Second Class Current in QCD Sum Rules

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Abstract

Induced tensor charge of the nucleon $g_T$, which originates from G-parity violation, is evaluated from QCD sum rules. We find that $g_T/g_A$ with $g_A$ being the axial charge is $-0.0152 \pm 0.0053$ which is proportional to u-d quark mass difference. This result is small compared to preliminary analysis of the experiment, but is consistent with the estimate in the MIT bag model.

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I. INTRODUCTION

Deriving the coupling constants from QCD [1] is one of the most important themes in hadron physics. Also, getting precise values of the coupling constants from the first principle will enable us to make more quantitative predictions for nuclear systems.

For extracting the hadronic quantities from QCD, QCD Sum Rules (QSR) discussed below are known to be one of the two powerful tools. (The other is lattice QCD simulations [2].) QSR was first proposed in a paper by Shifman Vainsthein, and Zakharov in 1978 [3], in which the main idea and the application to meson systems such as meson masses, decay rates and $\rho - \omega$ mixing, are shown. Later, many applications have been made [4,5]. The extension to the baryon systems was put forward by Ioffe [6]. QSR with external field was also proposed by Ioffe and Smilga [7], which will be discussed later.

In this paper we will evaluate induced tensor charge of the nucleon ($g_T$) with the help of axial charge ($g_A$) and nucleon sum rules. $g_T$ and $g_A$ are defined by the nucleon matrix element of the axial current, 

$$
\langle P(p_2)|A_\mu^5|N(p_1)\rangle
= \bar{u}_p(p_2)(\gamma_\mu \gamma_5 g_A + \frac{\gamma_5 g_P}{M_p + M_n} + i\frac{\gamma_5 \sigma_{\mu\nu} q^\nu}{M_p + M_n} g_T)u_n(p_1), \quad q = p_1 - p_2,
$$

(1)

where $M_p(M_n)$ is proton (neutron) mass, $A_\mu^5 = \bar{u}\gamma_\mu \gamma_5 d$ represents axial current and $u_p(u_n)$ reveals proton (neutron) wave function. $g_P$ is called induced pseudoscalar constant, which will not be examined in this paper. The deviation of $g_A$ from unity is a reflection of the underlying composite structure of the nucleon and there have been many studies on $g_A$, e.g., QCD sum rules [8,9], quark models [10], the bag model [11] and the skyrme model [12]. The difference of $g_A$ and $g_T$ is classified by $G$-parity which is the charge conjugation $C$ combined with the rotation of 180° around the y-axis in the isospin space

$$
G = Ce^{iI_5\pi},
$$

(2)

where $I_y$ is the rotational matrix around the y-axis in the isospin space.

Under the $G$-parity,

$$
G\bar{p}\gamma_\mu \gamma_5 n G^{-1} = -\bar{p}\gamma_\mu \gamma_5 n, \quad G\bar{p}\sigma_{\mu\nu} \gamma_5 n G^{-1} = \bar{p}\sigma_{\mu\nu} \gamma_5 n, \quad GA_\mu^5 G^{-1} = -A_\mu^5.
$$

(3)

The first current in eq.(3) with the same sign as that of $A_\mu^5$ under $G$-parity is called the first class current, while the second current in eq.(3) with the opposite sign as that of $A_\mu^5$ is referred to as the second class current [14].

There are two sources of $G$-parity violation in the standard model. One is from QED (electric charges of $u$ and $d$ quarks are different) and another is from the mass term in the QCD Hamiltonian (masses of $u$ and $d$ quarks are different). We will exclusively examine the latter effect in this thesis.

The mass term in the QCD Hamiltonian is written as

$$
H_{mass} = \frac{1}{2}(m_u + m_d)(\bar{u}u + \bar{d}d) + \frac{1}{2}(m_u - m_d)(\bar{u}u - \bar{d}d).
$$

(4)
where the light quark masses are determined from analyses of the hadron mass splittings in QCD sum rules [13];

\[ m_u(\mu = 1\text{GeV}) = (5.1 \pm 0.9)\text{MeV}, \quad m_d(\mu = 1\text{GeV}) = (9.0 \pm 1.6)\text{MeV}. \]

Under G-parity, the first term in eq.(4) does not change the sign, but the second term in eq.(4) changes sign, which means that \( g_T \) is represented as \( g_T \sim (m_u - m_d)/M_N \) since \( g_T \) is dimensionless. This implies that \( g_T \) will be much smaller than \( g_A \) because of the small \( u - d \) quark mass difference \( m_d - m_u \sim 5\text{MeV} \).

This rough estimate of \( g_T \) using the G-parity violation, however, may not be consistent with the analyses of the experimental data given by measuring the beta-ray angular distribution in aligned \( ^{12}\text{B} \) and \( ^{12}\text{N} \) [15]. In Ref. [15], results of the analyses using the experimental data are quoted as

\[ g_T/g_A = 0.14 \pm 0.10 \quad \text{in 1985}, \quad (5) \]
\[ g_T/g_A = -0.21 \pm 0.14 \quad \text{in 1992}. \quad (6) \]

This shows that \( g_T \) is of order 10% compared to \( g_A \), which is order of magnitude larger than the naive expectation. Although the experimental error bars are large and even the sign of \( g_T/g_A \) is not certain yet, the data poses a theoretical challenge to give more reliable estimate of \( g_T/g_A \).

It is in order here to show other examples of the G-parity violation which is more firmly established than \( g_T \) [16]:

1) Proton-neutron mass difference.
   Experimental mass difference is \( M_p - M_n = -1.29 \text{ MeV} \), in which the contribution of the \( u - d \) quark mass difference after subtracting the theoretical electromagnetic effect (0.76±0.3 MeV) is −2.05 MeV. This last number has been successfully reproduced in QSR calculations [17].

2) \( \rho^0 - \omega \) mixing.
   The \( \rho^0 - \omega \) mixing is defined by the covariant matrix element \( \langle \rho^0|H_{GPB}|\omega \rangle \) at the \( \rho^0 - \omega \) mass shell with \( H_{GPB} \) being the second term in eq.(4). The recent measurement of the \( e^+e^- \rightarrow \pi^+\pi^- \) shows an unambiguous determination of the \( \rho^0 - \omega \) mixing with negative sign, which ought to be dominated by the quark mass difference since the electromagnetic effect by \( \rho \rightarrow \gamma \rightarrow \omega \) is positive and small [18,19].

3) \( \pi^0 \) mass difference \( (m_{\pi^0} = 4.6 \text{ MeV}) \).
   This is a typical example of the electromagnetic G-parity violation. Theoretical estimate gives \( (m_{\pi^+} - m_{\pi^0})_{em} = 4.6 \pm 0.1 \text{ MeV} \), while the effect of the quark mass difference appears only in second order of the quark masses and is extremely small [20].

The ingredients of this paper are twofolds: i) to get QSR with an external field for \( g_T \) and \( g_A \) in section II-V, and ii) to predict the value of \( g_T \) relative to \( g_A \) in section VI.

As for i), we will adopt a method proposed by Ioffe and Smilga [7] and independently by Balitsky and Yung [21], in which two point functions with an external electromagnetic field strength \( F_{\mu\nu} \) is studied up to linear in \( F_{\mu\nu} \). The method has been applied for the magnetic moment of the nucleon and the results agree with the experimental data with a good accuracy. A method on two point functions with an axial-vector field \( Z_\mu \) was also
developed by Belyaev and Kogan [8], and later improved by Pasupathy et al. [9]. They have considered terms proportional to \( Z_\mu \) for evaluating the axial charge \( g_A \). The latter method with \( Z_\mu \) replaced by the vector potential \( A_\mu \) is, however, not suitable for studying magnetic moments since the explicit momentum transfer must be retained.

