regularization and there is no clear prescription to treat anomalies. Therefore we have to look for regularization schemes that explicitly stay at the physical space-time dimension.

The method of higher covariant derivatives (HCD) [2] seems well established. The advantage of this method is that gauge invariance (and BRST invariance) can be conserved explicitly. However the regularization is incomplete in the sense that it doesn’t regularize the one loop theory. A second regulator is therefore needed to take care of the one loop divergencies. The choice of this regulator is not straightforward. One could use dimensional regularization, but this conflicts somehow with the original motivation to avoid this regularization scheme, as this procedure can not be extended to gauge theories whose gauge invariance is linked to the dimensionality of space-time. Nevertheless it leads to consistent results [3]. Slavnov proposed as second regulator a combination of Pauli-Villars(PV) determinants [4]. This method was believed to solve the problem as it was thought to regularize the theory staying at the physical value of the space-time dimension. However in [5] it was pointed out that this method is not regularizing, which is a reason to discard it as a viable scheme. One needs a third regulator but the theory leads to inconsistent results and in the presence of matter even breaks unitarity[5, 6]. It seems that there exists no BRST invariant HCD PV regularization scheme.

However, the renormalizability of a non abelian gauge theory is not tied to the existence of a regularization scheme that formally preserves BRST invariance, but to the fact that the BRST symmetry is non anomalous[7]. So one can use a PV scheme that breaks BRST invariance. The calculation of the one loop effective action consists of several steps. After the introduction of the PV fields, one can calculate the breaking of BRST invariance. The result for pure Yang-Mills theories was partially obtained in [8]. Then one adds a counterterm to the action to preserve BRST invariance at the quantum level. Calculating with this new action one has a one loop BRST invariant regularized action and one can proceed like in any BRST invariant regularization scheme. In fact the inclusion of the counterterms can be seen as part of the regularization scheme.

The purpose of this paper is to present this method explicitly for pure Yang-Mills theories at one loop level, using the PV-method developped in [9], combined with HCD. We show that the inconsistencies of [5] are absent. Also at no point we need to introduce a preregulator. Working in the Batalin-Vilkovisky(BV) formalism [10, 9] to treat gauge theories implies a preferred choice for the Z-factors in multiplicative renormalization, different from the ones used in standard textbooks [11].

The paper is organized as follows. In section 2 the HCD regularized action is derived in the BV formalism and PV fields are introduced, according to [9]. Finally I comment on multiplicative renormalization in the BV scheme. Section 3 is devoted to the calculation of the one loop effective action and Renormalization Group coefficients using two regularization schemes. First the PV regularization without HCD is used and the Z-factors are presented in this theory illustrating the problem outlined above. This factors are used to calculate the
renormalization group coefficients. Secondly I present the results for the PV scheme combined with HCD. The conclusions are in section 4. The Feynman rules are collected in appendix A. Appendix B shows a sample calculation using PV regularization and explicitly staying at 4 dimensions during the whole calculation.

2 The regularized action

In this section the regularized action is derived. First the higher covariant derivatives are introduced in the action and the gauge fixed action is obtained. PV fields are introduced to regularize the one loop theory. Finally I comment on multiplicative renormalization.

2.1 The Higher Covariant Derivative Regularization

We consider pure Yang-Mills theory in 4 Euclidean dimensions and add a Higher Covariant Derivative (HCD) term:

$$S_{YM} = -\frac{1}{4} F^a_{\mu\nu} (1 - \frac{D^2}{\Lambda^2})^2 F^a_{\mu\nu}.$$  (1)

In this action $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{ab}_c A^b_\mu A^c_\nu$ is the field strength, $A^a_\mu$ the gauge field, $g$ the coupling constant and $f^{ab}_c$ the structure constants of the gauge algebra. The covariant derivative is $D^a_\mu = \delta^a_b \partial_\mu + f^{ab}_c A^c_\mu$. In (1) a trace over the algebra indices and an integral over Euclidean space-time is understood. I assume that the gauge group is a compact, simple Lie group. Therefore the structure constants can be taken to be totally antisymmetric in the three indices and are normalized so that $f^{ab}_c f^{bc}_d = \alpha^c a \delta^a_b$ where $c$ is the eigenvalue of the quadratic Casimir operator in the adjoint representation. For $SU(N)$, $c = N$.

