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GSI-Preprint-95-84
DEZEMBER 1995

KINETIC EQUATIONS WITH PAIRING CORRELATIONS

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Kinetic equations with pairing correlations

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December 20, 1995

Abstract

The Gorkov equations are derived for a general non-equilibrium system. The Gorkov factorization is generalized by the cumulant expansion of the 2-particle correlation and by a generalized Wick theorem in the case of a perturbation expansion. A stationary solution for the Green functions in the Schwinger-Keldysh formalism is presented taking into account pairing correlations. Especially the effects of collisional broadening on the spectral functions and Green functions is discussed. Kinetic equations are derived in the quasi-particle approximation and in the case of particles with width. Explicit expressions for the self-energies are given.

Keywords: Non-equilibrium field theory, kinetic equation, Wick theorem, BCS theory, superfluidity.

PACS: 05.60.+w, 24.10.Cn, 51.10.+v, 74.20.Fg.

1 Introduction

In heavy ion collisions at energies of E/A from 100 to 600 MeV the fragmentation of the fire-ball at the end of the reaction is of great interest in the last decade. Especially the formation of light fragments has been studied since aspects of the underlying microscopic dynamic are there more clearly visible. That shows up in the rather high production rate of deuterons [1]. This can be understood by the strong neutron-proton pairing in the $^3S_1 - ^3D_1$ channel (e.g. [2]).

This problem can be attacked theoretically in different ways. At first one can search for the deuteron pole in the T-matrix for proton-neutron scattering [3–5] and then implement this pole explicitly in the proton-neutron scattering cross sections in transport models [6]. Alternatively one can examine the neutron-proton pairing condensate in the Bardeen-Cooper-Schrieffer (BCS) approach for superfluid nuclear matter [2,7–9]. This idea is favoured by the fact that the BCS gap equation takes the form of the Schrödinger equation of the deuteron in the low density limit [10]. In the last time a couple of papers was concerned with

the description of the deuteron production in the BCS approach. In Ref. [2] also a dynamic approach is presented based on time-dependent Hartree-Fock-Bogoliubov equations. The result is a transport equation of nuclear matter with superfluidity as a possible channel.

Since nowadays kinetic equations are derived mostly in the Green function formalism fitted to non-equilibrium conditions ([11–13] and references therein), the aim of this paper is to derive a kinetic equation in the presence of pairing correlations in the framework of a non-equilibrium field theory. The knowledge of such a kinetic equation is not only interesting for the description of deuteron production in heavy ion collisions, but also in a dynamic description of neutron stars in which superfluidity occurs at densities in the range of roughly 0.1 to 0.6 nucl/fm^3 [8]

The paper is organized as follows. In Section 1 the Gorkov factorization is derived in the more general framework of the cumulant expansion for expectation values of products of field operators. The same idea is applied in Section 2, in which the Dyson equation of a non-equilibrium system with pairing correlations is derived starting from the perturbation expansion and using a generalized Wick theorem containing all kinds of initial correlations and normal and anomalous Green functions. That again generalizes the Gorkov factorization to arbitrary non-equilibrium systems. In the third Section the equilibrium solutions are discussed in the Schwinger-Keldysh formalism including pairing correlations. Especially the appearing of a width in the spectral function caused by collisional broadening in the medium is discussed. In Section 4 a kinetic equation is derived using the quasi-particle approximation. In Section 5 changes of the kinetic equations are mentioned stemming from the finite width of spectral functions. Finally explicit expressions for the self-energies are presented discussing also by that occasion the Feynman rules for systems with pairing correlations.

2 Gorkov factorization in non-equilibrium systems

The Gorkov equations are the formulation of the BCS theory in terms of Green functions. In standard textbooks the Gorkov equations are derived for a ground state or for a thermal system using the imaginary time formalism (e.g. [14]). Initially they were applied to describe superconducting in solid state physics.

But also in nuclear physics the BCS theory is of great importance because of the pairing correlation, which has been extensively studied in BCS formalism for the shell model (e.g. [14]). Furthermore BCS theory is applied to describe superfluidity in neutron stars. Several papers are concerned with the onset of superfluidity in symmetric and unsymmetric nuclear matter in different channels of the neutron-neutron and proton-neutron interaction [8,9]. As stated in the introduction deuteron production can be described within the BCS approach

[2,8]. Also superfluidity in finite nuclei was studied using the relativistic Walecka model [15].

In the present paper we want to extend this formalism to pairing correlations in non-equilibrium systems. We start therefore with a derivation of the Gorkov equations in the context of the closed-time-path (Schwinger-Keldysh) formalism ([11] and references therein). Two strategies are proposed: the first starts with the exact equations of motion for the normal and anomalous Green functions and factorizes then the 2-particle Green functions. The second uses the perturbation expansion in non-equilibrium and applies a suitable Wick theorem for the factorization of the time (contour) ordered products.

For simplicity a non-relativistic model is considered. We choose the common Lagrangian for fermions with a 2-particle potential

$$L = L_0 + L_I, \quad (1)$$

$$L_0 = \int d^3x \left(\psi^\dagger(x) \left(-i\partial_t - \frac{\Delta}{2m} \right) \psi(x) \right), \quad (2)$$

$$L_I = -\frac{J}{2} \int d^3x d^3y V(|\mathbf{x} - \mathbf{y}|) \psi^\dagger(x) \psi^\dagger(y) \psi(y) \psi(x). \quad (3)$$

Since non-equilibrium systems are considered, we make use of the Schwinger-Keldysh formalism. The time arguments lie on a directed time contour C that starts from $-\infty$, runs to $+\infty$ and back to $-\infty$ (see Fig. 1). Thereby the fact is reflected that the final state differs from the initial state by more than a phase factor. The Green function becomes contour ordered

$$iG(x, y) = \langle O_C[\psi_H(x) \psi_H^\dagger(y)] \rangle, \quad (4)$$

where the contour ordering operator O_C puts field operators with time arguments lying later on the time contour C to the left. $\langle . \rangle$ indicates the trace over the unknown statistical operator of the non-equilibrium system and the corresponding operator. In this contour ordered Green function the chronologically and anti-chronologically ordered Green function G_c and G_a and the 1-particle correlations $G_<$ and $G_>$ are comprised [11]. Furthermore we use the

Figure 1: The Schwinger-Keldysh time-path contour running from $-\infty$ to $+\infty$ and back to $-\infty$.

anomalous Green functions, necessary to describe pairing correlations, on the

closed-time-path contour

$$iF(x, y) = \langle O_C[\psi_H(x)\psi_H(y)] \rangle, \quad (5)$$

$$iF^\dagger(x, y) = \langle O_C[\psi_H^\dagger(x)\psi_H^\dagger(y)] \rangle. \quad (6)$$

