FADDEEV-JACKIW FORMALISM FOR A TOPOLOGICAL-LIKE OSCILLATOR IN PLANAR DIMENSIONS

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Abstract

The problem of a harmonic oscillator coupling to an electromagnetic potential plus a topological-like (Chern-Simons) massive term, in two-dimensional space, is studied in the light of the symplectic formalism proposed by Faddeev and Jackiw for constrained systems.


1 Introduction

The dynamics of gauge field theories with the Chern-Simons topological mass term in (2+1) dimensions [1] is a quite interesting subject and accordingly keeps being the goal of much investigation [2]. Besides its mathematical interest, Chern-Simons theories have been an important laboratory to explain some condensed matter phenomena like the fractional quantum Hall effect and high-$T_c$ behavior of superconductivity [3]. But, perhaps, its main characteristic is that unusual spin states and statistics appeared at the quantum mechanical level [4]. This feature has motivated the study of quantum mechanical systems in (2+1) dimensions trying to understand the role of anyons in quantum theory of Chern-Simons [5]- [12]. Recently, Dunne, Jackiw and Trugenberger [7] have studied a quantum mechanical oscillator model in close analogy to topological Chern-Simons systems (in a reduced phase space):

\[ L = \frac{B}{2} \vec{q} \times \dot{\vec{q}} - \frac{\kappa}{2} q^2. \] (1.1)

This model basically mimics the motion of a non-relativistic point particle in two dimensions under the influence of a perpendicular constant magnetic field. In that paper they discussed the relation of this model with conventional Chern-Simons theory (also with reduced phase space so that the Maxwell term vanishes). Here, we are going to discuss an analogous model starting from a simple charged harmonic oscillator which experiences an external electromagnetic field governed by a Chern-Simons term (instead of the usual Maxwell one). The justification for such a system is that at low energies the Chern-Simons term dominate over the Maxwell one [5] and this is the regime we are interested in (it is well known that the inclusion of a Maxwell term besides the Chern-Simons one suppresses fractional statistics [6]).

Our model can also be sought as an extension of the one discussed by Matsuyama with canonical quantization of a charged particle in the presence of an electromagnetic field plus a Chern-Simons term (without an oscillator potential). A relativistic version of this situation was also considered by Cortes, Gamboa and Velazquez [10]. The main
The goal of this work is to explore the analogy suggested in ref. [7], to study the Hamiltonian quantization of a harmonic oscillator coupled to the electromagnetic potential plus a topological Chern-Simons term in two dimensions, from the symplectic formalism point of view [13]. In this approach, the phase space is reduced in such a way that the Lagrangian depends on the first-order generalized velocities. The advantage of this linearization is that the non-null Dirac brackets are the elements of the inverse symplectic matrix [15]. The method becomes more involved when gauge fields come together, which is the case under analysis here, once the system gets constrained. In this case the symplectic matrix is singular and has no inverse unless a gauge-fixing term is included [16]. In this work, we want to shed some light on the symplectic formalism for constrained and unconstrained systems, taking first as an example the simple harmonic oscillator in section 2. Section 3 is devoted to discuss the oscillator coupling to a gauge field plus a Chern-Simons term. We finalize the paper in section 4 with the conclusions.

2 The symplectic formalism and the harmonic oscillator

In this section we intend to give the basic ideas of the Faddeev-Jackiw symplectic method [13] and discuss the quantization of the harmonic oscillator in this scheme. For a general review and application to other systems see for instance refs. [14]-[17].

We begin considering by a phase space $\Gamma(q^i, p^i), (i = 1, ..., N)$ such that its algebraic structure is characterized by a rank-two antisymmetric tensor $\omega_{\alpha\beta}$, whose components are

\[ \{q^i, q^j\} = 0 = \{p^i, p^j\}; \quad \{q^i, p^j\} = \delta_{ij}. \quad (2.1) \]

In this step, coordinates and respectively canonical momenta are assumed to be independent variables in $\Gamma(q^i, p^i)$.