Adopting QSR with the external field induces new parameters which are absent in ordinary QSR. These parameters reflects the response of QCD vacuum to the external field. For instance, \( \langle 0 | \bar{q} \gamma_\mu q | 0 \rangle_E \), which is identical to zero in the vacuum, acquires the non-zero value due to the presence of the external field. To evaluate these new condensates, QSR with the assumption of the vector dominance can be used [22].

There is another new feature of QSR with the external field compared to the ordinary one. The phenomenological side of the correlation function with external field takes the following double pole form near the nucleon resonance

\[
\langle 0 | \eta | N \rangle \langle N | J^E | N \rangle \langle N | \bar{\eta} | 0 \rangle (p^2 - M_N^2)^{-2},
\]

where \( J^E \) is a current coupled to the external field. Besides the double pole part which we are interested in, single poles, which expresses transition from the ground state to excited states, appears. Furthermore, the single pole term is not suppressed compared to the double pole term after applying the Borel transform. This bears no resemblance to the contribution of continuum which is exponentially suppressed by Borel transform. Hence we must take into account both the double pole and the single pole in phenomenological side on the same footing, which requires a procedure to subtract the single poles.

As for ii), one must remember that \( g_T \) originates from the \( G \)-parity violation induced by the u-d quark mass difference and the electromagnetism. The experimental value of \( g_T \) is still uncertain as we have mentioned above. Thereby we shall try to determine \( g_T \) from QSR with main emphasis on the effect of the u-d quark mass difference. Within our knowledge, no serious evaluation of \( g_T \) has been done so far except for a rough estimate using the MIT bag model [23,24]. We will therefore reexamine the bag model calculation also and compare it with our QSR result.

The paper is organized as follows. Section II–IV are devoted to derive \( g_T \) and \( g_A \) sum rules in QSR with the external field. In section V, we estimate the quark and induced condensates by using QSR. In section VI, we analyse the \( g_T \) sum rule and get its numerical number. In section VII, discussions and summary are made.

II. WEAK INTERACTION IN HADRONIC SIDE AND QCD SIDE

We start with the two point function with an external field;

\[
\Pi_E(p) = i \int d^4x e^{ip \cdot x} \langle 0 | T \eta_p(x) \bar{\eta}_n(0) | 0 \rangle_E = F_{\mu \nu} \Pi_{\mu \nu}(p),
\]

where ‘E’ denotes external field of weak boson \( W^+ \), \( F_{\mu \nu}(x) = \partial_\mu W^+_\nu(x) - \partial_\nu W^+_\mu(x) \), and \( \eta_p(\eta_n) \) corresponds to the proton (neutron) interpolating field defined as [6]

\[
\eta_p(x) = \epsilon_{abc}(u^a(x)C\gamma_\mu u^b(x))\gamma_5\gamma_\mu d^c(x), \quad \eta_n(x) = \eta_p(u \leftrightarrow d)
\]
The hadronic side of sum rules can be saturated by a process where neutron turns into proton by absorbing the $W^+$ boson, matrix element of which is shown in eq.(1). Hence we define an effective Lagrangian in which nucleon current couples to $W^+$ field as

$$L_{\text{had int}}^{\text{int}} = \frac{-g}{\sqrt{2}} j_\mu W^+ = \frac{-g}{\sqrt{2}} \vec{p} \left( g_A \gamma_\mu \gamma_5 W^+ + \frac{g_T}{M_p + M_n} \gamma_5 \sigma_{\mu \nu} \partial_\nu W^+ \right) n,$$

(11)

where $p$ (n) represents proton (neutron) field and $g$ is associated with Fermi constant as $G_F \sqrt{2} = \frac{g^2}{8M_W^2}$ with $M_W$ being $W^+$ boson mass in the Glashow-Weinberg-Salam model [25].

It is in order here to mention $g_p$. $g_p$ is associated with $g_A$ via PCAC and can be directly measured by the muon capture [24]. However, our external field does not pick up the contribution of $g_p$ because we are using the field strength $F_{\mu \nu}$ instead of the vector potential $W^+$ as an external field.

On the other hand, in the quark level, the interaction of quarks and the external field is written as

$$L_{\text{quark int}} = \frac{-g}{\sqrt{2}} j_\mu W^+ = \frac{-g}{\sqrt{2}} \bar{u} \gamma_\mu \gamma_5 d W^+_\mu,$$

(12)

where $u$ and $d$ are up and down quark respectively. The common factor $g/2\sqrt{2}$ in eq.(11) and in eq.(12) is obtained by comparing the V-A theory [26] with the Glashow-Weinberg-Salam model.

### III. HADRONIC SIDE FOR TWO POINT FUNCTION WITH EXTERNAL FIELD

In this section we examine the hadronic contribution to QSR with the external field. Firstly, we consider Fig.1, which shows that a neutron with momentum $p_1$ absorbs $W^+$ boson and turns into proton with momentum $p_2$. Using eq.(11), we may write down Fig.1 as

$$\text{Fig.1} = \frac{1}{p_2 - M_p} \left( g_A \gamma_\mu \gamma_5 + \frac{i \gamma_5 \sigma_{\mu \nu}}{M_p + M_n} g_T q_\nu \right) \frac{1}{p_1 - M_n} \left( \frac{\hat{q}}{2} + \hat{p} + M_p \right) \left( g_A \gamma_\mu \gamma_5 + \frac{i \gamma_5 \sigma_{\mu \nu}}{M_p + M_n} g_T q_\nu \right) \left( \frac{\hat{q}}{2} + \hat{p} + M_n \right),$$

(13)

where $p_2(p_1)$ is proton (neutron) momentum mentioned above, $p = \frac{p_1 + p_2}{2}$ and $q = p_1 - p_2$. In the limit of soft external momentum $q_\mu \rightarrow 0$, we keep only the terms proportional to $q_\mu$ in eq.(14) to extract $F_{\mu \nu}$. Then eq.(14) is reduced to

$$\frac{i \gamma_5 q_\nu}{(p^2 - M_n^2)(p^2 - M_p^2)} \left[ P_1 \hat{p} \sigma_{\mu \nu} + P_2 \sigma_{\mu \nu} \hat{p} + P_3 \sigma_{\mu \nu} + P_4 (\gamma_\mu p_\nu - \gamma_\nu p_\mu) \hat{p} \right] \equiv q_\nu \Gamma_{\mu \nu}(p),$$

(15)

where
\[ P_1 = -\frac{g_A}{2} - \frac{M_n}{M_n + M_p} g_T, \]  
\[ P_2 = -\frac{g_A}{2} + \frac{M_p}{M_n + M_p} g_T, \]  
\[ P_3 = -\frac{g_A}{2} (M_n - M_p) + \left( \frac{M_n M_p - \not{p}^2}{M_p + M_n} \right) g_T, \]  
\[ P_4 = \frac{2igT}{M_p + M_n}. \] (16)  