From the above Lagrangian L (Eqs. (1)-(3)), the equations of motion of the normal and anomalous Green functions can be derived ($D_x := i\partial_{t_x} + \frac{\Delta_x}{2m}$ and $D_y^\dagger := -i\partial_{t_y} + \frac{\Delta_y}{2m}$)

$$D_x G(x, y) = \delta_C(x - y) + ig \int_C d^3z V(|\mathbf{x} - \mathbf{z}|) \times \langle O_C(\psi^\dagger(z)\psi(z)\psi(x)\psi^\dagger(y)) \rangle, \quad (7)$$

$$D_y^\dagger G(x, y) = \delta_C(x - y) + ig \int_C d^3z V(|\mathbf{y} - \mathbf{z}|) \times \langle O_C(\psi(x)\psi^\dagger(y)\psi^\dagger(z)\psi(z)) \rangle, \quad (8)$$

$$D_x F(x, y) = +ig \int_C d^3z V(|\mathbf{x} - \mathbf{z}|) \times \langle O_C(\psi(z)^\dagger\psi(z)\psi(x)\psi(y)) \rangle, \quad (9)$$

$$D_y F(x, y) = +ig \int_C d^3z V(|\mathbf{y} - \mathbf{z}|) \times \langle O_C(\psi(x)\psi^\dagger(z)\psi(z)\psi(y)) \rangle, \quad (10)$$

$$D_x^\dagger F^\dagger(x, y) = +ig \int_C d^3z V(|\mathbf{x} - \mathbf{z}|) \times \langle O_C(\psi^\dagger(x)\psi^\dagger(z)\psi(z)\psi^\dagger(y)) \rangle, \quad (11)$$

$$D_y^\dagger F^\dagger(x, y) = +ig \int_C d^3z V(|\mathbf{y} - \mathbf{z}|) \times \langle O_C(\psi^\dagger(x)\psi^\dagger(y)\psi^\dagger(z)\psi(z)) \rangle, \quad (12)$$

where $\int_C$ denotes the integration along the closed-time-path contour and δ_C the δ -distribution on the contour [11].

The question is now how to factorize the full contour ordered 2-particle Green functions on the r.h.s. of Eqs. (7)-(12). This can be done by extracting the pure 2-particle correlations and separating off all 1-particle quantities. This procedure is formally carried out in the cumulant expansion. In the cumulant expansion a moment (expectation value) of n (ordered) field operators is expressed by the (ordered) irreducible correlations or (ordered) cumulants of these field operators. For the mathematical properties of cumulants we refer to Appendix A. The result is the following

$$\langle O_C(\chi_1 \dots \chi_{2n}) \rangle = \sum_{\mathbf{a}, \mathbf{p}} \sum_{r=1}^n (-1)^P \kappa_C\{2n\}_r. \quad (13)$$

$\kappa_C\{2n\}_r$ denotes the product of r ordered cumulants whose arguments are a partition of the $2n$ arguments of the moment into r distinct sets. If we deal with fermions, we want that the cumulants are physical i.e. complex functions and not Grassmann functions. Therefore the sum over all partitions, $\sum_{a.p.}$, is restricted to partitions which contain only sets of an even number of fermion field operators. The factor $(-1)^P$ stems from the permutations of fermion field operators to get the correct ordering. The field operators ψ and $\psi^\dagger$ are here abbreviated by χ . The contour ordered 2-point cumulant $\kappa_C\{2\}_1$ is the contour ordered Green function G , F or $F^\dagger$.

The application of the cumulant expansion (Eq. (13)) for the 4-point function in Eq. (7) leads to a generalization of the Gorkov factorization [16] to non-equilibrium systems

$$\begin{aligned} \langle O_C[\psi(z)^\dagger \psi(z) \psi(x) \psi^\dagger(y)] \rangle &= \kappa_C(\psi(z)^\dagger \psi(z) \psi(x) \psi^\dagger(y)) + G(x, y)G(z, z) \\ &\quad - (G(x, z)G(z, y) + F(x, z)F^\dagger(z, y)) \end{aligned} \quad (14)$$

Neglecting now the 4-point correlations we arrive finally at following equations of motion which are the generalization of the Gorkov equations to a non-equilibrium system. Note that all Green functions are contour ordered.

$$\begin{aligned} D_x G(x, y) &= -\delta_C(x - y) + ig \int_C d^3z V(|\mathbf{x} - \mathbf{z}|) [G(z, z)G(x, y) \\ &\quad - G(x, z)G(z, y) + F(x, z)F^\dagger(z, y)], \end{aligned} \quad (15)$$

$$\begin{aligned} D_y^\dagger G(x, y) &= -\delta_C(x - y) + ig \int_C d^3z V(|\mathbf{y} - \mathbf{z}|) [G(z, z)G(x, y) \\ &\quad - G(x, z)G(z, y) + F(x, z)F^\dagger(z, y)], \end{aligned} \quad (16)$$

$$\begin{aligned} D_x F(x, y) &= +ig \int_C d^3z V(|\mathbf{x} - \mathbf{z}|) [G(z, z)F(x, y) \\ &\quad - G(x, z)F(z, y) - F(x, z)G(y, z)], \end{aligned} \quad (17)$$

$$\begin{aligned} D_y F(x, y) &= +ig \int_C d^3z V(|\mathbf{y} - \mathbf{z}|) [G(z, z)F(x, y) \\ &\quad + F(x, z)G(y, z) - G(x, z)F(z, y)], \end{aligned} \quad (18)$$

$$\begin{aligned} D_x^\dagger F^\dagger(x, y) &= +ig \int_C d^3z V(|\mathbf{x} - \mathbf{z}|) [G(z, z)F^\dagger(x, y) \\ &\quad - G(z, x)F^\dagger(z, y) - F^\dagger(x, z)G(z, y)], \end{aligned} \quad (19)$$

$$\begin{aligned} D_y^\dagger F^\dagger(x, y) &= +ig \int_C d^3z V(|\mathbf{y} - \mathbf{z}|) [G(z, z)F^\dagger(x, y) \\ &\quad - G(z, x)F^\dagger(z, y) - F^\dagger(x, z)G(z, y)]. \end{aligned} \quad (20)$$

Up to now we have generalized the usual way of deriving the Gorkov equation on the one hand by extending it to non-equilibrium systems. On the other hand the Gorkov factorization was put into the more general context of a cumulant

expansion for moments which leads in a natural way to an inclusion of the anomalous Green functions.

3 Perturbation series containg pairing correlations

One lack of the above derivation of the Gorkov equations is that 2-particle correlations as resulting from the T-matrix approximation are neglected. Since the 2-particle correlations are known to be very important in nuclear matter, we present a second derivation using the perturbation expansion that leads finally to a generalized Dyson equation. Through the self-energies appearing in the Dyson equation, 2-particle correlations can be included.

The aim of the perturbation expansion is to express the exact Green function by a functional of free Green functions (G^0 , F^0 and $F^{\dagger 0}$) whose time evolution is known. For this purpose we substitute the Heisenberg field operator in the definition of the exact Green function by field operators in the interaction picture by means of the identity

$$\psi(t_f, \mathbf{x}) = U_I(t_0, t_f) \psi_I(t_f, \mathbf{x}) U_I(t_f, t_0) \quad (21)$$

where U_I is the time evolution operator in the interaction picture. The perturbation expansion of the exact Green function reads

$$iG(x, y) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_C d^4 1 \dots d^4 n \langle O_C [H_I(1) \dots H_I(n) \psi_I(x) \psi_I^\dagger(y)] \rangle. \quad (22)$$

The factorization of the expectation value of the contour ordered product of field operators is done by means of a generalized Wick theorem outlined in Appendix B. This Wick theorem completes a non-equilibrium Wick theorem [17] with respect to pairing correlations. This Wick theorem states that the expectation value of a contour ordered product of field operator factorizes into a product of free normal and anomalous Green functions and irreducible correlations or cumulants which stem from the initial correlations of the non-equilibrium system

$$\langle O_C [\chi_I(1) \dots \chi_I(2n)] \rangle = \sum_{a,p} \sum_{i=0}^n \sum_{r=1}^{n-i} \sum_{j+k+l=i} (-1)^P \kappa \{2(n-i)\}'_r G^j F^k F^{\dagger l}. \quad (23)$$

$\kappa \{n\}'_r$ denotes a product of r cumulants whose arguments are a partition of the n arguments into r distinct sets. The sets are restricted to contain only an even number fermion field operators neglecting sets containing two fermion field operators. $(-1)^P$ accounts for the permutations of fermion field operators to get the correct ordering.