From a mathematical point of view, we can consider the coordinates of phase space as $x^\alpha = x^\alpha(q^i, p^i), (\alpha = 1, ..., 2N)$ in such a way that the algebraic structure is determined by a rank-two antisymmetric tensor $\omega^{\alpha\beta}$, whose components are
where \( \det \omega^{\alpha \beta} \neq 0 \).

The tensor \( \omega^{\alpha \beta} \) permit us to rewrite the Poisson bracket of two functions \( A(q_i, p_i) \) and \( B(q_i, p_i) \) in a compact form

\[
\{ A(x), B(x) \} = \partial_\alpha A \omega^{\alpha \beta} \partial_\beta B ,
\]

(2.3)

where \( \partial_\alpha \equiv \partial/\partial x^\alpha \) and since \( \det \omega^{\alpha \beta} \neq 0 \), we can invert it. The inverse two-form is denoted by \( \omega_{\alpha \beta} \) and satisfy

\[
\omega_{\alpha \beta} \omega^{\beta \gamma} = \delta_{\alpha \gamma} ,
\]

(2.4)

such that its determinant is also non-singular. A two-form which obeys the relation (2.4) defines a symplectic structure which, on the other hand, gives rise to generalized (Dirac) brackets,

\[
\{ x^\alpha, x^\beta \}^*_{GB} \equiv \{ x^\alpha, x^\beta \}_D = (\omega^{\alpha \beta})^{-1} .
\]

(2.5)

In order to show explicitly the above result, let us consider a first order Lagrangian

\[
L = a_\alpha(x) \dot{x}^\alpha - V(x) .
\]

(2.6)

From the variational principle we get

\[
\int dt \left[ \left( \partial_\alpha a_\beta(x) - \partial_\beta a_\alpha(x) \right) \dot{x}^\beta - \partial_\alpha V(x) \right] = 0
\]

(2.7)

and we define the two-rank antisymmetric tensor

\[
\Omega_{\alpha \beta} \equiv \partial_\alpha a_\beta - \partial_\beta a_\alpha .
\]

(2.8)

At this step, there are two possibilities to deal with:
\textbf{a}) when }\det\Omega_{\alpha\beta} \neq 0, \text{ we can consider } \Omega_{\alpha\beta} \text{ as the symplectic matrix. In this way, the velocities can be obtained in a trivial manner}

\begin{equation}
\dot{x}^\alpha = \Omega^{\alpha\beta} \partial_\alpha V , \tag{2.9}
\end{equation}

where }\Omega^{\alpha\beta} \equiv (\Omega_{\alpha\beta})^{-1}. \text{ The Hamiltonian form corresponding to eq. (2.9) is the following}

\begin{equation}
\dot{x}^\alpha = \{x^\alpha, V(x)\} = \{x^\alpha, x^\beta\}_{GB} \partial_\beta V , \tag{2.10}
\end{equation}

therefore we can identify the generalized bracket

\begin{equation}
\{x^\alpha, x^\beta\}_{GB} = \Omega^{\alpha\beta} . \tag{2.11}
\end{equation}

On the other hand, since there are not constraints involved in this case (once the tensor }\Omega^{\alpha\beta} \text{ is invertible) we conclude that

\begin{equation}
\{x^\alpha, x^\beta\}_{GB} \equiv \{x^\alpha, x^\beta\}_D = \Omega^{\alpha\beta} , \tag{2.12}
\end{equation}

\emph{i. e.}, in the symplectic formalism the Dirac brackets are associated to the elements of the matrix }\Omega^{\alpha\beta} \text{ (inverse of }\Omega_{\alpha\beta}).