By using eq.(11), we evaluate eq.(8) in first order of the external field:

\[ - \int d^4x e^{ip \cdot x} \langle 0 | T(\eta_p(x) \bar{\eta}_n(0) L_{\text{had}} | 0) \rangle = -\frac{g}{2\sqrt{2}} \lambda_n \lambda_p \int d^4y \left( \frac{1}{2\pi} \right)^4 \int d^4l e^{i(p-l) \cdot y} W^+(y) \]  
\times \frac{1}{\not{l} - M_n} \left( g_A \gamma_\mu \gamma_5 + \frac{i\gamma_5 \sigma_{\mu\nu}}{M_p + M_n} g_T (l - p)_\nu \right) \]  
\[ = -\frac{g}{2\sqrt{2}} \lambda_n \lambda_p \int d^4y \int d^4q e^{-iq \cdot y} q^\nu \Gamma_{\mu\nu}(p) W^+(y) \]  
\[ = -\frac{ig}{4\sqrt{2}} \lambda_n \lambda_p \Gamma_{\mu\nu} F_{\mu\nu}(0) \]  
\[ = \frac{g\gamma_5 \lambda_n \lambda_p}{4\sqrt{2}(p^2 - M_n^2)(p^2 - M_p^2)} F_{\mu\nu}(0) \]  
\[ \times [P_1 \not{p} \sigma_{\mu\nu} + P_2 \sigma_{\mu\nu} \not{p} + P_3 \sigma_{\mu\nu} + P_4 (\gamma_\mu p_\nu - \gamma_\nu p_\mu) \not{p}] , \] (20)

where \( P_1, P_2, P_3 \) and \( P_4 \) are defined in eq.(16)-eq.(19), \( \lambda_n \) and \( \lambda_p \) are defined as \( \langle 0 | \eta | N \rangle = \lambda_n u(p) \) with \( u(p) \) being the nucleon Dirac spinor. Apart from the terms above, we must take into account two other contributions. One is the single pole caused by a transition of nucleon to resonance states as follows,

\[ \Pi_E(p) \sim \lambda_N \lambda_{N^*} \frac{1}{\not{p} - M_N} H_{NN^*} \frac{1}{\not{p} - M_{N^*}} , \] (21)

where \( N(N^*) \) is the nucleon (excited states, e.g., \( N(1440) \)), and \( H_{NN^*} \) is a transition matrix from the nucleon to the excited states. As we have mentioned, the single pole is not suppressed compared to the double pole after the Borel transform. Since we are not interested in the single poles, we will subtract them, using a procedure shown later. The other hadronic contribution is a continuum starting at a threshold \( S_0 \), which contains only the excited states. The continuum can be exponentially suppressed by applying the Borel transform.

**IV. OPE FOR TWO POINT FUNCTION WITH EXTERNAL FIELD**

In ordinary QSR with no external fields, only Lorenz invariant operators survive in OPE. On the other hand, the external field induces new condensates. Relevant condensates up to dimension 6 in our case read
\[
\langle 0 | \bar{u} \gamma_5 \sigma_{\mu \nu} d | 0 \rangle_E = (m_u - m_d) F_{\mu \nu} \frac{g}{2 \sqrt{2}} \chi(0),
\]
(22)
\[
g_s \langle 0 | \bar{u} \gamma_5 G_{\mu \nu} d | 0 \rangle_E = \langle \bar{d} d - \bar{u} u \rangle_0 F_{\mu \nu} \frac{g}{2 \sqrt{2}} \kappa(0),
\]
(23)
\[
g_s \epsilon_{\mu \nu \rho \omega} \langle 0 | \bar{d} G_{\rho \omega} u | 0 \rangle_E, = \langle \bar{d} d - \bar{u} u \rangle_0 i F_{\mu \nu} \frac{g}{2 \sqrt{2}} \xi(0).
\]
(24)

The above condensates are non-vanishing because the QCD vacuum is distorted by the external field and the Lorenz invariance is broken. Note also that taking \( m_u = m_d \) makes all the above terms vanish. Using the fixed point gauge, we can rewrite the
\[
i \langle 0 | T(\pi(x) \bar{\eta}_n(0)) | 0 \rangle_E
\]
as follows:
\[
i \langle 0 | T(\pi(x) \bar{\eta}_n(0)) | 0 \rangle_E = 4i \epsilon_{a b c d e f} \langle 0 | \gamma_5 \gamma_\mu i S_{q}^{c e} (x) \gamma_\nu C \hat{E}_{a d} (x) C \gamma_\mu i S_{a}^{b f} \gamma_\nu \gamma_5 | 0 \rangle_E
\]
(25)

with
\[
i S_{q}^{a b}(x) = \langle 0 | T(q^a(x) q^b(0)) | 0 \rangle
\]
\[
= \frac{i \bar{x}^2 x^4}{2 \pi^2 x^4} \delta^{a b} + \frac{i x_\alpha}{8 \pi^2 x^2} (e^\nu)^{a b} \hat{G}_{\nu \rho} \gamma_5 \gamma_\rho - \frac{m \delta^{a b}}{4 \pi^2 x^2} + \langle \chi_q^a(x) \chi_q^b(0) \rangle_0,
\]
(26)
\[
i E^{a b}(x) = \langle 0 | T(u^a(x) d^b(0)) | 0 \rangle_E
\]
\[
= \frac{i g}{2 \sqrt{2}} \delta^{a b} \frac{x_\lambda}{8 \pi^2 x^2} \bar{F}_{\lambda \mu \nu} \gamma_\mu + \langle \chi_u^a(x) \chi_d^b(0) \rangle_E
\]
(27)

\[-(m_u - m_d) \left\{ \frac{1}{32 \pi^2} ( \log(-x^2 \Lambda^2 / 4) + 2 \gamma_E) \gamma_5 \sigma_{\mu \nu} F^{\mu \nu} + \frac{i}{16 \pi^2 x^2} F_{\mu \nu} \gamma_5 \gamma_\mu \chi d \rho \right\},\]

where \( i E^{a b}(x) \) is calculated by eq.(12) with the first order perturbation, \( \bar{F}_{\mu \nu} = \frac{1}{2} \epsilon_{\mu \nu \rho \omega} F^{\rho \omega} \),

and \( \Lambda^2 \) is an infrared cut off parameter.

\[
\langle \chi_u^a(x) \chi_d^b(0) \rangle_E \text{ in eq.(27) expresses non-perturbative condensate under the external field, while other terms are coupled directly to the quark propagator. To calculate the nonperturbative terms, we shall compare it with } \langle \chi_q^a(x) \chi_q(0) \rangle_0 \text{ which is expanded as}
\]
\[
\langle \chi_u^a(x) \chi_d^b(0) \rangle_0 = \langle 0 | q^a(x) q^b(0) | 0 \rangle \quad \text{(28)}
\]
\[
= -\frac{\delta^{a b}}{12} \langle 0 | q q | 0 \rangle - \frac{\delta^{a b} x^2}{192} \langle 0 | g_s q \sigma \cdot G q | 0 \rangle + \cdots \quad \text{(29)}
\]

In the case above, we have retained only the Lorentz scalar operators. In contrast, \( \langle \chi_u^a(x) \chi_d^b(0) \rangle_E \) is expanded only by the Lorenz tensor terms corresponding to the induced condensates eq.(22) – eq.(24). Hence
\[
\langle \chi_u^a(x) \chi_d^b(0) \rangle_E = -\frac{1}{24} \delta^{a b} \gamma_5 \sigma_{\mu \nu} \langle \bar{u} \gamma_5 \sigma_{\mu \nu} d \rangle_E - \frac{x_\rho x_\omega}{48} \gamma_5 \sigma_{\mu \nu} \langle \bar{u} \gamma_5 \sigma_{\mu \nu} D_\rho D_\omega d \rangle_E + \cdots
\]
\[
= -\frac{1}{24} \frac{g}{\sqrt{2}} \delta^{a b} \gamma_5 \sigma_{\mu \nu} F^{\mu \nu} (m_u - m_d) \chi(0)
\]
\[
- \frac{g}{2 \sqrt{2}} \frac{1}{3^2 2^5} \gamma_5 \sigma_{\mu \nu} \langle \bar{d} d - \bar{u} u \rangle_0 \times \left\{ \langle \kappa(0) - \xi(0) \rangle x^2 F_{\mu \nu} - (2 \kappa(0) + \xi(0)) x_\mu x_\omega F_{\nu \omega} \right\} + \cdots \quad \text{(30)}
\]

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We turn to carry out OPE for eq.(25) with diagrams Fig. 2(a)-(j). For the chiral odd structures, we impose \( m_u = m_d \). This induces \( P_1 = P_2 = -g_A/2 \) in eq.(20) due to \( g_T = 0 \). The chiral odd structure can be applied to estimate the axial-charge \( (g_A) \) with \( m_u = m_d \).