In order to construct the quantum theory one needs to fix the gauge and gauge invariance is turned into BRS invariance [12]. In the following we will use the Batalin-Vilkovisky formalism (BV) [10, 9] to construct the gauge fixed action. The field content of the quantum theory is $\Phi^A = \{A^a_\mu, c^a, b^a\}$, where $c^a$ will be the ghosts and $b^a$ the antighosts. Now antifields are introduced—which act as sources for BRS-transformations—and the extended action is

$$S_{ext} = -\frac{1}{4} F^a_{\mu\nu} (1 - \frac{D^2}{\Lambda^2})^2 F^a_{\mu\nu} + A^a_\mu D^a_\mu b^a + \frac{1}{2} c^a f^{ab}_c b^c - \frac{1}{2\alpha} b^a b^a.$$  (2)

This action satisfies the master equation $(S_{ext}, S_{ext}) = 0$, which expresses the BRS invariance. Notice the introduction of a non minimal term $-\frac{1}{2\alpha} b^a b^a$ to fix the gauge. This can be done by shifting the antifields $A^a_\mu$ and $b^a$ and is in fact a canonical transformation on the variables $\{\Phi^A, \Phi^*_A\}$ w.r.t. the antibracket. Then the gauge fixed action will automatically satisfy the master equation.

As generating function for the gauge fixing canonical transformation we take $F(\Phi^A, \Phi^*_A) = \Phi^A \Phi^*_A + b(1 - \frac{\partial^2}{\partial^2 A}) \partial_\mu A^\mu$. This means that the antifields are redefined as

$$\Phi^*_A = \Phi^*_A + \frac{\partial}{\partial^2 \Phi^A} F(\Phi, \Phi^*).$$  (3)

2
This leads to the following extended gauge fixed action

\[ S_{gf}(\Phi, \Phi^*) = -\frac{1}{4} F^a_{\mu\nu}(1 - \frac{D^2}{\Lambda^2})^2 F^{\mu\nu}_a - \frac{1}{2\alpha}(\partial_\mu A^a_\mu)(1 - \frac{\partial^2}{\Lambda^2})(\partial_\nu A^{a\nu} + b_\mu \partial_\mu (1 - \frac{\partial^2}{\Lambda^2}) D^\mu c^a + A^a_\mu D^\mu c^a + \frac{1}{2} c^a f_{bc} c^b c^c - \frac{1}{\alpha} b^a_\mu (1 - \frac{\partial^2}{\Lambda^2}) \partial_\mu A^{a\mu} - \frac{1}{2\alpha} b^a_\mu b^{*a}. \]  

(4)

This action satisfies the master equation, i.e. \((S_{gf}, S_{gf}) = 0\) which implies BRS invariance of \(S_{gf}(\Phi, \Phi^*)\) under:

\[ \delta_{BRS} A^{a\mu} = (A^{a\mu}, S)|_{\Phi^* = 0} = (D^\mu b^b c^b, \]  

(5)

\[ \delta_{BRS} b^a = (b^a, S)|_{\Phi^* = 0} = -\frac{1}{\alpha} (1 - \frac{\partial^2}{\Lambda^2}) \partial_\mu A^{a\mu}, \]  

(6)

\[ \delta_{BRS} c^a = (c^a, S)|_{\Phi^* = 0} = \frac{1}{2} f_{bc} c^b c^c. \]  

(7)

A number of comments are now in order.

**Comment 1**: This theory is finite for all higher loop diagrams, except for the one loop (sub)diagrams. If one denotes the superficial degree of divergence (SDD) for a 1PI diagram \(G\) as \(\omega(G)\) then one has for this theory in 4 dimensions that

\[ \omega(G) = 4 - 4(L - 1) - E_A - \frac{7}{2} E_{gh} \]  

(8)

Now only the one-loop (sub)diagrams with 2, 3, 4 external gauge fields \((E_A)\) and 0 external ghosts \((E_{gh})\) are divergent. So in order to regularize the one-loop theory one needs another regularization. Dimensional regularization could be used and everything works out well [3]. However then the introduction of the HCD becomes superfluous and moreover one of the reasons to look for a HCD regularization scheme was to avoid the potential problems with dimensional regularization.

**Comment 2**: The choice of this HCD term leads to propagators which factorize (see appendix A) and are very convenient in Feynman diagram calculations because the usual manipulations can be applied. I also used a gauge fixing term of the form

\[ -\frac{1}{2\alpha} \partial_\mu A^a_\mu (1 - \frac{\partial^2}{\Lambda^2})^2 \partial_\nu A^{a\nu}, \]  

(9)

so that the gluon propagators are behaving like \(1/p^6\) (i.e. the regularization is not destroyed by the gauge-fixing) and have a factorizing denominator (see appendix A). Another advantage of the factorization of the denominators is that one doesn’t need dimensional regularization techniques to calculate diagrams. In ref. [5] dimensional regularization techniques were used not only because the scheme of Slavnov was not fully regularizing, but also to calculate the diagrams that were regularized.