Let us illustrate this unconstrained case with the example of the (one-dimensional) harmonic oscillator

\begin{equation}
L = \frac{m}{2} \left( \dot{q}^2 - \omega^2 q^2 \right) . \tag{2.13}
\end{equation}

In order to apply the symplectic formalism we should first linearize the quadratic term }\dot{q}^2. \text{ Following the procedure adopted in a recent paper [17], we have}

\begin{equation}
\dot{q}^2 \longrightarrow 2p, \dot{q} - p^2 . \tag{2.14}
\end{equation}

Here, }p \text{ is an auxiliary variable. So, the first-order Lagrangian becomes
\[ L = mp\dot{q} - V \, , \quad (2.15) \]

where \( V \) is the symplectic potential

\[ V = \frac{m}{2} \left( p^2 + \omega^2 q^2 \right) . \quad (2.16) \]

Thus, from eq. (2.6) we conclude that

\[ a_q = mp \, ; \quad a_p = 0 \quad (2.17) \]

and consequently the tensor \( \Omega_{\alpha\beta} \) is given by

\[ \Omega_{\alpha\beta} = \Omega_{qp} = -\frac{\partial a_q}{\partial p} + \frac{\partial a_p}{\partial q} = -m = -\Omega_{pq} . \quad (2.18) \]

The matrix \( \Omega_{\alpha\beta} \) is naturally

\[ \Omega_{\alpha\beta} = m \begin{pmatrix} 0 & -1 \\ +1 & 0 \end{pmatrix} \quad (2.19) \]

This is the symplectic matrix whose inverse permit us to identify the brackets

\[ \{q, p\} = \frac{1}{m} ; \quad \{q, q\} = \{p, p\} = 0 . \quad (2.20) \]

The first Poisson bracket in eq. (2.20) has been written in unusual form. This happened because \( p \) is an auxiliary variable and not the canonical momentum associated with variable \( q \). In order to find the usual canonical relation we should go back to the Lagrangian (2.15). The canonical momentum is defined in the usual manner

\[ P = \frac{\partial L}{\partial \dot{q}} = mp . \quad (2.21) \]

Therefore, we get

\[ \{q, P\} = 1 ; \quad \{q, q\} = \{P, P\} = 0 , \quad (2.22) \]
which are the canonical Poisson bracket relations.

b) The second possibility occurs if \( \det \Omega_{\alpha\beta} = 0 \). In this case we cannot identify \( \Omega_{\alpha\beta} \) as the symplectic matrix. This feature reveals that the system under consideration is constrained [16]. An alternative manner to circumvent this problem is to use the constraints conveniently to change the coefficients \( a_{\alpha}(q) \) in the first-order Lagrangian (2.6) and consequently obtain a final two-rank tensor which could be identified with the symplectic matrix.

In the present case, we can build up an eigenvalue equation with matrix \( \Omega_{\alpha\beta} \) and \( m (m = 1, \ldots, M < 2N) \) eigenvectors \( v^\alpha \) such that

\[
v_m^\alpha \Omega_{\alpha\beta} = 0 \, . \tag{2.23}
\]

From eqs. (2.7) and (2.23) we can write

\[
v_m^\alpha \partial_\alpha V \equiv \Sigma_m = 0 \, , \tag{2.24}
\]

which defines possible constraints \( \Sigma_m \). By imposing that \( \Sigma_m \) does not evolve in time, we arrive at

\[
\dot{\Sigma}_m = (\partial_\alpha \Sigma_m) \dot{q}^\alpha = 0 \tag{2.25}
\]

and since \( \dot{\Sigma}_m \) is linear with \( \dot{q}^\alpha \) we can incorporate this factor into the Lagrangian (2.6) by means of Lagrange multipliers \( \lambda_\alpha \). So, by considering the rescale

\[
\bar{a}_\alpha = a_\alpha + \lambda_\beta \partial_\beta \Sigma \, , \tag{2.26}
\]

where \( a_\alpha \) is the original coefficient, we get a new two-rank antisymmetric tensor \( \bar{\Omega}_{\alpha\beta} \) in such a way that