On the other hand, the chiral even structures in eq.(25) are evaluated up to linear in \( (m_u - m_d) \), which leads to \( g_T \) sum rule.

Let us examine each contribution in Fig.2 more closely. The coefficient of \( F_{\mu\nu} \) with the chiral odd structure given in Fig.2(a) reads

\[
\text{Fig.2(a)} = - \frac{g}{2\sqrt{2}} \frac{6}{\pi^2 x^8} x^\lambda \gamma_\lambda \tilde{F}_\lambda. \tag{31}
\]

The coefficient of \( F_{\mu\nu} \langle \frac{\alpha_s}{\pi} G^2 \rangle \) with the chiral odd structure given in Fig.12(b) reads

\[
\text{Fig.2(b)} = - \frac{g}{2\sqrt{2}} \frac{1}{32\pi^4 x^4} x^\lambda \tilde{F}_\lambda \gamma_\lambda \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_0, \tag{32}
\]

where \( \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_0 = 0.012(\text{GeV}^2) \) is called gluon condensate, whose value is determined from the analysis of heavy quark system based on QSR. The coefficient of \( F_{\mu\nu} \langle \bar{q}q \rangle^2 \) with the chiral odd structure given in Fig.12(c) reads

\[
\text{Fig.2(c)} = - \frac{g}{2\sqrt{2}} \frac{1}{12\pi^2 x^2} x^\lambda \tilde{F}_\lambda \gamma_\lambda. \tag{33}
\]

The coefficient of \( (m_u - m_d) F_{\mu\nu} \) coupled directly to the propagator given in Fig.2(d) reads

\[
\text{Fig.2(d)} = -i \frac{g}{2\sqrt{2}} \frac{3}{2\pi^2 x^6} x^\lambda \tilde{F}_\lambda \gamma_\lambda (m_u - m_d). \tag{34}
\]

Fig.2(f) also gives a coefficient of \( (m_u - m_d) F_{\mu\nu} \). However, it causes the infrared divergence which must be absorbed into the same dimensional operator shown in Fig.2(e). The coefficient of \( \langle \bar{d}\gamma_5 \sigma_{\mu\nu} u \rangle_E \), which is dimension 3 and is given in Fig.2(e), reads

\[
\text{Fig.2(e)} = i \frac{1}{\pi^4 x^8} \gamma_5 \bar{\gamma}_6 \gamma_\mu \bar{x} \langle \bar{d}\gamma_5 \sigma_{\rho\omega} u \rangle_E. \tag{35}
\]

The coefficient of \( \langle d\bar{d} - \bar{u}u \rangle_0 F_{\mu\nu} \) given in Fig.2(g) reads

\[
\text{Fig.2(g)} = -i \frac{g}{2\sqrt{2}} \frac{1}{2\pi^4 x^6} x^\lambda \tilde{F}_\lambda \gamma_\lambda \langle d\bar{d} - \bar{u}u \rangle_0. \tag{36}
\]

The coefficient of \( g_s \langle \bar{d}\gamma_5 G_{\mu\nu} u \rangle_E \) and \( g_s \epsilon^{\mu\nu\omega\rho} \langle \bar{d}G_{\rho\omega} u \rangle_E \) given in Fig.12(h) and (i) read

\[
\text{Fig.2(h) and (i)} = i \frac{\gamma_5 \sigma_{\rho\omega}}{12\pi^4 x^4} [g_s \langle \bar{d}\gamma_5 G_{\rho\omega} u \rangle_E + ig_s \epsilon^{\rho\omega\mu\nu} \langle \bar{d}G_{\mu\nu} u \rangle_E] + \frac{\gamma_5 \gamma_\rho x_\omega - \gamma_\omega x_\rho \bar{x}}{4\pi^4 x^6} [g_s \langle \bar{d}\gamma_5 G_{\rho\omega} u \rangle_E + \frac{1}{2} g_s i \epsilon^{\rho\omega\mu\nu} \langle \bar{d}G_{\mu\nu} u \rangle_E], \tag{37}
\]

where the contribution of Fig.2(h) is zero because \( \langle 0 | \bar{d}u | 0 \rangle = 0 \) and in Fig.2(i) the gluon field emitted by a soft quark interacts \( W^+ \) field via quark condensate.

The coefficient of \( g_s \langle \bar{d}\gamma_5 G_{\mu\nu} u \rangle_E \) and \( g_s \epsilon^{\mu\nu\omega\rho} \langle \bar{d}G_{\rho\omega} u \rangle_E \) given in Fig.2(j) reads
where the gluon field emitted by a hard quark interacts $W^+$ field via quark condensate.

In summary, we obtain the following formula

$$\Pi_E(p) = \frac{g}{2\sqrt{2}} F_{\mu\nu} [Q_1 \gamma_5 (\not{p} \sigma_{\mu\nu} + \sigma_{\mu\nu} \not{p}) + \{Q_2 \gamma_5 \sigma_{\mu\nu} + Q_3 \gamma_5 (\mu_\nu - \gamma_\mu p_\nu) \not{p}\} (m_u - m_d)],$$

where we have used a relation $\langle \bar{d}d - \bar{u}u \rangle_0 = C_m (m_u - m_d)$ which will be discussed below.

V. THE ESTIMATE OF THE QUARK AND INDUCED CONDENSATES

Before setting up the sum rules, we need to estimate the magnitude of the quark and induced condensates. For the quark condensate, we utilize Finite Energy Sum Rules (FESR) [27,28] for nucleon mass, where we look for the optimal quark condensate which reproduce the nuclon mass within the standard values of the condensate $\langle \bar{q}q \rangle_0 (1\text{GeV}^2) = -(225 \pm 25\text{MeV})^3$.