For the present purpose we neglect the initial correlations so that only a products of normal and anomalous Green functions remain. Up to the first order in the perturbation expansion we get the following expressions of the Green functions

$$\begin{aligned}
iG(\mathbf{x}, y) &= iG^0(\mathbf{x}, y) - g \int_C d^3 1 d^4 2 V(|\mathbf{1} - \mathbf{2}|) \\
&\times \left[G^0(\mathbf{x}, 1) (-G^0(2, 2)G^0(1, y) + G^0(1, 2)G^0(2, y) \right. \\
&\quad \left. - F^0(1, 2)F^{\dagger 0}(2, y) \right) \\
&- F^0(\mathbf{x}, 1) \left(-G^0(2, 2)F^{\dagger 0}(1, y) + G^0(2, 1)F^{\dagger 0}(2, y) \right. \\
&\quad \left. + F^{\dagger 0}(1, 2)G^0(2, y) \right) \Big] . \quad (24)
\end{aligned}$$

$$\begin{aligned}
iF(\mathbf{x}, y) &= iF^0(\mathbf{x}, y) - g \int_C d^3 1 d^4 2 V(|\mathbf{1} - \mathbf{2}|) \\
&\times \left[G^0(\mathbf{x}, 1) (-G^0(2, 2)F^0(1, y) + G^0(1, 2)F^0(2, y) \right. \\
&\quad \left. + F^0(1, 2)G^0(y, 2) \right) \\
&- F^0(\mathbf{x}, 1) (G^0(2, 2)G^0(y, 1) - G^0(2, 1)G^0(y, 2) \\
&\quad \left. + F^{\dagger 0}(1, 2)F^0(2, y) \right) \Big] . \quad (25)
\end{aligned}$$

$$\begin{aligned}
iF^{\dagger}(\mathbf{x}, y) &= iF^{\dagger 0}(\mathbf{x}, y) - g \int_C d^3 1 d^4 2 V(|\mathbf{1} - \mathbf{2}|) \\
&\times \left[G^0(1, \mathbf{x}) \left(-G^0(2, 2)F^{\dagger 0}(1, y) + G^0(2, 1)F^{\dagger 0}(2, y) \right. \right. \\
&\quad \left. \left. + F^{\dagger 0}(1, 2)G^0(2, y) \right) \right. \\
&\left. + F^{\dagger 0}(\mathbf{x}, 1) (-G^0(2, 2)G^0(1, y) + G^0(1, 2)G^0(2, y) \right. \\
&\quad \left. - F^0(1, 2)F^{\dagger 0}(2, y) \right) \Big] . \quad (26)
\end{aligned}$$

$$\begin{aligned}
iG(y, \mathbf{x}) &= iG^0(y, \mathbf{x}) - g \int_C d^3 1 d^4 2 V(|\mathbf{1} - \mathbf{2}|) \\
&\times \left[-G^0(1, \mathbf{x}) (G^0(2, 2)G^0(y, 1) - G^0(2, 1)G^0(y, 2) \right. \\
&\quad \left. + F^{\dagger 0}(1, 2)F^0(y, 2) \right) \\
&- F^{\dagger}(\mathbf{x}, 1) (-G^0(2, 2)F^0(1, y) + G^0(1, 2)F^0(2, y) \\
&\quad \left. + F^0(1, 2)G^0(y, 2) \right) \Big] . \quad (27)
\end{aligned}$$

Moreover we added the expansion of $G(y, \mathbf{x})$ to the set of equations for the later purpose to combine the four Green functions in a matrix formulation.

The perturbation series can be summed up by the Dyson equation. In the presence of pairing correlations anomalous self-energies σ and $\sigma^{\dagger}$ appear which

are later specified by some approximations. The Dyson equations are

$$G(x, y) = G^0(x, y) + \int_C d^4 1 d^4 2 \left[G^0(x, 1) (\Sigma(1, 2) G(2, y) + \sigma(1, 2) F^\dagger(2, y)) + F^0(x, 1) (\sigma^\dagger(1, 2) G(2, y) - \Sigma(2, 1) F^\dagger(2, y)) \right], \quad (28)$$

$$F(x, y) = F^0(x, y) + \int_C d^4 1 d^4 2 \left[G^0(x, 1) (\Sigma(1, 2) F(2, y) - \sigma(1, 2) G(y, 2)) + F^0(x, 1) (\Sigma(2, 1) G(y, 2) + \sigma^\dagger(1, 2) F(2, y)) \right], \quad (29)$$

$$F^\dagger(x, y) = F^{\dagger 0}(x, y) + \int_C d^4 1 d^4 2 \left[G^0(1, x) (\Sigma(2, 1) F^\dagger(2, y) - \sigma^\dagger(1, 2) G(2, y)) + F^{\dagger 0}(x, 1) (\Sigma(1, 2) G(2, y) + \sigma(1, 2) F^\dagger(2, y)) \right], \quad (30)$$

$$G(y, x) = G^0(y, x) + \int_C d^4 1 d^4 2 \left[G^0(1, x) (\Sigma(2, 1) G(y, 2) + \sigma^\dagger(1, 2) F(2, y)) + F^{\dagger 0}(x, 1) (\sigma(1, 2) G(y, 2) - \Sigma(1, 2) F(2, y)) \right], \quad (31)$$

We emphasize that all Green functions and self-energies are contour ordered. If we write out the contour ordered functions as well as the contour integral, a matrix notation is convenient with

$$\mathbf{G} = \begin{pmatrix} G_{<} & G_{<}^\dagger \\ G_{>} & G_{>}^\dagger \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \Sigma_{<} & \sigma_{<} \\ \sigma_{>} & \Sigma_{>} \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (32)$$

The Dyson equation, restricted to contributions from normal functions, takes then following form

$$\mathbf{G}(x, y) = \mathbf{G}^0(x, y) + \int d^4 1 d^4 2 \mathbf{G}^0(x, 1) \sigma^3 \Sigma(1, 2) \sigma^3 \mathbf{G}(2, y), \quad (33)$$

where the fat symbols indicate matrices due to the Schwinger-Keldysh formalism. σ^3 accounts for the minus sign stemming from the time integration along the lower branch of the Schwinger-Keldysh contour on which the time arguments run from $+\infty$ to $-\infty$ (see Fig. 1).