\[
\bar{\Omega}_{\alpha\beta} = \partial_\alpha \bar{a}_\beta - \partial_\beta \bar{a}_\alpha \, . \tag{2.27}
\]
After completing this step if $\tilde{\Omega}_{\alpha\beta}$ is still vanishing we must repeat the above strategy until we find a non-singular matrix. As has been pointed in ref. [16], for systems which involve gauge fields it may occur that the matrix is singular and the eigenvectors $v^m_\alpha$ do not lead to any new constraints. Since the main goal of this procedure is to obtain the symplectic tensor it is necessary to choose some gauge condition. Such a case will be discussed in next section.

3 Chern-Simons Oscillator

Let us now extend our previous discussion to the problem of a two-dimensional harmonic oscillator coupled to electromagnetic field plus a Chern-Simons term. This system is described by the Lagrangian

$$L^{(0)} = \frac{m}{2} \left[ \dot{q}_i(t)\dot{q}^i(t) - \omega^2 q_i(t)q^i(t) \right] - \int d^2 x eA_0(t, \vec{x}) \delta(\vec{x} - \vec{q}) + \int d^2 x A_i(t, \vec{x}) \delta(\vec{x} - \vec{q}) \dot{q}^i(t) + \theta \int d^2 x \epsilon_{\mu\nu\rho} A_\mu(t, \vec{x}) \partial_\nu A_\rho(t, \vec{x}) \quad (3.1)$$

where $q_i(t)$ is the particle coordinate with charge $-e$ on the plane $(i = 1, 2)$, $A_\mu(t, \vec{x})$ is the electromagnetic potential $(\mu = 0, 1, 2)$, $\theta$ is the Chern-Simons parameter, $\epsilon_{012} = e^{012} = 1$ and $g^{\mu\nu} = diag(- + +)$. In order to proceed with the symplectic quantization of this system we linearize the kinetic term as was done for the simple harmonic oscillator, eq.(2.14), so we get

$$L^{(0)} = m \left[ p_i(t) - \frac{e}{m} A_i(t, \vec{q}) \right] \dot{q}^i(t) - \theta \int d^2 x \epsilon_{ij} \dot{A}^i(t, \vec{x}) \dot{A}^j(t, \vec{x}) - V^{(0)} \quad (3.2)$$

where we used the fact that $A_i(t, \vec{q}) = \int d^2 x A_i(t, \vec{x}) \delta(\vec{x} - \vec{q})$ and defined the potential

$$V^{(0)} = \frac{m}{2} \left[ p_i(t)\dot{p}^i(t) + \omega^2 q_i(t)\dot{q}^i(t) \right] + eA_0(t, \vec{q}) + 2\theta \int d^2 x \epsilon_{ij} \partial^i A^j(t, \vec{x})A_0(t, \vec{x}) \quad (3.3)$$

Once the Lagrangian (3.2) has the general symplectic form (2.6) we can identify the coefficients
\[ a_{q_i}^{(0)}(t) = m p_i(t) - e A_i(t, \vec{q}) \]  
(3.4)

\[ a_{A_i}^{(0)}(t, \vec{x}) = - \theta \epsilon_i j A_j(t, \vec{x}) \]  
(3.5)

\[ a_{p_i}^{(0)}(t) = 0 ; \quad a_{A_0}^{(0)} = 0 \]  
(3.6)

and calculate the matrix elements using its definition, eq. (2.8),

\[ \Omega_{q_i p_j}^{(0)} = - m \delta_{ij} \]  
(3.7)

\[ \Omega_{q_i A_j(t, \vec{y})}^{(0)} = e \delta_{ij} \delta(\vec{y} - \vec{q}) \]  
(3.8)

\[ \Omega_{A_i(t, \vec{x}) A_j(t, \vec{y})}^{(0)} = 2 \theta \epsilon_i j \delta(\vec{x} - \vec{y}) , \]  
(3.9)