**Comment 3**: This regularized theory is manifestly local as it is obtained from a local functional (1) through local canonical transformations. In this it differs from the action of Faddeev and Slavnov [4] who have a manifestly nonlocal action in the auxiliary field. Because we wanted to keep the action manifestly local there are also higher derivatives in the ghost action and the BRS-transformation of the antighost.
2.2 One loop Pauli-Villars regularization

In order to regularize the one loop diagrams I will adopt PV regularization. For the introduction
of PV-fields I follow the general approach of [9]. Denote first $S_{AB} \equiv \frac{\partial}{\partial \Phi^A} S \frac{\partial}{\partial \Phi^B}$. The PV action
is given by

$$S_{PV}(\Phi, \Phi^*, \Phi_i) = \frac{1}{2} \sum_{i=1}^{N} \Phi_i^A S_{AB} \Phi_i^B - \frac{1}{2} M_i^2 \Phi_i^A T \Phi_i^B,$$

(10)

with $\Phi_i$ the PV copies, $T$ an invertible matrix and $N$ the number of PV copies needed. For
more information about the statistics of the PV fields and the number of copies needed see
[9]. The BRS-transformation of the PV fields $\{\Phi_i^A\}$ are defined as

$$\delta_{BRS} \Phi_i^A \equiv K_{iB}^A \Phi_i^B,$$

where

$$K_{iB}^A = \frac{\partial}{\partial \Phi_i^A} S \frac{\partial}{\partial \Phi^B}. \quad \text{This yields}$$

$$\delta_{BRS} A_i^\mu = \partial^\mu c_i + f_a^{bc} A_i^b \xi_i^c,$$

(11)

$$\delta_{BRS} b_i^a = -\frac{1}{\alpha} \partial_\mu A_i^a \xi_i^\mu,$$

(12)

$$\delta_{BRS} c_i^a = f_a^{bc} b_i^b c_i^c.$$

(13)

For one loop calculations it is very convenient to adopt a formal\(^1\) integration rule,

$$\int D\Phi_i e^{-\frac{1}{2} \Phi_i^A M_{AB} \Phi_i^B} = s \text{Det}(M_{AB})^{-\frac{1}{2} c_i},$$

(14)

where the $c_i$ have to satisfy certain relations in order to regularize. The fully regularized path
integral is now given by

$$Z_{\text{reg}}[J_A, \Phi_A^*] = \int D\Phi_i \prod_{i=1}^{N} D\Phi_i^A \exp -\frac{1}{\hbar} \{ S_{gf} + S_{PV} + J^A \Phi_i \}$$

(15)

There are some important comments on this PV action.

Comment 1: In this PV action all the original vertices are copied and coupled quadratically
to the PV fields, i.e. every PV vertex contains two PV fields. This action, apart from the
mass-term, is invariant under the BRS-transformations (11 – 13). Although it is not necessary
to introduce the vertices $b_i^a (1 - \frac{\partial_\mu^2}{\alpha^2}) f_a^{bc} A_i^b \xi_i^c$ and $b_i^a f_a^{bc} (1 - \frac{\partial_\mu^2}{\alpha^2}) A_i^b c_i^c$ for regularization, they are
needed for BRS invariance under the defined transformations for the PV fields. With these
transformations it is obvious that the measure in the path integral is invariant if $\sum_{i=1}^{N} c_i = -1$. The BRS-variation of the total measure in the path integral is

$$\delta_{BRS} \left\{ D\Phi_i \prod_{i=1}^{N} D\Phi_i^A \right\} = \left( 1 + \sum_{i=1}^{N} c_i \right) K^A_A = 0.$$

(16)

In order to prove this in the theory of Faddeev and Slavnov, again the introduction of a pre-
regulator is needed [13].

\(^1\) We can of course introduce PV-fields with a well defined statistics. A boson with mass $M$ leads to $c_i = 1$ and a fermion pair with mass $2M$ leads to $c_i = -2$. 
Comment 2: The theory is not anymore regularized for the higher loop diagrams that contain a one loop subdiagram with external PV fields[13]. I will not go further into this here.

Comment 3: The theory is regularized at one loop level and is not manifestly BRS invariant, because the mass term —and only the mass term— for the PV fields breaks BRS invariance. This does not mean however that renormalizability is destroyed, because one can introduce at any higher loop level a counterterm to restore gauge invariance. Part of the one loop counterterm for this regularization scheme without HCD is obtained in [8] for the Feynman gauge \( \alpha = 1 \). There, only the finite part of the one loop counterterm was calculated without taking into account the antifield dependent part. Here I present the total one loop counterterm and remark that there is no antifield dependence. In [8] the heat kernel expansion is used to calculate the breaking of BRS invariance \( A = (\Gamma, \Gamma) \), where \( \Gamma \) is the Legendre transform w.r.t. the sources of (15). With the same method one can easily obtain the infinite part. \( A \) is then given by

\[
A = \frac{1}{(4\pi)^2} \text{str}(Ja_2) - \sum_{i=1}^{N} c_1 M_i^2 \log \frac{M_i^2}{\mu^2} \text{str}(Ja_1) + \sum_{i=1}^{N} c_1 M_i^2 \log \frac{M_i^2}{\mu^2} \text{str}(Ja_0),
\]  

(17)

with \( a_n \) the Seeley-DeWitt coefficients (cfr. [8]).