If we use the equations of motion of the free Green functions (see Appendix C), we get following equations of motion

$$D_x \mathbf{G}(x, y) = \sigma^3 \delta(x - y) + \sigma^3 \int d^4 1 \left[\Sigma(x, 1) \sigma^3 \mathbf{G}(1, y) + \sigma(x, 1) \sigma^3 \mathbf{F}^\dagger(1, y) \right], \quad (34)$$

$$D_y^\dagger \mathbf{G}(x, y) = \sigma^3 \delta(x - y) + \int d^4 1 \left[\mathbf{G}(x, 1) \sigma^3 \mathbf{\Sigma}(1, y) + \mathbf{F}(x, 1) \sigma^3 \boldsymbol{\sigma}^\dagger(1, y) \right] \sigma^3, \quad (35)$$

$$D_x \mathbf{F}(x, y) = \sigma^3 \int d^4 1 \left[\mathbf{\Sigma}(x, 1) \sigma^3 \mathbf{F}(1, y) - \boldsymbol{\sigma}(x, 1) \sigma^3 \mathbf{G}(y, 1) \right], \quad (36)$$

$$D_y \mathbf{F}(x, y) = \int d^4 1 \left[-\mathbf{G}(x, 1) \sigma^3 \boldsymbol{\sigma}(1, y) + \mathbf{F}(x, 1) \sigma^3 \mathbf{\Sigma}(y, 1) \right] \sigma^3, \quad (37)$$

$$D_x^\dagger \mathbf{F}^\dagger(x, y) = \sigma^3 \int d^4 1 \left[-\boldsymbol{\sigma}^\dagger(x, 1) \sigma^3 \mathbf{G}(1, y) + \mathbf{\Sigma}(1, x) \sigma^3 \mathbf{F}^\dagger(1, y) \right], \quad (38)$$

$$D_y^\dagger \mathbf{F}^\dagger(x, y) = \int d^4 1 \left[-\mathbf{G}(1, x) \sigma^3 \boldsymbol{\sigma}^\dagger(1, y) + \mathbf{F}^\dagger(x, 1) \sigma^3 \mathbf{\Sigma}(1, y) \right] \sigma^3, \quad (39)$$

$$D_x^\dagger \mathbf{G}(y, x) = \sigma^3 \delta(x - y) + \sigma^3 \int d^4 1 \left[\mathbf{\Sigma}(1, x) \mathbf{G}(y, 1) + \boldsymbol{\sigma}^\dagger(x, 1) \sigma^3 \mathbf{F}(1, y) \right], \quad (40)$$

$$D_y \mathbf{G}(y, x) = \sigma^3 \delta(x - y) + \int d^4 1 \left[\mathbf{G}(1, x) \sigma^3 \mathbf{\Sigma}(y, 1) + \mathbf{F}^\dagger(x, 1) \sigma^3 \boldsymbol{\sigma}(1, y) \right] \sigma^3. \quad (41)$$

A glance at Eqs. (15)-(20) shows the same structure of the equations of motion if we take a corresponding Hartree-Fock self-energy in Eqs. (34)-(41).

4 Equilibrium solutions of the equations of motion

In this Section we want to derive the equilibrium properties of the normal and anomalous Green functions in the Schwinger-Keldysh formalism. We expect a generalization of the known expressions for the Green functions in terms of the spectral function, the distribution function and the retarded and advanced Green functions. We specially take into account the finite damping width of the particles which leads to off-shell contributions in medium and is of particular importance for the in-medium propagation (see e.g. [18,19]). The resulting expressions extended for non-equilibrium systems are applied to derive kinetic

equations in the subsequent Sections. First we start with the equations of motion of the Wigner transformed Green functions. The Wigner transform of the Eqs. (34)-(41) is

$$\left(\frac{i\partial_T}{2} + \omega + \frac{i\mathbf{p}\nabla_R}{2m} - \frac{p^2}{2m}\right)\mathbf{G} = \sigma^3\left[1 + \Sigma\sigma^3\mathbf{G} + \sigma\sigma^3\mathbf{F}^\dagger + \frac{i}{2}\{\Sigma, \sigma^3\mathbf{G}\} + \frac{i}{2}\{\sigma, \sigma^3\mathbf{F}^\dagger\}\right], \quad (42)$$

$$\left(-\frac{i\partial_T}{2} + \omega - \frac{i\mathbf{p}\nabla_R}{2m} - \frac{p^2}{2m}\right)\mathbf{G} = \left[1 + G\sigma^3\Sigma + \mathbf{F}\sigma^3\sigma^\dagger + \frac{i}{2}\{\mathbf{G}, \sigma^3\Sigma\} + \frac{i}{2}\{\mathbf{F}, \sigma^3\sigma^\dagger\}\right]\sigma^3, \quad (43)$$

$$\left(\frac{i\partial_T}{2} + \omega + \frac{i\mathbf{p}\nabla_R}{2m} - \frac{p^2}{2m}\right)\mathbf{F} = \sigma^3\left[\Sigma\sigma^3\mathbf{F} - \sigma\sigma^3\mathbf{G}_- + \frac{i}{2}\{\Sigma, \sigma^3\mathbf{F}\} - \frac{i}{2}\{\sigma, \sigma^3\mathbf{G}_-\}\right], \quad (44)$$

$$\left(\frac{i\partial_T}{2} + \omega - \frac{i\mathbf{p}\nabla_R}{2m} - \frac{p^2}{2m}\right)\mathbf{F} = \left[-\mathbf{G}\sigma^3\sigma + \mathbf{F}\sigma^3\Sigma_- + \frac{i}{2}\{\mathbf{G}, \sigma^3\sigma\} + \frac{i}{2}\{\mathbf{F}, \sigma^3\Sigma_-\}\right]\sigma^3, \quad (45)$$

$$\left(-\frac{i\partial_T}{2} + \omega + \frac{i\mathbf{p}\nabla_R}{2m} - \frac{p^2}{2m}\right)\mathbf{F}^\dagger = \sigma^3\left[-\sigma^\dagger\sigma^3\mathbf{G} + \Sigma_-\sigma^3\mathbf{F}^\dagger - \frac{i}{2}\{\sigma^\dagger, \sigma^3\mathbf{G}\} + \frac{i}{2}\{\Sigma_-, \sigma^3\mathbf{F}^\dagger\}\right], \quad (46)$$

$$\left(-\frac{i\partial_T}{2} + \omega - \frac{i\mathbf{p}\nabla_R}{2m} - \frac{p^2}{2m}\right)\mathbf{F}^\dagger = \left[-\mathbf{G}_-\sigma^3\sigma^\dagger + \mathbf{F}^\dagger\sigma^3\Sigma + \frac{i}{2}\{\mathbf{G}_-, \sigma^3\sigma^\dagger\} + \frac{i}{2}\{\mathbf{F}^\dagger, \sigma^3\Sigma\}\right]\sigma^3, \quad (47)$$

$$\left(-\frac{i\partial_T}{2} + \omega + \frac{i\mathbf{p}\nabla_R}{2m} - \frac{p^2}{2m}\right)\mathbf{G}_- = \sigma^3\left[1 + \Sigma_-\sigma^3\mathbf{G}_- + \sigma^\dagger\sigma^3\mathbf{F} + \frac{i}{2}\{\Sigma_-, \sigma^3\mathbf{G}_-\} + \frac{i}{2}\{\sigma, \sigma^3\mathbf{F}^\dagger\}\right], \quad (48)$$

$$\left(\frac{i\partial_T}{2} + \omega - \frac{i\mathbf{p}\nabla_R}{2m} - \frac{p^2}{2m}\right)\mathbf{G}_- = \left[1 + \mathbf{G}_-\sigma^3\Sigma_- + \mathbf{F}^\dagger\sigma^3\sigma + \frac{i}{2}\{\mathbf{G}_-, \sigma^3\Sigma_-\} + \frac{i}{2}\{\mathbf{F}^\dagger, \sigma^3\sigma\}\right]\sigma^3, \quad (49)$$

where we dropped the arguments (p, R) of each function. For $\mathbf{G}(-p, R)$ and $\Sigma(-p, R)$, the Wigner transforms of $\mathbf{G}(y, x)$ and $\Sigma(y, x)$, we used the abbreviation $\mathbf{G}_-$ respectively Σ_- . $\{A, B\}$ is the Poisson bracket: $\partial_p A \partial_R B - \partial_R A \partial_p B$ resulting from the gradient expansion of the Wigner transformed integrals in Eqs. (34)-(41).