while the others are vanishing. This way, we construct the matrix with the convention

\[ y^{\alpha} = (\vec{q}, \vec{p}, \vec{A}, A_0) \]

\[ \Omega^{(0)}_{\alpha \beta} = \begin{pmatrix}
0 & - m \delta_{ij} & e \delta_{ij} \delta(\vec{y} - \vec{q}) & 0 \\
- m \delta_{ij} & 0 & 0 & 0 \\
e \delta_{ij} \delta(\vec{y} - \vec{q}) & 0 & 2 \theta \epsilon_i j \delta(\vec{x} - \vec{y}) & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix} \]  
(3.10)

which is obviously singular. Following the steps reviewed in the previous section, we determine the non-trivial zero-modes associated with this singular matrix, solving the equation
\[ \Omega^{(0)}_{\alpha\beta} v^{(0)\beta} = 0 ; \quad v^{(0)\beta} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}, \] (3.11)

so that \( a = b = c = 0 \) and \( d \) remains arbitrary. Since these zero-modes also satisfy

\[ v^{(0)\beta} \partial_\beta V^{(0)} = 0 , \] (3.12)

we have

\[ d \frac{\partial V^{(0)}}{\partial A_0} = d (e + 2 \theta \epsilon_{ij} \partial^i A^j) = 0 \] (3.13)

and since \( d \) is arbitrary we are led with a constraint:

\[ \Sigma^{(0)} = e + 2 \theta \epsilon_{ij} \partial^i A^j = 0 . \] (3.14)

The next step is to remove the singularity from the symplectic matrix by including this constraint into the Lagrangian (3.2), so we write:

\[ L^{(1)} = L^{(0)} + \Sigma^{(0)} \dot{\lambda} , \] (3.15)

where \( \lambda = \lambda(\vec{x}) \) is a Lagrange multiplier and an integration over space is assumed for the last term of the above equation and for the following throughout. The generalized potential is now given by

\[ V^{(1)} = V^{(0)} \bigg|_{\Sigma^{(0)} = 0} = \frac{m}{2} \left( p^2 + \omega^2 q^2 \right) , \] (3.16)

which coincides with the one for the simple harmonic oscillator, eq. (2.16). To calculate the new symplectic matrix we must obtain its coefficients \( a_\alpha \). In fact, they are the same as given by eqs. (3.4)-(3.6) with the addition of the one corresponding to \( \lambda \):

\[ a^{(1)}_\lambda = e + 2 \theta \epsilon_{ij} \partial^i A^j \] (3.17)
and the exclusion of $A_0$, since $A_0$ is no longer an explicit dynamical variable of the problem. The matrix elements are therefore given by eqs. (3.7)-(3.9) with the addition

$$\Omega^{(1)}_{\lambda, \lambda} = 2\theta\epsilon_{ij}\partial^j\delta(\vec{x} - \vec{y})$$

(3.18)

and the others are still vanishing. This way we have the new (symplectic) matrix ($y^\alpha = (\vec{q}, \vec{p}, \vec{A}, \lambda)$)

$$\Omega^{(1)}_{\alpha\beta} = \begin{pmatrix} 0 & -m\delta_{ij} & e\delta_{ij}\delta(\vec{y} - \vec{q}) & 0 \\ m\delta_{ij} & 0 & 0 & 0 \\ -e\delta_{ij}\delta(\vec{y} - \vec{q}) & 0 & 2\theta\epsilon_{ij}\delta(\vec{x} - \vec{y}) & 2\theta\epsilon_{ij}\partial^i\delta(\vec{x} - \vec{y}) \\ 0 & 0 & -2\theta\epsilon_{ij}\partial^j\delta(\vec{x} - \vec{y}) & 0 \end{pmatrix}$$

(3.19)