For pure Yang-Mills (17) yields

\[
\mathcal{A} = -\frac{1}{(4\pi)^2} \sum_{i=1}^{N} c_1 M_i^2 \log \frac{M_i^2}{\mu^2} \text{str}(c\partial_\alpha A^\alpha) + \frac{1}{(4\pi)^2} \frac{1}{12} \text{str}[c(-4\partial^\nu A_\mu A_\nu A^\mu + 4\partial^\nu A^\mu \cdot (\partial_\mu A_\nu - \partial_\nu A_\mu)) + 4(\partial^\nu A_\nu)^2 + 8A_\mu \partial^\nu A_\nu A_\mu - 4A_\mu \Box A_\mu + 2\Box A_\nu A_\nu].
\]  

(18)

The one loop BRS invariance can be restored by a counterterm \( \tilde{M}_1 \) in the action such that \( \mathcal{A} = (\tilde{M}_1, S) \). The counterterm is then

\[
\tilde{M}_1 = \frac{1}{(4\pi)^2} \left[ \frac{1}{2} \sum_{i=1}^{N} c_1 M_i^2 \log \frac{M_i^2}{\mu^2} \text{tr}(A_\alpha A^\alpha) + \frac{1}{12} \text{tr}\left(\frac{3}{2}(\partial_\mu A^\mu)^2 - \frac{1}{2} \partial_\mu A_\nu \cdot \partial^\mu A^\nu - 2A^\alpha A^\beta \partial_\mu A_\nu + \frac{3}{2} A_\mu A_\nu A^\mu A^\nu - \frac{1}{2} A^2 A^2 \right) \right].
\]  

(19)

If one now adds this counterterm to the action (4), \( S = S_{gf} + \hbar \tilde{M}_1 \) one obtains the effective action by performing the usual Legendre transformation. This action we denote by

\[
\tilde{\Gamma}^{(1)}(z^d_0; g_0) = S(z^d_0; g_0) + \hbar \tilde{M}_1,
\]  

(20)

where \( z^d \equiv \{ \Phi^{Acl}, \Phi^*_A \} \). The subscript 0 denotes that we have taken bare quantities. This effective action is one-loop BRS-invariant and satisfies a Zinn-Justin equation \( (\tilde{\Gamma}^{(1)}, \tilde{\Gamma}^{(1)}) = 0 + \mathcal{O}(h^2) \), which generates the 1-loop Ward identities.

So one can perfectly adopt PV regularization for gauge theories if one is willing to destroy manifest gauge invariance and introduce counterterms. So after introducing a counterterm, the theory is again manifestly gauge invariant at one loop level. Renormalization can be done in
the usual way and consistent results for the one loop $\beta$-function and anomalous dimension of the fields are obtained. This will be shown in section 3.

Comment 4: It seems that there is no consistent gauge invariant PV-regularization scheme. In [4] a gauge invariant PV action was considered, which differs from 10. However, then the regulariztion of the theory is destroyed and leads to inconsistent results [5].

2.3 Multiplicative renormalization and BV

Suppose that one performs a multiplicative renormalization, as I will do in section 3. Then one starts from the extended gauge fixed action (4), where all quantities are taken to be 'bare' quantities. The gauge-fixing parameter $\alpha$ and the coupling constant $g$ get renormalized by defining

$$g_0 = Z_g g_R, \quad (21)$$
$$\alpha_0 = Z_\alpha \alpha_R. \quad (22)$$

For the fields and antifields we have

$$A_\mu^0 = Z_A^{1/2} A_R; \quad A^*_\mu_0 = \frac{1}{Z_A^{1/2}} A^*_\mu_R; \quad (23)$$
$$c_0 = Z_c^{1/2} c_R; \quad c^*_0 = \frac{1}{Z_c^{1/2}} c^*_R; \quad (24)$$
$$b_0 = Z_b^{1/2} b_R; \quad b^*_0 = \frac{1}{Z_b^{1/2}} b^*_R. \quad (25)$$

In fact this amounts to a canonical transformation of fields and antifields with generating function $F(\Phi^A, \Phi^A_*) = Z_\Phi \Phi^A \Phi^A_*$ and a redefinition of the parameters [14]. Now certain relations between the $Z$-factors can be derived. In fact in (4) some terms do not get radiative corrections, i.e. $-\frac{1}{\alpha} b^*(1 - \frac{\alpha^2}{32}) \partial_\mu A^\mu$ and $-\frac{1}{2\alpha} b^{*2}$. From this one can conclude that

$$Z_b = \frac{1}{Z_\alpha}, \quad (26)$$
$$Z_b = \frac{1}{Z_A}. \quad (27)$$

This implies that one cannot choose $Z_b = Z_c$ as is done in previous literature. Because then one would have

$$Z_c = Z_b = \frac{1}{Z_A} \quad (28)$$

and this does not hold, as can be seen from explicit one loop calculations in the following section.