Since we want to consider the equilibrium limit, the derivatives vanish

$$(\omega - \epsilon_p)\mathbf{G} = \sigma^3[1 + \Sigma\sigma^3\mathbf{G} + \sigma\sigma^3\mathbf{F}^\dagger], \quad (50)$$

$$(\omega - \epsilon_p)\mathbf{F} = \sigma^3[\Sigma\sigma^3\mathbf{F} - \sigma\sigma^3\mathbf{G}_-], \quad (51)$$

$$(-\omega - \epsilon_p)\mathbf{F}^\dagger = \sigma^3[-\sigma^\dagger\sigma^3\mathbf{G} + \Sigma_- \sigma^3\mathbf{F}^\dagger], \quad (52)$$

$$(-\omega - \epsilon_p)\mathbf{G}_- = \sigma^3[1 + \Sigma_- \sigma^3\mathbf{G}_- + \sigma^\dagger\sigma^3\mathbf{F}], \quad (53)$$

with $\epsilon_p = \frac{p^2}{2m}$. It is possible to write these equations in matrix form

$$\begin{pmatrix} \omega - \epsilon_p & 0 \\ 0 & -\omega - \epsilon_p \end{pmatrix} \begin{pmatrix} \mathbf{G} & \mathbf{F} \\ \mathbf{F}^\dagger & -\mathbf{G}_- \end{pmatrix} = \sigma^3 + \sigma^3 \begin{pmatrix} \Sigma & \sigma \\ \sigma^\dagger & -\Sigma_- \end{pmatrix} \times \sigma^3 \begin{pmatrix} \mathbf{G} & \mathbf{F} \\ \mathbf{F}^\dagger & -\mathbf{G}_- \end{pmatrix}. \quad (54)$$

This matrix structure is called Gorkov structure from now on. It appears in addition to the Schwinger-Keldysh structure of the equation discussed above. It can be understood in terms of doubling the degrees of freedom by writing

$$\psi(x) \longrightarrow \psi(x) = \begin{pmatrix} \psi(x) \\ \psi^\dagger(x) \end{pmatrix}, \quad \psi^\dagger(y) \longrightarrow \psi^\dagger(y) = \begin{pmatrix} \psi^\dagger(y) \\ \psi(y) \end{pmatrix}. \quad (55)$$

Then the Green function can be replaced by

$$\begin{aligned} \mathbf{G}(x, y) \longrightarrow \mathbf{G}(x, y) &= \langle O_C[\psi(x)\psi^\dagger(y)] \rangle = \begin{pmatrix} \mathbf{G}(x, y) & \mathbf{F}(x, y) \\ \mathbf{F}^\dagger(x, y) & \overline{\mathbf{G}}(x, y) \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{G}(x, y) & \mathbf{F}(x, y) \\ \mathbf{F}^\dagger(x, y) & -\mathbf{G}(y, x) \end{pmatrix}. \end{aligned} \quad (56)$$

The matrices from the Gorkov structure are indicated by a bar over the symbols. This procedure is similar to the doubling of the representations in thermal field theory which leads on an alternative way to the above introduced Schwinger-Keldysh structure [20].

If we use the abbreviation $E = \begin{pmatrix} \omega - \epsilon_p & 0 \\ 0 & -\omega - \epsilon_p \end{pmatrix}$, we can finally write Eq. (54) as

$$1_{ss'} E_{gg'} \mathbf{G}_{g's's\bar{g}} = 1_{g\bar{g}} \sigma_{s\bar{s}}^3 + \sigma_{s's}^3 \Sigma_{gs',g's''} 1_{g'g''} \sigma_{s''s'''}^3 \bar{\mathbf{G}}_{g''s'''\bar{s}\bar{g}}, \quad (57)$$

where we have written out the indices for a better understanding of the matrix structure. The indices $s, s', \dots$ refer to the indices with respect to the Schwinger-Keldysh structure and $g, g', \dots$ respectively to the Gorkov structure. We see that the presence of a thermal or a non-equilibrium state and the presence of pairing correlations gives rise to two different matrix structures of the Green functions and the self-energies. In the following we want to solve this equation.

This task is closely related to diagonalizing the whole equation as shown in Appendix D for the case of the contour ordered Green function in which pairing correlations are not included. Since it is not known from the beginning that only one transformation matrix with respect to the Schwinger-Keldysh structure containing one distribution function is sufficient to diagonalize the equation with respect to the Schwinger-Keldysh structure, we proceed differently. We derive a equation containing only retarded respectively advanced functions, solve this equation and show that, if this solution is transformed with respect to the transformation $\mathbf{T}$ described in Appendix D, the result is a solution of the general equation. From Eq. (50) we get subsequent equations for the causal Green function and the 1-particle correlations

$$(\omega - \epsilon_p)G_{>} = 1 + \Sigma_R G_{>} - \Sigma_{<} G_{>} + \sigma_R F_{>}^\dagger - \sigma_{<} F_{>}^\dagger, \quad (58)$$

$$(\omega - \epsilon_p)G_{<} = -\Sigma_R G_{<} - \Sigma_{<} G_{<} + \sigma_R F_{<}^\dagger - \sigma_{<} F_{<}^\dagger, \quad (59)$$

$$(\omega - \epsilon_p)G_{<} = 1 + \Sigma_{>} G_{<} - \Sigma_a G_{<} + \sigma_{>} F_{<}^\dagger - \sigma_a F_{<}^\dagger. \quad (60)$$

Subtracting these equations from each other, we obtain

$$(\omega - \epsilon_p)G_R = 1 + \Sigma_R G_R + \sigma_R F_R^\dagger, \quad (61)$$

$$(\omega - \epsilon_p)G_A = -1 + \Sigma_A G_A + \sigma_A F_A^\dagger. \quad (62)$$

G_R is the retarded and G_A the advanced Green function. Analogous equations can be derived for F , $F^\dagger$ and $\bar{G}$. Using $\mathbf{G}_0 = \begin{pmatrix} G_R & 0 \\ 0 & G_A \end{pmatrix}$ and $\mathbf{\Sigma}_0 = \begin{pmatrix} \Sigma_R & 0 \\ 0 & \Sigma_A \end{pmatrix}$ all these equation can be combined as

$$E\mathbf{G}_0 = 1 + \mathbf{\Sigma}_0\mathbf{G}_0 \quad (63)$$

Since these quantities are diagonal in the Schwinger-Keldysh indices, it remains only the task to diagonalize this equation separately for the retarded respectively advanced functions with respect to the Gorkov structure. This can be done as in the common BCS theory by a Bogoliubov transformation. Here it is carried out for the retarded functions

$$\begin{pmatrix} \omega - \epsilon_p & 0 \\ 0 & \omega + \epsilon_p \end{pmatrix} \begin{pmatrix} G_R & F_R \\ F_R^\dagger & \bar{G}_R \end{pmatrix} = 1 + \begin{pmatrix} \Sigma_R & \sigma_R \\ \sigma_R^\dagger & \bar{\Sigma}_R \end{pmatrix} \begin{pmatrix} G_R & F_R \\ F_R^\dagger & \bar{G}_R \end{pmatrix}. \quad (64)$$