As we can easily check this matrix is still singular. Furthermore, the search for non-trivial zero-modes would be unfruitful since here the potential is simply $(m^2)(p^2 + \omega^2 q^2)$. Therefore, following ref. [17], we are going to fix the gauge, which we choose to be the Weyl one ($A_0 = 0$). Once $A_0$ is absent from the Lagrangian (3.15) and noting the equivalence $A_0 = \dot{\lambda}$ we introduce a Lagrange multiplier $\eta = \eta(\vec{x})$ for $\lambda$:

$$L^{(2)} = L^{(1)} + \eta \dot{\lambda} ,$$

(3.20)

so that the new coefficients are given by

$$a^{(2)}_{\lambda} = e + 2\theta\epsilon_{ij}\partial^j A^i + \eta ; \quad a^{(2)}_{\eta} = 0$$

(3.21)

and the others are unchanged. This way, collecting the coefficients we get for the symplectic matrix: ($y^\alpha = (\vec{q}, \vec{p}, \vec{A}, \lambda, \eta)$)
\[ \Omega_{\alpha \beta}^{(2)} = \begin{pmatrix} 0 & -m\delta_{ij} & e\delta_{ij}\delta(\vec{y} - \vec{q}) & 0 & 0 \\ m\delta_{ij} & 0 & 0 & 0 & 0 \\ -e\delta_{ij}\delta(\vec{y} - \vec{q}) & 0 & 2\theta\epsilon_{ij}\delta(\vec{x} - \vec{y}) & 2\theta\epsilon_{ij}\partial^i\delta(\vec{x} - \vec{y}) & 0 \\ 0 & 0 & -2\theta\epsilon_{ij}\partial^j\delta(\vec{x} - \vec{y}) & 0 & -\delta(\vec{x} - \vec{y}) \\ 0 & 0 & 0 & \delta(\vec{x} - \vec{y}) & 0 \end{pmatrix} \]

which is not singular. Its inverse can be readily obtained and we find

\[ \Omega^{\alpha \beta} = \frac{1}{2m\theta} \begin{pmatrix} 0 & 2\theta\delta^{ij} & 0 & 0 & 0 \\ -2\theta\delta^{ij} & 0 & e,\epsilon^{ij}\delta(\vec{x} - \vec{q}) & 0 & 2\theta e\partial^j\delta(\vec{x} - \vec{q}) \\ 0 & e\epsilon^{ij}\delta(\vec{x} - \vec{q}) & m\epsilon^{ij}\delta(\vec{x} - \vec{y}) & 0 & 2m\theta\partial^i\delta(\vec{x} - \vec{y}) \\ 0 & 0 & 0 & 0 & 2m\theta\delta(\vec{x} - \vec{y}) \\ 0 & 2\theta e\partial^i\delta(\vec{x} - \vec{q}) & 2m\theta\partial^i\delta(\vec{x} - \vec{y}) & -2m\theta\delta(\vec{x} - \vec{y}) & 0 \end{pmatrix} \] (3.23)

From the elements of the above matrix we find the Dirac brackets of the theory:

\[ \{q_i, p_j\} = \frac{1}{m} \delta_{ij} \] (3.24)

\[ \{p_i, A_j\} = \frac{e}{2m\theta} \epsilon_{ij}\delta(\vec{x} - \vec{q}) \] (3.25)

\[ \{p_i, \eta\} = \frac{e}{m} \partial_i\delta(\vec{x} - \vec{q}) \] (3.26)

\[ \{A_i, A_j\} = \frac{1}{2\theta} \epsilon_{ij}\delta(\vec{x} - \vec{y}) \] (3.27)
\[ \{ A_i, \eta \} = \partial_i \delta(\vec{x} - \vec{y}) \]  
\[ \{ \lambda, \eta \} = \delta(\vec{x} - \vec{y}) \]