Remark that we're not using the standard notations for the $Z$-factors. In our notation $Z_g = Z_1$ and $Z_A = Z_3$.
3 The one loop effective action

In this section I will calculate the one loop effective action using two regularization schemes. First I use Pauli-Villars without introducing Higher Covariant Derivatives. In this scheme I calculate the $Z$-factors and check the statements in section 2.3 by explicit calculations. With these $Z$-factors the renormalization group coefficients are obtained, consistent with other regularization schemes. Secondly I will present the calculations with the Higher Covariant Derivative Pauli Villars scheme and obtain the same results.

3.1 One loop effective action using Pauli-Villars regularization

For the explicit calculations in this section I work in the Feynman gauge $\alpha = 1$. The Feynman rules are given in appendix A.

The divergent part (for $M_i^2 \to \infty$) of $\bar{\Gamma}^{(1)}$, which I denote by $\Gamma^{(1)}_{\text{div}}$ can be absorbed in the $Z$-factors, from which the renormalization group coefficients are computed.

We start by computing three 1PI Green functions, i.e. the vacuum polarization tensor $\Pi^{ab}_{\mu\nu}(p,M_i)$, the ghost-selfenergy $\Omega^{ab}(p,M_i) = \langle b^a(-p)c^b(p) \rangle$ and the antighost-gluon-ghost vertex $V^{abc}_{\mu}(p_1,p_2) = \langle b^a(p_2)A^b_{\mu}(p_1)c^c(-p_1 - p_2) \rangle$. The one loop corrections to $\Pi^{ab}_{\mu\nu}(p,M_i)$ are given by the Feynman diagrams of fig.1 and result in

$$\Pi^{ab}_{\mu\nu}^{(1)}(p,M_i) = \lim_{M_i \to \infty} \frac{g_0^2c_v}{16\pi^2} \left\{ \frac{5}{3} \sum_{i=1}^{N} c_i \log \left( \frac{M_i^2}{\kappa^2} \right) + \Pi_{\text{fin}}^{(1)} \right\} (p^2g_{\mu\nu} - p_{\mu}p_{\nu}) \delta^{ab} \quad (29)$$

where $\kappa$ is the renormalization scale and $\Pi_{\text{fin}}^{(1)}$ collect all finite terms for $M_i \to \infty$. An explicit calculation is lined out in appendix B. For the other two 1PI Green functions $\Omega^{ab}(p)$ and $V^{abc}(p)$ the Feynman diagrams that give the one loop corrections, are depicted in figs. 2 and 3 respectively. They result in

$$\Omega^{ab}^{(1)}(p) = \lim_{M_i \to \infty} \frac{g_0^2c_v}{16\pi^2} \left\{ \frac{1}{2} \sum_{i=1}^{N} c_i \log \left( \frac{M_i^2}{\kappa^2} \right) + \Omega_{\text{fin}}^{(1)} \right\} p^2 \delta^{ab} \quad (30)$$

$$V^{abc}^{(1)}(p_1,p_2) = \lim_{M_i \to \infty} \frac{g_0^2c_v}{16\pi^2} \left\{ \frac{1}{2} \sum_{i=1}^{N} c_i \log \left( \frac{M_i^2}{\kappa^2} \right) + V_{\text{fin}}^{(1)} \right\} g_0 f_{bc}^{\mu} i p_{2\mu}. \quad (31)$$

In (29-31) the $c_i$ satisfy the relations $\sum_{i=1}^{N} c_i = -1$ and $\sum_{i=1}^{N} c_i M_i = 0$ (see appendix B). From this functions one can read of all the independent $Z$-factors:

$$Z_A = 1 - \frac{5}{3} \lim_{M_i \to \infty} \frac{g_0^2c_v}{16\pi^2} \sum_{i=1}^{N} c_i \log \left( \frac{M_i^2}{\kappa^2} \right)$$

$$Z_b^{1/2} Z_c^{1/2} = 1 - \frac{1}{2} \lim_{M_i \to \infty} \frac{g_0^2c_v}{16\pi^2} \sum_{i=1}^{N} c_i \log \left( \frac{M_i^2}{\kappa^2} \right)$$