The solution is

$$\begin{pmatrix} G_R & F_R \\ F_R^\dagger & \bar{G}_R \end{pmatrix} = \frac{\begin{pmatrix} \omega + \epsilon_p - \bar{\Sigma}_R & -\sigma_R \\ -\sigma_R^\dagger & \omega - \epsilon_p - \Sigma_R \end{pmatrix}}{(\omega - \epsilon_p - \Sigma_R)(\omega + \epsilon_p - \bar{\Sigma}_R) - \sigma_R \sigma_R^\dagger}. \quad (65)$$

The poles of these Green function are

$$\omega = E_\pm = S \pm W, \quad (66)$$

with

$$S = \frac{1}{2}(\Sigma_R + \overline{\Sigma}_R), \quad A = \frac{1}{2}(\Sigma_R - \overline{\Sigma}_R), \quad W = \sqrt{(\epsilon_p + A)^2 + \sigma_R \sigma_R^\dagger}. \quad (67)$$

By means of separating these poles in the Green functions and introducing the parameters of the Bogoliubov transformation

$$u^2 = \frac{\epsilon + A + W}{2W}, \quad v^2 = \frac{\epsilon + A - W}{2W}, \quad u^2 - v^2 = 1, \quad (68)$$

the Greens function can finally be written in the well-known form

$$G_R = \left(\frac{u^2}{\omega - E_+ + i\epsilon} + \frac{v^2}{\omega - E_- + i\epsilon} \right), \quad (69)$$

$$F_R = -uv \left(\frac{1}{\omega - E_+ + i\epsilon} - \frac{1}{\omega - E_- + i\epsilon} \right), \quad (70)$$

$$F_R^\dagger = -uv \left(\frac{1}{\omega - E_+ + i\epsilon} - \frac{1}{\omega - E_- + i\epsilon} \right) \quad (71)$$

$$\tilde{G}_R = \left(\frac{v^2}{\omega - E_+ + i\epsilon} + \frac{u^2}{\omega - E_- + i\epsilon} \right). \quad (72)$$

These equations can be combined in a matrix equation using the famous Bogoliubov transformation

$$\hat{G}_R = B \begin{pmatrix} \frac{1}{\omega - E_+ + i\epsilon} & 0 \\ 0 & \frac{1}{\omega - E_- + i\epsilon} \end{pmatrix} B^{-1}, \quad \text{with } B = \begin{pmatrix} u & v \\ -v & u \end{pmatrix}. \quad (73)$$

A similar equation is valid for the advanced Green function with $+i\epsilon$ replaced by $-i\epsilon$. With that we found a solution of Eq. (63). We transform now $\mathbf{G}_0$ with respect to the transformation $\mathbf{T}$ described in Appendix D in connection with the diagonalization of the contour ordered Green function. We achieve for each Green function the known contour ordered Green functions by the transformation $\mathbf{G} = \sigma^3 \mathbf{T} \mathbf{G}_0 \mathbf{T}^{-1}$

$$G_c(p) = (1 - f(\omega))\hat{G}_R(p) + f(\omega)\hat{G}_A(p), \quad (74)$$

$$G_<(p) = -f(\omega)(\hat{G}_R(p) - \hat{G}_A(p)), \quad (75)$$

$$G_>(p) = (1 - f(\omega))(\hat{G}_R(p) - \hat{G}_A(p)), \quad (76)$$

$$G_d(p) = -f(\omega)\hat{G}_R(p) - (1 - f(\omega))\hat{G}_A(p). \quad (77)$$

If we examine the effect of this transformation on Eq. (63), we get

$$\sigma^3 \mathbf{T} \mathbf{E} \mathbf{G}_0 \mathbf{T}^{-1} = \sigma^3 \mathbf{T} \mathbf{T}^{-1} + \sigma^3 \mathbf{T} \dot{\Sigma} \mathbf{T} \mathbf{T}^{-1} \sigma^3 \sigma^3 \mathbf{G}_0 \mathbf{T}^{-1}, \quad (78)$$

which is equal to Eq. (57) because of the transformation properties of σ^3 under $\mathbf{T}$ (see Appendix D) and the fact that $\hat{E}$ and $\mathbf{T}$ commute. Therefore $\mathbf{G} =$

$\sigma^3 \mathbf{T} \mathbf{G}_0 \mathbf{T}^{-1}$ is a solution of the full equation. It should be emphasized that it is not clear from beginning that only one distribution function is necessary for the equilibrium solution of this equation. On the other hand this can be supported by the fact that the distribution functions of G and $\bar{G}$ respectively F and $F^\dagger$ must be equal (see Appendix C). Furthermore normal and anomalous Green functions are described by the same Matsubara frequencies in the imaginary time formalism [FeWa71].

In the last part we want to study the behaviour of the Green functions with respect to medium properties i.e. when the particles have a width due to collisional broadening. In this case the self-energies get an imaginary part which causes then the width of the spectral functions. This leads to physically crucial off-shell contributions to the propagation of particles in medium (see e.g. [18,19]).

A glance at the Eqs. (69)-(72) shows that these expressions are only valid for real self-energies. In medium the Bogoliubov parameters u and v as well as $E_\pm$ become complex. So the orthogonal Bogoliubov transformation must be replaced by a unitary one. Thus the poles of the retarded Green functions in Eqs. (69)-(72) move from the real axis into the complex plane, so that we have to substitute this expression for the retarded Green function by one which ensures that the cuts and poles of the retarded and advanced Green functions remain on the real axis. Therefore we use the prescription to fold the spectral function with the corresponding free Green function to get the full Green function [21]

$$G_R(\omega, \mathbf{p}) = i \int \frac{d\omega'}{2\pi} \frac{A(\omega', \mathbf{p})}{\omega' - \omega + i\epsilon}, \quad G_A(\omega, \mathbf{p}) = i \int \frac{d\omega'}{2\pi} \frac{A(\omega', \mathbf{p})}{\omega' - \omega - i\epsilon}, \quad (79)$$

with

$$\tilde{A}(\omega, \mathbf{p}) = (G_R - G_A) = B \begin{pmatrix} A_+ & 0 \\ 0 & A_- \end{pmatrix} B^{-1} = \begin{pmatrix} A_G & A_F \\ A_{F^\dagger} & A_{\bar{G}} \end{pmatrix}. \quad (80)$$

A_G , A_F , $A_{F^\dagger}$ and $A_{\bar{G}}$ have following explicit form

$$A_G = |u|^2 A_+ + |v|^2 A_-, \quad (81)$$

$$A_F = -uv^* A_+ + u^* v A_-, \quad (82)$$

$$A_{F^\dagger} = -u^* v A_+ + uv^* A_-, \quad (83)$$

$$A_{\bar{G}} = |v|^2 A_+ + |u|^2 A_-, \quad (84)$$

where A_+ and A_- are the spectral functions corresponding to the two different energy-poles

$$A_+ = \frac{\text{Im}E_+}{(\omega - \text{Re}E_+)^2 + (\text{Im}E_+)^2}, \quad A_- = \frac{\text{Im}E_-}{(\omega - \text{Re}E_-)^2 + (\text{Im}E_-)^2}. \quad (85)$$

Therefore in the presence of pairing correlations the usual nucleon spectral function must be replaced by a sum over two spectral functions corresponding to the

two energy-poles. Something similar is observed if the nucleon spectral function is calculated in the T-matrix approximation. Within this approach a bound state is found which leads to a pole in the 2-particle Green function and an additional peak in the nucleon spectral function, which is identified with the deuteron [4,8]. This is closely related to the $n-p$ condensation in the ${}^3S_1 - {}^3D_1$ channel since the gap equation becomes the deuteron Schrödinger equation in the low density limit [8,10].