From ref. [28], we get the FESR for nucleon as follows:

$$64\pi^4 \lambda_N^2 = \frac{3}{4} S_N^3 + 2\pi^2 \left(\frac{\alpha_s}{\pi} G^2\right)_0 S_N + \frac{128}{3} \pi^4 \langle \bar{q}q \rangle_0^2,$$

$$64\pi^4 \lambda_N^2 M_N = -8\pi^2 \langle \bar{q}q \rangle S_N^2 + \frac{32}{9} \pi^4 \langle \bar{q}q \rangle_0 \left(\frac{\alpha_s}{\pi} G^2\right)_0,$$

$$64\pi^4 \lambda_N^2 M_N^2 = \frac{S_N^4}{4} + \pi^2 \left(\frac{\alpha_s}{\pi} G^2\right)_0 S_N^2 - \frac{128}{9} \pi^4 \langle \bar{q}q \rangle_0^2 \frac{\alpha_s}{\pi} S_N,$$

where $\lambda_N$ is defined above, and $S_N$ is the continuum threshold of nucleon sum rules. Solving eq.(45)-(47) numerically, we get results in Table 1. Hence we will utilize the following numbers in the analyses of $g_T$ sum rule later;

$$\langle \bar{q}q \rangle(\text{GeV}^3) = (-0.2185)^3, \quad S_N(\text{GeV}^2) = 1.6, \quad \lambda_N(\text{GeV}^3) = 0.0188.$$  

<table>
<thead>
<tr>
<th>$\langle \bar{q}q \rangle_0$</th>
<th>$(-0.250\text{GeV})^3$</th>
<th>$(-0.230\text{GeV})^3$</th>
<th>$(-0.2185\text{GeV})^3$</th>
<th>$(-0.210\text{GeV})^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_N(\text{GeV}^2)$</td>
<td>2.27</td>
<td>1.84</td>
<td>1.60</td>
<td>1.42</td>
</tr>
<tr>
<td>$\lambda_N(\text{GeV}^3)$</td>
<td>0.0296</td>
<td>0.0224</td>
<td>0.0188</td>
<td>0.0162</td>
</tr>
<tr>
<td>$M_N(\text{GeV})$</td>
<td>1.15</td>
<td>1.024</td>
<td>0.940</td>
<td>0.872</td>
</tr>
</tbody>
</table>
Table 1: $S_N, \lambda_N, M_N$ obtained from Eq.(45) ~ (47) with four different values of $\langle \bar{q}q \rangle_0$.

Now we turn to the calculation of induced condensates with the help of QSR. We first expand eq.(22) in terms of $W^+$ up to first order.

$$
\langle \bar{d} \gamma_5 \sigma_{\mu\nu} u \rangle_E = -i \frac{g}{2\sqrt{2}} \int d^4 x \langle 0| T(\bar{d} \gamma_5 \sigma_{\mu\nu} u(0) \bar{u} \gamma_\rho \gamma_5 d(x))|0 \rangle W^+(x) \tag{49}
$$

$$
= -i \frac{g}{2\sqrt{2}} \int d^4 x \ Pi_{\mu\nu\rho}(x) W^+_\rho(x),
$$

where $Pi_{\mu\nu\rho}(x) = \langle 0| T(\bar{d} \gamma_5 \sigma_{\mu\nu} u(0) \bar{u} \gamma_\rho \gamma_5 d(x))|0 \rangle$. \(\tag{50}\)

To estimate $Pi_{\mu\nu\rho}(x)$, we expand eq. (50) in terms of the local operators up to dimension 5, whose diagrams are Fig.3(a)-(c), and retain the terms proportional to quark mass. Then we get the following equation:

$$
Pi_{\mu\nu\rho}(q) = (-q_\mu g_{\rho\nu} + q_\nu g_{\mu\rho})(m_u - m_d) \chi(q^2) \tag{51}
$$

with

$$
\chi(q^2) = \frac{3}{8\pi^2} \log(-q^2) + \left(\frac{1}{q^2} + \frac{m_0^2}{3q^4}\right) C_m, \tag{52}\)

where the first, second and third term on the right hand side correspond to Fig.3(a), (b) and (c), respectively. $m_0^2 = 0.8(\text{GeV}^2)$ is defined by $\langle 0| g_s \bar{q} \sigma \cdot G q |0 \rangle = m_0^2 \langle 0| \bar{q} q |0 \rangle$ \[30\]. Using eq.(50) and eq.(51), we reach the result defined in eq.(22):

$$
\langle \bar{d} \gamma_5 \sigma_{\mu\nu} u \rangle_E = \frac{g}{2\sqrt{2}} F_{\mu\nu}(0)(m_u - m_d) \chi(0), \tag{53}\)

where we replace $F_{\mu\nu}(x)$ by $F_{\mu\nu}(0)$ because the field strength is assumed to be constant. For the phenomenological side in eq.(52), we assume that $\chi(q^2)$ is saturated by $a_1$ meson with mass 1260(\text{MeV}), which is the lowest state coupled to both pseudovector and pseudotensor states, and the continuum starting at $S_\chi$:

$$
\frac{1}{\pi} \text{Im} \chi(s) = f_\chi \delta(s - m_{a_1}^2) - \frac{3}{8\pi^2} \theta(s - S_\chi). \tag{54}\)

Then, we get

$$
\chi(0) = \frac{f_\chi}{m_{a_1}^2} - \frac{3}{8\pi^2} \int_0^{\Lambda^2} \theta(s - S_\chi) \frac{ds}{s - m_{a_1}^2}, \tag{55}\)

where $\Lambda^2 = 1(\text{GeV}^2)$ is taken as a characteristic scale of separating the perturbative and the non-perturbative part in $\langle \bar{d} \gamma_5 \sigma_{\mu\nu} u \rangle_E$. Matching eq.(52) and eq.(54) using FESR, we get two sum rules,

$$
n = 0, \quad \frac{3}{8\pi^2} S_\chi + C_m = f_\chi, \tag{56}\)

$$
n = 1, \quad \frac{3}{16\pi^2} S_\chi^2 + \frac{m_0^2}{3} = f_\chi m_{a_1}^2. \tag{57}\)

$f_\chi$ is rewritten as
To obtain $f_{X}$, we take $C_{m}$ (GeV$^{2}$) = $(-0.0307) - (-0.0223)$ which makes the proton-neutron mass difference within the interval $1.95$GeV $\leq (M_{n} - M_{p}) \leq 2.41$GeV where the electromagnetic effect are subtracted out [29]. Thus we get

$$\chi(0) = (-0.0337) - (-0.0470)$$

As for $g_{s} \langle \bar{d} \gamma_{5} G_{\mu\nu} u \rangle_{E}$ and $g_{s} \langle \bar{d} \epsilon_{\mu\nu\rho\omega} G^{\rho\omega} u \rangle_{E}$, we make OPE up to dimension 7 and get the following results:

$$g_{s} \langle \bar{d} \gamma_{5} G_{\mu\nu} u \rangle_{E} = \frac{g}{2\sqrt{2}} C_{m}(m_{u} - m_{d}) F_{\mu\nu} \kappa(0),$$

with $\kappa(q^{2}) = \frac{m_{a}^{2}}{12\, q^{2}} + \frac{\pi^{2}}{36q^{4}} \langle \alpha_{s} G^{2} \rangle_{0},$

$$g_{s} \langle \bar{d} \epsilon_{\mu\nu\rho\omega} G^{\rho\omega} u \rangle_{E} = i \frac{g}{2\sqrt{2}} C_{m}(m_{u} - m_{d}) F_{\mu\nu} \xi(0),$$

with $\xi(q^{2}) = -\frac{m_{a}^{2}}{6q^{2}} + \frac{1}{18q^{4}} \pi^{2} \langle \alpha_{s} \pi G^{2} \rangle_{0},$

where graphs for OPE are shown in Fig. 4. For phenomenological part, we adopt two pole approximation namely,

$$\kappa(q^{2}) = \frac{f_{a_{1}}}{m_{a_{1}}^{2} - q^{2}} + \frac{f_{a_{2}}}{m_{a_{2}}^{2} - q^{2}},$$

where $m_{a_{1}}$ is $a_{1}$ meson mass appeared above, $m_{a_{2}}$ is $a_{2}$ meson mass (1360MeV) which is the tensor meson, and $f_{a_{1}}$ and $f_{a_{2}}$ represent the pole residues. The same approximation is also adopted for $\xi(q)$. Equating the OPE side to the phenomenological side and comparing the coefficients up to $q^{4}$ to determine $f_{a_{1}}$ and $f_{a_{2}}$, we get $\kappa(0) = -0.079$, $\xi(0) = 0.163$.