$$Z_g Z_b^{1/2} Z_c^{1/2} Z_A^{1/2} = 1 + \frac{1}{4} \lim_{M_i \to \infty} \frac{g_0^2c_v}{16\pi^2} \sum_{i=1}^{N} c_i \log \left( \frac{M_i^2}{\kappa^2} \right). \quad (32)$$
This yields
\[ Z_{A}^{1/2} = Z_{\alpha} = 1 - \frac{5}{6} \lim_{M_{i} \to \infty} \frac{g_{0}^{2}c_{v}}{16\pi^{2}} \sum_{i=1}^{N} c_{i} \log \left( \frac{M_{i}^{2}}{\kappa^{2}} \right) \]
\[ Z_{b}^{1/2} = 1 + \frac{5}{6} \lim_{M_{i} \to \infty} \frac{g_{0}^{2}c_{v}}{16\pi^{2}} \sum_{i=1}^{N} c_{i} \log \left( \frac{M_{i}^{2}}{\kappa^{2}} \right) \]
\[ Z_{c}^{1/2} = 1 - \frac{4}{3} \lim_{M_{i} \to \infty} \frac{g_{0}^{2}c_{v}}{16\pi^{2}} \sum_{i=1}^{N} c_{i} \log \left( \frac{M_{i}^{2}}{\kappa^{2}} \right) \]
\[ Z_{g} = 1 + \frac{11}{6} \lim_{M_{i} \to \infty} \frac{g_{0}^{2}c_{v}}{16\pi^{2}} \sum_{i=1}^{N} c_{i} \log \left( \frac{M_{i}^{2}}{\kappa^{2}} \right) . \] (33)

As a check on these results, I calculate the one loop correction to the 1PI-function \( \langle A_{\mu}^{*}(p)c^{b}(-p) \rangle \). There is only the contribution of the diagram in fig. 4. The result is
\[ \langle A_{\mu}^{*}(p)c^{b}(-p) \rangle^{(1)} = - \lim_{M_{i} \to \infty} \frac{g_{0}^{2}c_{v}}{16\pi^{2}} \left\{ \frac{1}{2} \sum_{i=1}^{N} c_{i} \log \left( \frac{M_{i}^{2}}{\kappa^{2}} \right) + \text{fin} \right\} i p_{\mu} \delta^{ab} \] (34)

and from this I derive that
\[ \frac{Z_{c}^{1/2}}{Z_{A}^{1/2}} = 1 - \frac{1}{2} \lim_{M_{i} \to \infty} \frac{g_{0}^{2}c_{v}}{16\pi^{2}} \sum_{i=1}^{N} c_{i} \log \left( \frac{M_{i}^{2}}{\kappa^{2}} \right) , \] (35)

which is in agreement with (33).

Finally I come to the computation of the renormalization group coefficients. It is known that if one denotes by \( \Gamma_{R}(p, \kappa, g) \) the renormalized 1PI Green functions for the Yang-Mills theory with \( n_{A} \) external lines of the fields \( \Phi^{A} \) in the Feynman gauge \( \alpha = 1 \), the renormalization group equation takes the form
\[ \left[ \kappa \frac{\partial}{\partial \kappa} + \beta(g) \frac{\partial}{\partial g} + \frac{1}{2} \sum_{A} \gamma_{A}(g) n_{A} \right] \Gamma_{R}(p, \kappa, g) = 0. \] (36)

In multiplicative renormalization one then has that
\[ \beta(g) \equiv \kappa \frac{\partial g}{\partial \kappa} = \kappa g_{0} \frac{\partial Z_{g}^{-1}}{\partial \kappa} \]
\[ \gamma(g) \equiv -\kappa \frac{\partial \ln(Z\Phi)}{\partial \kappa} . \] (38)

For the case I treat here, I obtain
\[ \beta(g) = -\frac{11}{3} \frac{c_{v}}{16\pi^{2}} g^{3} + \mathcal{O}(g^{4}) ; \]
\[ \gamma_{A}(g) = \frac{10}{3} \frac{c_{v}}{16\pi^{2}} g^{2} + \mathcal{O}(g^{3}) ; \]
\[ \gamma_{b}(g) = -\frac{10}{3} \frac{c_{v}}{16\pi^{2}} g^{2} + \mathcal{O}(g^{3}) ; \]
\[ \gamma_{c}(g) = \frac{16}{3} \frac{c_{v}}{16\pi^{2}} g^{2} + \mathcal{O}(g^{3}) . \] (39)

These are the standard results for \( \beta(g) \) and \( \gamma_{A}(g) \).
3.2 One loop effective action using HCD-PV regularization

Now I use the full regularization scheme defined in section 2 to compute the one-loop 1PI-functions.