It must be emphasized that the two degrees of freedom due to the Schwinger-Keldysh and the Gorkov structure decouple only formally for the retarded and advanced functions (Eqs. (61), (62)) since in the calculation of the retarded and advanced self-energies the medium and pairing correlation properties enter simultaneously

5 Kinetic equation in quasi-particle approximation

In this Section we attack the main aim of this paper, the derivation of a kinetic equation for systems with pairing correlations. That is done in this Section for the case of the quasi-particle approximation, i.e. the spectral function is a δ -distribution in energy. The above spectral function has then the following form

$$A(\omega, \mathbf{p}) = B \begin{pmatrix} \delta(\omega - E_+) & 0 \\ 0 & \delta(\omega - E_-) \end{pmatrix} B^{-1}. \quad (86)$$

In a kinetic equation the 1-particle density of a system under non-equilibrium conditions is studied. Therefore the above expressions for the 1-particle correlations (Eqs. (75), (76)) must be extended for non-equilibrium conditions. We make the ansatz that all different normal and anomalous 1-particle correlations are governed by one distribution function

$$G_{<}(p, R) = -f(p, R)A_G(p, R), \quad G_{>}(p, R) = (1 - f(p, R))A_G(p, R), \quad (87)$$

$$F_{<}(p, R) = -f(p, R)A_F(p, R), \quad F_{>}(p, R) = (1 - f(p, R))A_F(p, R), \quad (88)$$

$$F_{<}^\dagger(p, R) = -f(p, R)A_{F^\dagger}(p, R), \quad F_{>}^\dagger(p, R) = (1 - f(p, R))A_{F^\dagger}(p, R), \quad (89)$$

$$\bar{G}_{<}(p, R) = -f(p, R)A_{\bar{G}}(p, R), \quad \bar{G}_{>}(p, R) = (1 - f(p, R))A_{\bar{G}}(p, R). \quad (90)$$

The above ansatz is however the simplest choice being possible. An improvement in which different distribution function are used is deferred to future publications.

Since we use only one distribution function only one kinetic equation for f is necessary. This is derived from the equations of motion of $G_{<}$, $G_{>}$ and A_G

$$\begin{aligned} \left(i\partial_T + i\frac{\mathbf{p}}{m} \nabla_R \right) G_{<} &= \Sigma_R G_{<} + \Sigma_{<} G_A - G_R \Sigma_{<} - G_{<} \Sigma_A \\ &+ \sigma_R F_{<}^\dagger + \sigma_{<} F_A^\dagger - F_R \sigma_{<}^\dagger - F_{<} \sigma_A^\dagger \end{aligned}$$

This term is a derivation of the collision term and contributes therefore only in second order of the gradient expansion. It is neglected. The final kinetic equation reads after the ω -integration and after inserting the explicit form of the spectral functions (see Eq. (86))

$$\begin{aligned}
& u^2 \left(\partial_T + \frac{\mathbf{p}}{m} \nabla_R \right) f \Big|_{\epsilon=E_+} \\
& + v^2 \left(\partial_T + \frac{\mathbf{p}}{m} \nabla_R \right) f \Big|_{\epsilon=E_-} \\
& - u^2 \{ \text{Re} \Sigma_R, f \} \Big|_{\epsilon=E_+} - v^2 \{ \text{Re} \Sigma_R, f \} \Big|_{\epsilon=E_-} \\
& + uv \left[\{ \text{Re} \sigma_R, f \} \Big|_{\epsilon=E_+} - \{ \text{Re} \sigma_R, f \} \Big|_{\epsilon=E_-} \right] = -iu^2 [(1-f)\Sigma_< + f\Sigma_>] \Big|_{\epsilon=E_+} \\
& - iv^2 [(1-f)\Sigma_< + f\Sigma_>] \Big|_{\epsilon=E_-} \\
& + iuv [(1-f)\sigma_< + f\sigma_>] \Big|_{\epsilon=E_+} \\
& - iuv [(1-f)\sigma_< + f\sigma_>] \Big|_{\epsilon=E_-} \quad (99)
\end{aligned}$$

Finally we want to discuss the changes in the kinetic equation due to the pairing correlations. At first we get an anomalous contribution to the drift term as well as to the collision term. Both terms are proportional to uv . Since v^2 is similar to a smeared Fermi distribution [14] they contribute at the surface of the Fermi sea where the pairs are built. That means that we have a correction of the force for particles bound in pairs.

In the common part of the kinetic equation the contributions are split into two parts. The first is proportional to v^2 , has the 1-particle energy E_- and delivers for vanishing pairing correlations ($v^2 \rightarrow -1$, $u^2 \rightarrow 0$ for $\epsilon_p < \epsilon_F$) the known terms. The other term, proportional to u^2 , makes the particles sensible for the region above the smeared Fermi sea due to the energy E_+ . It opens in the collision term in principle a new scattering channel and tends therefore for particles in the Fermi sea to increase the medium cross section since u^2 contributes mainly above the Fermi sea where there is nearly no Pauli blocking.

The main difference to the transport equation in [2] is that here only one distribution function is used instead of the two different density matrices for the normal and anomalous sector. Therefore only one kinetic equation is necessary instead of two coupled equations. The effects of the pairing correlation is here totally described by the spectral functions which are integrated out. Therefore the structure of the kinetic equation becomes much simpler.