while the others are vanishing. The last three of the above Dirac brackets coincide with those given by Barcelos-Neto and de Souza [17] for the pure Chern-Simons theory, in the \( A_0 = 0 \) gauge. The first three relations can be rewritten in terms of the canonical momentum of the particle, as was done for the simple oscillator in previous section:

\[ P_i = \frac{\partial L}{\partial \dot{q}_i} = m p_i - e A_i , \]

so that we find

\[ \{ q_i, P_j \} = \delta_{ij} ; \quad \{ p_i, A_j \} = 0 ; \quad \{ p_i, \eta \} = 0 \]

which are in agreement with Matsuyama [8]. From the Lagrangian \( L^{(2)} \), eq. (3.20), and the Euler-Lagrange equations we find the equations of motion, which hold strongly at the operator level:

\[ m \ddot{p}_i - e \dot{A}_i + m \omega^2 q_i = 0 \]  
\[ p_i = \dot{q}_i \]  
\[ e \dot{q}_i - 2 \theta \epsilon_{ij}(\dot{A}_j - \partial^i \dot{\lambda}) = 0 \]  
\[ \dot{\eta} + 2 \theta \epsilon_{ij} \partial^i A^j = 0 \]  
\[ \dot{\lambda} = 0 \]
In particular, the first two equations characterize the harmonic oscillator motion in the presence of an external electromagnetic field and the fourth equation defines the Lagrange multiplier $\eta$. Substituting last equation in the third we have a powerful relation

$$\dot{q}_i = \frac{2\theta}{e} \epsilon_{ji} \dot{A}^j,$$  \hspace{1cm} (3.37)

or explicitly

$$\dot{q}_i(t) = \frac{2\theta}{e} \epsilon_{ji} \int d^2x \dot{A}^j(t, \bar{x}) \delta(\bar{x} - \bar{q})$$

$$= \frac{2\theta}{e} \epsilon_{ji} \dot{A}^j(t, \bar{q}),$$  \hspace{1cm} (3.38)

which can be integrated in time giving up to a constant

$$A_j(t, \bar{q}) = \frac{e}{2\theta} \epsilon_{ji} q^i(t).$$  \hspace{1cm} (3.39)

This relation shows that the electromagnetic potential corresponds to the one of a singular magnetic field

$$B = \epsilon_{ij} \partial^i A^j$$

$$= \epsilon_{ij} \frac{\partial}{\partial x_i} A^j(t, \bar{q})$$

$$= \frac{e}{2\theta} \delta(\bar{x} - \bar{q}),$$  \hspace{1cm} (3.40)

which gives a flux

$$\phi = \int d^2x B = \frac{e}{2\theta}.$$  \hspace{1cm} (3.41)

As is well known [4], [12], this particular magnetic flux implies a fractional spin for the particle just described, since $\theta$ can assume any value, while the kinetic angular momentum has only integer values.
4 Conclusions

We have studied in this paper the Faddeev-Jackiw quantization for a simple (unconstrained) oscillator and also an oscillator coupling to a gauge field with a Chern-Simons term. This last example is naturally constrained by virtue of the presence of gauge fields. Apart from its academic interest we can mention that the Chern-Simons oscillator constitutes a very interesting model, in particular for bringing fractional statistics. Naturally, this is not surprising, but here we have an alternative to the Dunne-Jackiw-Trugenberger model. Our construction was also inspired in a work of Matsuyama where a charged particle couples to electromagnetic field and Chern-Simons term (without an oscillator potential). The main differences from his work to ours is that he worked with canonical quantization, in the Coulomb gauge, while we used a symplectic formalism in another gauge \((A_0 = 0)\). This also bring us a bonus which indicates that in this model fractional statistics is not a gauge artifact reaching the same conclusion as the one obtained by Foerster and Girotti for the pure Chern-Simons theory [6]. Besides, we have included a harmonic potential, which does not change the symplectic structure so our analysis can be readily extended to other potentials.

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