**VI. ANALYSIS AND NUMERICAL RESULT**

From eq.(41) and (20), we have found that there exist four sum rules corresponding to the tensor structures $\hat{p} \sigma_{\mu\nu}$, $\sigma_{\mu\nu} \hat{p}$, $\sigma_{\mu\nu}$, and $(\gamma_{\mu} p_{\nu} - \gamma_{\nu} p_{\mu}) \hat{p}$. We take only two sum rules among them to evaluate $g_{A}$ and $g_{T}$.

As mentioned above, sum rule for $g_{A}$ is obtained from the chiral odd structure in the chiral limit. On the other hand, sum rule for $g_{T}$ can be deduced from the part proportional to $(\gamma_{\mu} p_{\nu} - \gamma_{\nu} p_{\mu}) \hat{p}$ since $P_{1}$ in eq.(20) does not contain $g_{A}$. The sum rule obtained from the tensor structure $\sigma_{\mu\nu}$ is not suitable, since in the phenomenological side the contribution of the term with $g_{A}$ is comparable to that of the term with $g_{T}$, and both terms vanish in the chiral limit.

Matching eq.(20) and eq.(41) and making Borel transform, we obtain $g_{A}$ and $g_{T}$ sum rules in Borel sum rules (BSR) [3] as
\[
\left( \frac{g_A}{M^2} + A_{sp} \right) = -\frac{e^{M_N^2/2\lambda_N^2}}{\lambda_p\lambda_n} \left[ \frac{1}{8\pi^4} M^4 E_1 \left( \frac{S_A}{M^2} \right) - \frac{1}{16\pi^2} \left( \frac{\alpha_s}{\pi} G^2 \right)_0 - \frac{2}{3} \frac{\langle \bar{q}q \rangle_0^2}{M^2} \right],
\]

(65)

\[
\left( \frac{1}{M^2 M_p + M_n} + T_{sp} \right) = -\left( \frac{m_u - m_d}{\lambda_p\lambda_n} \right) M^2 e^{M_N^2/2\lambda_N^2} \left[ \frac{1}{32\pi^4} - \frac{\chi(0)}{12\pi^2} \right] \frac{C_m}{8\pi^2 M^2} + M^2 \frac{g_{m_s}}{M_N},
\]

(66)

where \( E_n(x) = 1 - (1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!})e^{-x} \), \( A_{sp} \) and \( T_{sp} \) represent the contribution of single poles coming from the nucleon - the resonance transition discussed above. To subtract \( A_{sp} \) and \( T_{sp} \), we multiply the operator \( \frac{\partial}{\partial(1/M^2)} \) to both sides of eq.(65) and (66).

Assuming that \( A_{sp} \) and \( B_{sp} \) are independent of the Borel mass \( M^2 \), we obtain the final sum rules

\[
g_A = \frac{e^{M_N^2/2\lambda_N^2}}{\lambda_p^2/4\pi^4} M^6 \times \left[ E_2 \left( \frac{S_A}{M^2} \right) - \frac{M_N^2}{2M^2} E_1 \left( \frac{S_A}{M^2} \right) - \frac{\pi^2 M_N^2}{4M^6} \left( \frac{\alpha_s}{\pi} G^2 \right)_0 + \frac{8\pi^4 \langle \bar{q}q \rangle_0^2}{3M^6} \right] \left\{ 1 + M_N^2 \right\},
\]

(67)

\[
g_T = \left( \frac{m_u - m_d}{\lambda_N^2} \right) M^2 e^{M_N^2/2\lambda_N^2} \times \left[ \frac{M^2 E_0 \left( \frac{S_T}{M^2} \right)}{M^2 E_1 \left( \frac{S_T}{M^2} \right)} \left( \frac{1}{32\pi^4} - \frac{\chi(0)}{12\pi^2} \right) + \frac{C_m}{8\pi^2 M^2} M_N^2 \right],
\]

(68)

where \( S_A(S_T) \) is the threshold for \( g_A(g_T) \), and \( \lambda_n = \lambda_p \equiv \lambda_N \) and \( M_n = M_p \equiv M_N \) are taken since \( m_u - m_d \) is extracted out in (66).

Here we make analyses of \( g_T \) sum rule. I) We make a Borel analysis using nucleon sum rules according to the procedures shown in ref. [3], and apply the FESR to get the qualitative understanding. II) We make a Borel analysis on the ratio \( g_T/g_A \) which is directly related to the experimental data, using \( g_A \) sum rule.

I) First of all, we write down two nucleon sum rules [6] in order to get rid of the coefficient \( e^{M_N^2/2\lambda_N^2}/\lambda_N^2 \) in eq.(68):

\[
4\pi^4 \lambda_N^2 e^{-M_N^2/M^2} = \frac{M^6}{8} E_2(x_N) + \frac{\pi^2 M^2}{8} \left( \frac{\alpha_s}{\pi} G^2 \right)_0 E_0(x_N) + \frac{8\pi^4}{3} \langle \bar{u}u \rangle^2,
\]

(69)

\[
4\pi^4 \lambda_N^2 M_N e^{-M_N^2/M^2} = -\pi^2 \langle \bar{d}d \rangle M^4 E_1(x_N) + \frac{2\pi^4}{9} \langle \bar{d}d \rangle \left( \frac{\alpha_s}{\pi} G^2 \right)_0,
\]

(70)

where \( x_N = S_N/M^2 \). We call eq. (69) (eq.(70)) even (odd) sume rule since it contains only even (odd) dimensional operators.

Because the formulas obtained from BSR depend on the unphysical parameter, i.e., the Borel mass \( M^2 \), we must adopt a Borel window, \( M_{min.}^2 < M^2 < M_{max.}^2 \), in which \( g_T \) is independent of \( M^2 \) within this range. In other words, the threshold \( S_T \) is chosen to make the Borel curve as flat as possible in the Borel window. To obtain the Borel window, we take \( 1 - E_0(S_T/M^2) \geq 30\% \) at \( S_T = 2.0 \) in eq.(66) as the upper limit and get \( M_{max.}^2 = 1.66 \). However, we can not get the suitable minimum in Borel window since the contribution of
the condensate term in eq.(68) is the same magnitude as that of the perturbative term.
The multiplication of the operator for subtracting the single pole makes the contribution
of the perturbative term reduced because of \( \mathcal{O}(1/M^2)M^2e^{M^2/M^2} = 0 \) at \( M^2 = M_N^2 \). Thus,
we utilize the lower limit of the nucleon sum rule (69) where the second and third terms
is less than 30 % compared to the perturbative term. Thus we arrives at a Borel window
\( 1.22 \text{GeV}^2 \leq M^2 < 1.66 \text{GeV}^2 \), and we carry out the Borel analysis on \( g_T \) with \( C_m = -0.0337 \) and \(-0.0223 \) to search the optimal threshold in the Borel window.

The results are summarized in Table 2 and Borel curves with the optimal threshold
are shown in Fig.5 (a) and (d), which show that the magnitude of \( g_T \) represents a good agreement with the result from BSR.