The strategy is as follows. I have redone the calculations of the previous subsection without knowing the counterterm to restore BRS-invariance. This counterterm has to be local. The contributions to the $Z$-factor are non-local. So if one is only interested in these coefficients, one doesn’t need the exact form of the counterterm. In order to do the very extensive algebra, I used the Mathematica package HIP [15]. I took the limits in the order $\lim_{\Lambda \to \infty} \lim_{M_i \to \infty}$. The results are then

$$
\Pi_{\mu\nu}^{(1)}(p, M_i) = \lim_{\Lambda \to \infty} \lim_{M_i \to \infty} \frac{g_0^2 c_v}{16\pi^2} \left\{-\frac{19}{12} \log \left(\frac{\Lambda^2}{\kappa^2}\right) + \frac{1}{12} \sum_{i=1}^{N} c_i \log \left(\frac{M_i^2}{\kappa^2}\right)\right\} p^2 g_{\mu\nu} \delta^{ab} \\
+ \frac{1}{12} \sum_{i=1}^{N} c_i \log \left(\frac{M_i^2}{\kappa^2}\right) \right\} \right\} + \kappa - \text{independent} + \text{fin},
$$

$$
\Omega^{ab}^{(1)}(p) = - \lim_{\Lambda \to \infty} \frac{g_0^2 c_v}{16\pi^2} \left\{ \frac{1}{2} \log \left(\frac{\Lambda^2}{\kappa^2}\right) \right\} p^2 \delta^{ab} + \text{fin},
$$

$$
V_{\mu abc}^{(1)}(p, k) = - \lim_{\Lambda \to \infty} \frac{g_0^2 c_v}{16\pi^2} \left\{ \frac{1}{2} \log \left(\frac{\Lambda}{\kappa^2}\right) \right\} g_0 f_{\mu a}^{\hat{a}} i p_{\hat{a}} + \text{fin}.
$$

One can easily see that equations (40-42) lead to the same Renormalization Group coefficients (39). So the HCD-PV regularization scheme as it is presented here leads to consistent results.

4 Conclusions

In this paper I have derived a Higher Covariant Derivative Pauli Villars regularization scheme for pure Yang-Mills within the context of the Batalin Vilkovisky formalism. This was done in two steps. First I introduced the Higher Covariant derivatives and did the gauge fixing. This regularizes all higher loop diagrams. In order to regularize one loop (sub)diagrams PV fields are added to the theory. Here BRS invariance is broken explicitly, but the counterterm for the case of PV has been presented. When calculating the one loop effective action everything is consistent with other regularization schemes. The BV scheme implies a preferred choice for the $Z$-factors.

The remaining question, which I didn’t address here, is “What about the higher loop diagrams?” It is clear that one has to introduce more PV fields for the higher loop diagrams, even an infinite number of generations. There is a priori no objection to that because PV is applied in a perturbative context. One drawback is that when you introduce second order PV fields they not only have to couple to first order PV fields but also to the original fields. The use of Higher Covariant Derivatives makes the algebra very complicated (cfr. appendix A) and doesn’t regularize all higher loop diagrams. Therefore, in spite of the consistency exhibited in this paper, one could have some doubt about the usefulness of Higher Covariant Derivative regularization.
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I wish to thank W. Troost and A. Van Proeyen for introducing me to this problem and for useful suggestions.

A Feynman rules

The Feynman rules associated with (15) are:

**Propagators**

\[
\frac{\delta^{ab}\Lambda^4 g_{\mu\nu}}{p^2 (p^2 + \Lambda^2)^2}
\]

\[
\frac{\delta^{ab}\Lambda^4 g_{\mu\nu}}{(p^2 + M_i^2) (p^2 + \Lambda^2)^2}
\]

\[
\frac{\delta^{ab}\Lambda^2}{p^2 (p^2 + \Lambda^2)}
\]

These Feynman rules are given for the Feynman gauge \(\alpha = 1\). As we pointed out in section 2, the denominators factorize in factors that are quadratic in \(p\). This gives the opportunity to use standard manipulations (feynman-parameters etc.) to calculate diagrams (cfr. Appendix B).

**Vertices**

\[
- \frac{ig}{\Lambda^4} S_3 \left\{ f_{a_2a_3}^{a_1} \left[ \Lambda^4 p_{1\mu_2} g_{\mu_3\mu_1} - p_1^4 p_{1\mu_2} g_{\mu_1\mu_2} 
\right.ight.
\]

\[
\left. + \Lambda^2 p_{1\mu_2} (p_1^2 g_{\mu_3\mu_1} + p_{1\mu_3} p_{3\mu_1})
\right.
\]

\[
\left. p_1^2 (p_3 - p_1)_{\mu_2} (p_{1\mu_3} p_{3\mu_1} - p_1 \cdot p_3 g_{\mu_1\mu_3}) \right\}
\]
where $S_3$ is the symmetrization operator with respect to the indices 1, 2 and 3.

\[ -g^2 (A^a_{\mu_1} A^a_{\mu_4} - ig f^a_{bc} \Lambda^2 (p^2 + p^2) p_{\mu}) \]

Here also $S_4$ means symmetrization with respect to 1, 2, 3 and 4.

I do not exhibit the higher order vertices here, because they don’t show up in the calculations at one-loop order.