This implies that adopting \( S_T = 2M_N^2 \sim 2.5M_N^2 \) as shown in Table 4 makes the first term
with the threshold \( S_T \) small compared to the second term. Neglecting the first term and
utilizing \( \lambda_N^2 = 4\langle \bar{q}q \rangle_0^2 \) and \( M_N = \left( -\frac{25\pi^2}{2} \langle \bar{q}q \rangle_0 \right)^{\frac{1}{2}} \) obtained by FESR, we arrive at

\[
\frac{g_T}{2M_N} = \frac{(m_u - m_d)}{\lambda_N^2} \left[ \left( \frac{1}{2}S_T^0 - M_N^2 S_T \right) \left( \frac{1}{32\pi^4} - \frac{\chi(0)}{12\pi^2} - \frac{C_m}{8\pi^2}M_N^2 \right) \right].
\] (71)

This implies that adopting \( S_T = 2M_N^2 \sim 2.5M_N^2 \) as shown in Table 4 makes the first term
with the threshold \( S_T \) small compared to the second term. Neglecting the first term and
utilizing \( \lambda_N^2 = 4\langle \bar{q}q \rangle_0^2 \) and \( M_N = \left( -\frac{25\pi^2}{2} \langle \bar{q}q \rangle_0 \right)^{\frac{1}{2}} \) obtained by FESR, we arrive at

\[
g_T = \frac{25}{32} \frac{\langle \bar{d}d - \bar{u}u \rangle_0}{\langle \bar{u}u \rangle_0} = \frac{25}{32} C_m (m_u - m_d)/\langle \bar{u}u \rangle_0,
\] (72)

which gives \( g_T = (-0.00896) - (-0.00651) \) when \(-0.0307 \leq C_m \leq -0.0223 \) is used. This
represents a good agreement with the result from BSR.

<table>
<thead>
<tr>
<th>\langle \bar{q}q \rangle_0 (S_N)</th>
<th>( C_m = -0.0307 )</th>
<th>( C'_m = -0.0223 )</th>
<th>( C''_m = -0.0307 )</th>
<th>( C'''_m = -0.0223 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_T )</td>
<td>( g_T^{even} (S_T^{even}) )</td>
<td>( g_T^{odd} (S_T^{odd}) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-0.2185 ) (1.60)</td>
<td>(-0.0106 ) (2.15)</td>
<td>(-0.00413 ) (1.74)</td>
<td>(-0.0163 ) (2.62)</td>
<td>(-0.00719 ) (2.00)</td>
</tr>
</tbody>
</table>

Table 2: \( g_T^{even} (g_T^{odd}) \) and its threshold \( S_T^{even} (S_T^{odd}) \) using even (odd) nucleon sum rule with
two different value of \( C_m \) where \( \langle \bar{q}q \rangle_0 \) is in GeV^3 unit, and the thresholds \( S_N, S_T \) are in
GeV^2 unit. \( \langle \bar{q}q \rangle_0 = (-0.2185 \text{GeV})^3 \) reproduces nucleon mass.

II) As it is customary to take the ratio of \( g_T \) and \( g_A \), we make Borel analysis on \( g_T/g_A \)
by taking the ratio of eq.(67) and eq.(68). For the threshold \( S_A \), we take \( S_A \) which satisfies
the experimental number of \( g_A (=1.25) \) in FESR, and get \( S_A = 1.68 \text{GeV}^2 \). Then FESR
for \( g_A \) reads

\[
g_A = \frac{1}{4\pi^4} \frac{1}{\lambda_N^2} \left[ \frac{1}{6} S_A^3 - \frac{M_N^2}{4} S_A^2 - \frac{\pi^2 M_N^2}{4} \left( \frac{\alpha_s}{\pi} \right) G^2 + \frac{8\pi^4}{3} \langle \bar{q}q \rangle^2 \right],
\] (73)

where we have used \( \lambda_N \) and \( \langle \bar{q}q \rangle \) in eq. (48). After searching optimal threshold in the
above window, we get the results in Table 3. Corresponding Borel curves are given in Fig.
6 where we use \( \langle \bar{q}q \rangle_0 (\text{GeV}^2) = (-0.2185)^3 \) to compare the results with those in case I).
Note that replacing \( M_{min}^2 = 1.22 \) by \( M_{min}^2 = 0.61 \) which is obtained from \( g_A \) sum rule has

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the same qualitative results with the window used in case I) and the quantitative change is within 15%.

Table 3 shows that

\[ g_T/g_A = -0.0152 \pm 0.0053 \quad (74) \]

which will be the one to be compared with experimental value.

<table>
<thead>
<tr>
<th>( \langle \bar{q}q \rangle_0 (S_A) )</th>
<th>( C_m = -0.0307 )</th>
<th>( C_m = -0.0223 )</th>
<th>( g_T (S_T) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-0.2185)^4 (1.68)</td>
<td>-0.0205 (2.97)</td>
<td>-0.00983 (2.22)</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: \( g_T/g_A \) and its threshold \( S_T \) with two different values of \( C_m \) where \( \langle \bar{q}q \rangle_0 \) is in GeV^3 unit, the thresholds \( S_T, S_A \) are in GeV^2 unit and \( C_m \) is in GeV^2 unit. \( \langle \bar{q}q \rangle_0 = (-0.2185)^3 \) reproduces nucleon mass.

Here we mention the uncertainty of our results originating from \( C_m \). \( \chi(0) \) grows as \( C_m \) becomes small, which changes the sign of \( g_T \) from negative to positive. Hence determining \( g_T \) in QSR does not become quite accurate unless \( \langle \bar{d}d - \bar{u}u \rangle_0 \) is precisely determined. Also \( g_A \) and \( g_T \) are rather sensitive to \( \langle \bar{q}q \rangle_0 \), thus we need to know its accurate value.

VII. DISCUSSIONS AND SUMMARY

So far, we have calculated \( g_T \) in QSR and gotten Table 2 and 3, and we found that

i) \( g_T \) is of order \( (m_u - m_d) \).

ii) Its sign based on the definition of eq.(1) is negative.

iii) \( g_T/g_A \) ranges from -0.0205 to -0.00983, i.e., \( g_T/g_A \approx 3 \sim 4(m_u - m_d)/M_N \) which is much smaller than the preliminary experimental value.

iv) By using FESR, we get the analytic formula for \( g_T \):

\[ g_T = \frac{25}{32} \frac{\langle \bar{d}d - \bar{u}u \rangle_0}{\langle \bar{u}u \rangle_0} = \frac{25}{32} \frac{C_m(m_u - m_d)}{\langle \bar{u}u \rangle_0}. \quad (75) \]

As mentioned in the introduction, the MIT bag model has been utilized so far to calculate \( g_T \). In this model, we get the following result up to \( O(m_u - m_d) \) (see Appendix A for the detailed calculations):

\[ g_T = 0.041M_N(m_u - m_d)R^2, \quad (76) \]

with \( R \) being the bag radius. Note that the difference of the bag radius between the proton and the neutron is neglected. By taking \( R = 1.085 \) fm [31], which reproduces the proton mass, we obtain \( g_T = -0.00455 \) which is consistent with the result obtained by QSR. Since the obtained result is rather sensitive to the bag radius, one should take this number only qualitatively.
Another effect to $g_T$, which we must take into account, is the electromagnetic effect. Rough estimate using a hadronic model shows that this effect is smaller than that of the u-d quark mass difference as in the case of the $\rho - \omega$ mixing and the p-n mass difference. Thus our conclusion eq.(74) will not be changed qualitatively by the electromagnetic effect. Nevertheless, more detailed study of QSR with the electromagnetic effect must be done.

In summary we have examined the induced tensor, $g_T$, in QSR with the external field, and gotten $g_T/g_A = -0.0152 \pm 0.0053$ which is smaller than preliminary experimental numbers by one order of magnitude. (current experimental number is ranging from $0.14 \pm 0.10$ to $-0.21 \pm 0.14$ [15]). However, the experiments and its analyses remain uncertain in order to compare with result obtained in this paper [15,24,32]. Our result should be checked in future beta-decay experiments to understand the G-parity violation.

ACKNOWLEDGMENTS

I would like to thank T. Hatsuda for his useful and valuable discussions throughout this work. Also, I would like to thank M. Doi and M. Morita for drawing my attention to the induced tensor.
APPENDIX A: ESTIMATE OF INDUCED TENSOR IN THE MIT BAG MODEL

We define the matrix element as

\[ \langle p(p_2) | J_5^\mu(x) | n(p_1) \rangle = \bar{u}_p(p_2) \left( \frac{i\gamma_5 \sigma_{\mu\nu}}{M_n + M_p} g_T q_\nu \right) u_n(p_1) \]  
(A1)

where \( q = p_1 - p_2 \).

I) \[
\frac{\partial}{\partial q} \langle p | J^\mu_5(x) | n \rangle |_{q=0} = \frac{g_T}{2M_N} \bar{u}_p \sigma u_n = \frac{g_T}{2M_N} s-f |p| \bar{\sigma} \tau^+ |n \rangle s-f,  
\]  
(A2)

where the subscript denotes a spin-flavor matrix element and, we use

\[ \langle p | J^\mu_5(x) | n \rangle = \frac{g_T}{2M_N} \bar{q}_p \left( \begin{array}{cc} \bar{\sigma} & 0 \\ 0 & \bar{\sigma} \end{array} \right) u_n \]

II) The axial current in the MIT bag model is

\[ J^\mu_5(x) = \sum_i \bar{q}_i(x) \gamma_\mu \gamma_5 \tau^+ q_i(x) \]  
(A3)

From eq.(A2), we obtain

\[ \frac{\partial}{\partial q} \langle p | J^\mu_5(x) | n \rangle |_{q=0} = \frac{\partial}{\partial q} \int d^3 x \langle p | J^\mu_5(x) | n \rangle e^{-i\bar{q} \cdot x} |_{q=0} = -i \int d^3 x \bar{q} \langle p | J^\mu_5(x) | n \rangle. \]  
(A4)

Hence,

\[ \int_{bag} d^3 x \bar{q} u(x) \gamma_\mu \gamma_5 q_d(x) \]  
(A5)

\[ = \frac{N_u N_d}{4\pi} \int_{bag} d^3 x \bar{q} \prod \left\{ \frac{E_u + m_u}{E_u} \sqrt{E_d - m_d} \right\} j_0(x_u r/R) i\bar{\sigma} \cdot \hat{r} j_1(x_d r/R) - (u \leftrightarrow d) \]

where

\[ q(x) = \frac{1}{\sqrt{4\pi}} \left[ \sqrt{\frac{E_u + m_u}{E_u}} j_0(x_u r/R) \right], \quad E(m, R) = \frac{1}{R} [x^2 + (mR)^2]^{1/2} \]

\[ N_q^{-2}(x) = R^3 j_0^2(x) \frac{2E(E - 1/R) + m/R}{E(E - m)}, \quad \tan x = \frac{x}{1 - mR - [x^2 + (mR)^2]^{1/2}}, \]

where \( j_n(x) \) is spherical Bessel function and \( R \) is Bag radius and \( r = |\bar{x}| \).

To show the effect of \( u - d \) quark mass difference, we expand eq.(A5) up to linear in \( m_u - m_d \):
(Eq. A5) = \frac{(m_u - m_d) R N^2}{x_0 - 1} \frac{4\pi}{4\pi} \int_{\text{bag}} d^3 x \ r j_0(x_0r/R) j_1(x_0r/R)

= - \frac{(m_u - m_d) N^2}{2(x_0 - 1)} \frac{4\pi}{4\pi} \int_{\text{bag}} d^3 x \ r^2 \ (j_0^2(x_0r/R) + j_1^2(x_0r/R)) \quad \text{(A6)}

where

\tan x_0 = \frac{x_0}{1 - x_0}, \quad E(m, R) = \frac{x_0}{R} + \frac{m}{2(x_0 - 1)} + O(m^2). \quad \text{(A7)}

Thus, we get

\frac{\partial}{\partial \bar{q}} \langle p|j_0^5(x)|n\rangle_{\bar{q}}

\times \frac{1}{3} \langle p|\sum_i \bar{\sigma}_i \cdot \tau_i^+|n\rangle_{s-f} \frac{(m_u - m_d) N^2}{4\pi(x_0 - 1)}

= \left( R \int d^3 x \ r j_0(x_0r/R) j_1(x_0r/R) - \frac{1}{2} \int d^3 x \ r^2 \ (j_0^2(x_0r/R) + j_1^2(x_0r/R)) \right). \quad \text{(A8)}

Equating eq.(A2) to eq.(A8), we arrive at

\frac{g_r}{2M_N} = -(m_u - m_d) \frac{5R^2}{36x_0(x_0^2 - 1)^2} \left[ \frac{1}{x_0} - \frac{17}{6} + \frac{8}{3} x_0 - \frac{2}{3} x_0^2 \right], \quad \text{(A9)}

where x_0 = 2.04 and we have used

\langle p|\sum_i \bar{\sigma}_i \cdot \tau_i^+|n\rangle_{s-f} = \frac{5}{3} \langle p|\bar{\sigma} \cdot \tau^+|n\rangle_{s-f}. \quad \text{(A10)}
Figure Captions

Fig. 1
A schematic illustration that neutron absorbs $W^+$ boson and turns into proton.

Fig. 2
OPE for $\Pi_E(p)$, where for the chiral odd structures $\Pi_E(p)$ is expanded up to dimension 8 with $m_u = m_d$, while for the chiral even structures $\Pi_E(p)$ is expanded up to dimension 5, and up to linear in $(m_u - m_d)$. Dashed lines denote the external field, wavy lines denote gluon lines, and broken lines denote the quark/gluon condensate.

Fig. 3
OPE up to dimension 5 for $\chi(p)$ sum rules. Wavy lines denote gluon lines and broken lines denote the quark/gluon condensate.

Fig. 4
OPE up to dimension 7 for $\kappa(p)$ and $\xi(p)$ sum rules. Wavy lines denote gluon lines and broken lines denote the quark/gluon condensate.

Fig. 5 (a), (b)
$g_T^{\text{even}}$ and $g_T^{\text{odd}}$ with the optimal threshold $S_T$ as a function of the Borel mass squared $M^2$. $S_T$ is also shown in GeV$^2$ unit. (a) ((b)) corresponds to $g_T^{\text{even}}$ ($g_T^{\text{odd}}$). The solid (dashed) line corresponds to $C_m(\text{GeV}^2) = -0.0306 (-0.0223)$.

Fig. 6
$g_T/g_A$ with the optimal threshold $S_T$ as a function of the Borel mass squared $M^2$. $S_T$ is also shown in GeV$^2$ unit. The solid (dashed) line corresponds to $C_m(\text{GeV}^2) = -0.0306(-0.0223)$. 

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REFERENCES