**B A sample computation**

In this appendix I will analyse the explicit calculation of the one loop Feynman diagrams. My mean point here is to show that one can stay at space-time dimension four during the whole calculation. As an example I take the first line of fig. 1.
After doing the algebra the momentum integral is

\[ I = \frac{1}{2} f_{cd}^a f_{cd}^b \int \frac{d^4k}{(2\pi)^4} G(\mu, \nu, p, k) \]

\[ \sum_{i=1}^{N} c_i \frac{1}{(k^2 + M_i^2)(k^2 + \Lambda^2)^2((k + p)^2 + M_i^2)((k + p)^2 + \Lambda^2)^2} - \]

\[ \frac{1}{k^2(k^2 + \Lambda^2)^2(k + p)^2((k + p)^2 + \Lambda^2)^2}. \] (43)

In order to perform this step, we have to demand that

\[ \sum_{i=1}^{N} c_i = -1. \] (44)

Because the denominators are factorizing one can now use the standard Feynman parameter procedure to combine the factors,

\[ I = \frac{6}{2} f_{cd}^a f_{cd}^b \int \frac{d^4k}{(2\pi)^4} \int_{0}^{1} dx \int_{0}^{1} dy G(\mu, \nu, p, k) y(1-y) \]

\[ \sum_{i=1}^{N} c_i \frac{1}{(k^2 + 2xk \cdot p + xp^2 + M_i^2)^2(k^2 + 2yk \cdot p + yp^2 + \Lambda^2)^4} - \]

\[ \frac{1}{(k^2 + 2xk \cdot p + xp^2)^2(k^2 + 2yk \cdot p + yp^2 \Lambda^2)^4}. \] (45)

In order to increase the degree of the denominator we combine the terms through,

\[ \frac{1}{(k + a)^n} - \frac{1}{(k + b)^n} = n \int_{a}^{b} \frac{dx}{(k + x)^{n+1}}. \] (46)

Applying this twice, demanding that

\[ \sum_{i=1}^{N} c_i M_i^2 = 0, \] (47)

and using another Feynman parameter, you arrive at

\[ I = \frac{\Gamma(8)}{2} f_{cd}^a f_{cd}^b \int \frac{d^4k}{(2\pi)^4} \int_{0}^{1} dx \int_{0}^{1} dy \sum_{i=1}^{N} c_i \int_{0}^{M_i^2} da \int_{0}^{M_i^2} db \int_{0}^{1} dz G(\mu, \nu, p, k) z^3(1-z)^3 y(1-y) \]

\[ \frac{1}{(k^2 + 2zk \cdot p + xp^2 + 2(1-z)yk \cdot p + (1-z)yp^2 + zb + (1-z\Lambda^2)^8)}. \] (48)

Note that in the last time you apply (46) the renormalization scale \( \kappa \) comes in. Finally one shifts \( k \) to \( k' = k - (zx(1-z)y)p \). This can be done because the \( k \)-integral is now finite. Now all integrals can be interchanged and performed starting with the momentum integral. Once again I’d like to stress that all manipulations were done in four dimensions.
References


Figures

\[ \int d^4k \] \hspace{1cm} \begin{array}{c}
A^a_{\mu} \\
\vdash \end{array} \hspace{1cm} p \hspace{1cm} \begin{array}{c}
k + p \hspace{0.5cm} A^b_{\mu} \\
\vdash \end{array} \hspace{1cm} p \hspace{1cm} \begin{array}{c}
k \hspace{0.5cm} A^a_{\mu} \\
\vdash \end{array} + \sum_{i=1}^{N} c_i \begin{array}{c}
k \hspace{0.5cm} A^a_{\mu} \\
\vdash \end{array} \hspace{1cm} p \hspace{1cm} \begin{array}{c}
k + p \hspace{0.5cm} A^b_{\mu} \\
\vdash \end{array} \hspace{1cm} p \hspace{1cm} \begin{array}{c}
k \hspace{0.5cm} A^a_{\mu} \\
\vdash \end{array} \hspace{1cm} p
\]

fig 1. One loop contributions to the vacuum polarization tensor

\[ \int d^4k \] \hspace{1cm} \begin{array}{c}
p \hspace{1cm} A^a_{\mu} \hspace{1cm} \vdash \hspace{1cm} A^b_{\mu} \hspace{1cm} \vdash \hspace{1cm} p \end{array}

\text{counterterm to restore BRS-invariance}

fig 2. One loop contributions to the ghost selfenergy
\[
\int d^4k \quad k - p_1 \quad k \quad + \sum_{i=1}^{N} c_i \quad k - p_1 \quad k
\]

\[
\int d^4k \quad k - p_1 \quad k \quad + \sum_{i=1}^{N} c_i \quad k - p_1 \quad k
\]

fig. 3: One loop contributions to the ghost-gluon-ghost vertex.

\[
\int d^4k \quad A^\alpha_\mu \quad p \quad k \quad p \quad + \sum_{i=1}^{N} c_i \quad A^\alpha_\mu \quad k \quad p
\]

fig 4.: One loop contributions to \( \langle A^\alpha_\mu \partial^\mu c^\alpha \rangle \)