6 Kinetic equation for particles with width

The common methods for solving the kinetic equations makes use of the quasi-particle approximation, in which the particles are put on the mass-shell. Nevertheless we want to treat briefly the kinetic equations of particles with width in the presence of pairing correlations. We keep the ansatz that all 1-particle correlations are described by one distribution function. We can therefore start the discussion from Eq. (93). We apply similar transformations as above to get

$$\begin{aligned}
-iA_G \left(\partial_T + \frac{\vec{p}}{m} \vec{\nabla}_R \right) f \\
+ iA_G \{ \text{Re} \Sigma_R, f \} = & -A_G((1-f)\Sigma_< + f\Sigma_>) \\
& + [(1-f)(\sigma_< + \sigma_>^*) + f(\sigma_> + \sigma_<^*)] \text{Re} F_R \\
& - i[(1-f)(\sigma_< - \sigma_>^*) + f(\sigma_> - \sigma_<^*)] \text{Im} F_R \\
& - 2\text{Im} \Sigma_R \{ \text{Re} G_R, f \} \\
& + \frac{i}{2} \{ \text{Re} \sigma_R, \text{Re} F_< \} (1-2f) - \frac{1}{2} \{ \text{Re} \sigma_R, \text{Im} F_< \} \\
& + \frac{1}{2} \{ \text{Im} \sigma_R, \text{Re} F_< \} + \frac{i}{2} \{ \text{Im} \sigma_R, \text{Im} F_< \} (1-2f) \\
& - \frac{i}{2} \{ \text{Re} \sigma_R, \text{Re} F_> \} (1-2f) - \frac{1}{2} \{ \text{Re} \sigma_R, \text{Im} F_> \} \\
& + \frac{1}{2} \{ \text{Im} \sigma_R, \text{Re} F_> \} \\
& - \frac{i}{2} \{ \text{Im} \sigma_R, \text{Im} F_> \} (1-2f). \tag{100}
\end{aligned}$$

The last term is neglected since it leads in quasi-particle approximation to a term in second order of the derivatives. We get again the terms of the above kinetic equation but now extended to particles with width: the spectral function is now a function containing two peaks. It can be calculated according to Eqs. (80)-(85). Instead of handling particles which have two sharp energies we have a particle which consist of a strength function over off-shell states.

7 Approximations for the self-energies

Up to now the kinetic equations where studied assuming that the self-energies are known. In this Section we want to close this hole by studying specific expressions for the self-energies for systems with pairing correlations.

In this context we must take account of the Schwinger-Keldysh and of the Gorkov structure in the self-energies. The Schwinger-Keldysh structure can be respected by making use of the contour-integration.

The Gorkov structure is best taken into account by extending the usual Feynman rules for a diagrammatic representation with extra symbols for the

anomalous Green functions. If we read all diagrams from below to above the common Green function is a line with an arrow indicating upward. The Green function F can be symbolized by a line where two arrows at each end indicate into the exterior of the line, while for $F^\dagger$ they indicate into the interior. $\bar{G}$ is a line with one arrow indicating downward. The symbols can be comprised in the rule that a field operator ψ within a Green function at a vertex is depicted by an arrow on the line of the Green function, indicating toward the vertex, while the arrow for $\psi^\dagger$ indicates away from the vertex. Furthermore at each vertex one arrow has to indicate toward and one away from the vertex. The symbols are graphically depicted in Fig. 2. Similar Feynman rules were set up for boson Green functions in the presence of a condensate [14]. In the following

$$G = \begin{array}{c} \blacktriangleup \\ \hline \blacktriangledown \end{array}, \quad F = \begin{array}{c} \blacktriangleup \\ \hline \blacktriangleup \end{array}, \quad F^\dagger = \begin{array}{c} \blacktriangledown \\ \hline \blacktriangledown \end{array}, \quad \bar{G} = \begin{array}{c} \blacktriangledown \\ \hline \blacktriangledown \end{array}$$

Figure 2: The elements of the normal and anomalous Green functions for Feynman diagrams

we study in the present context the Hartree, Fock and Born self-energies. In the case of the Hartree and the Fock self-energies we derive the explicit expressions from the perturbation expansion, while the Born self-energies are only treated in graphical manner. The full expressions for the perturbation series up to the first order is given in Eqs. (24)-(27). They are depicted in Figs. 3 and 4. For the Hartree self-energies we get one single expression

$$\Sigma^H(x) = -g \int d^3y V(|\mathbf{x} - \mathbf{y}|) G_+(y, y). \quad (101)$$

In the case of the Fock self-energy all kinds of anomalous self-energies can

Figure 3: Hartree-type contributions in the perturbation series of the Green function G .

appear (confer Fig. 4)

$$\Sigma^F(x, y) = -g V(|\mathbf{x} - \mathbf{y}|) G(x, y), \quad (102)$$

$$\sigma^F(x, y) = gV(|\mathbf{x} - \mathbf{y}|)F(x, y), \quad (103)$$

$$\sigma^\dagger F(x, y) = gV(|\mathbf{x} - \mathbf{y}|)F^\dagger(x, y), \quad (104)$$

$$\bar{\Sigma}^F(x, y) = gV(|\mathbf{x} - \mathbf{y}|)\bar{G}(x, y). \quad (105)$$

Note that all these Green function carry Schwinger-Keldysh indices while the

Figure 4: Fock-type contributions in the perturbation series of the Green function G .

Hartree self-energy is singular on the contour [11]. For the Born approximation also all anomalous components appear. Furthermore we get an additional contribution from the anomalous sector (see Fig. 5). The explicit result is

$$\Sigma^B(x, y) = -g^2 \int d^3 1 d^3 2 V(|\mathbf{x} - \mathbf{1}|) V(|\mathbf{y} - \mathbf{2}|) [G(1, 2)G(2, 1) - F(1, 2)F^\dagger(2, 1)] G(x, y), \quad (106)$$

$$\sigma^B(x, y) = -g^2 \int d^3 1 d^3 2 V(|\mathbf{x} - \mathbf{1}|) V(|\mathbf{y} - \mathbf{2}|) [G(1, 2)G(2, 1) - F(1, 2)F^\dagger(2, 1)] F(x, y), \quad (107)$$

$$\sigma^\dagger B(x, y) = -g^2 \int d^3 1 d^3 2 V(|\mathbf{x} - \mathbf{1}|) V(|\mathbf{y} - \mathbf{2}|) [G(1, 2)G(2, 1) - F(1, 2)F^\dagger(2, 1)] F^\dagger(x, y), \quad (108)$$

$$\bar{\Sigma}^B(x, y) = -g^2 \int d^3 1 d^3 2 V(|\mathbf{x} - \mathbf{1}|) V(|\mathbf{y} - \mathbf{2}|) [G(1, 2)G(2, 1) - F(1, 2)F^\dagger(2, 1)] \bar{G}(x, y). \quad (109)$$

The anomalous self-energies can be interpreted if we look on the context in which they appear in the kinetic equation. E.g. the self-energy σ appears together with $F^\dagger$. Since $\langle \psi\psi \rangle$ is the pairing condensate, the term $\sigma F^\dagger$ describes the propagation of a pair driven by the self-energy σ . Thus, in the Fock and Born self-energy we have terms in which the particles alone and particles in pairs are propagated. The Born term describes the scattering of particles. Therefore contributions from the anomalous Green functions in the Born term results from an additional contribution due to the scattering between particles and particles in pairs.

Figure 5: Feynman diagrams for the contributions to normal Born self-energy Σ^B and to the anomalous Born self-energy σ^B .

8 Conclusion

The contents of this paper is a derivation of a kinetic equation of a system of fermions with pairing correlations. Since the Green function formalism has a close relation to standard quantum field theory and allows to give a most fundamental derivation, we start with Green functions in the Schwinger-Keldysh formalism following many other papers on kinetic equations (e.g. [12,13] and references therein). In the presence of pairing correlations a lot of new features must be considered.

The derivation starts from a perturbation expansion of the Green functions for an arbitrary non-equilibrium system. The explicit perturbation series is formulated by the help of a generalized Wick theorem. This Wick theorem is based on the cumulant expansion of a contour ordered product of field operators taking into account all initial correlations existing in the non-equilibrium system as well as normal and anomalous 1-particle Green functions. Thus the Gorkov equation is embedded in the more general context of the cumulant expansion. The appearing initial correlations are here neglected.

Dyson equations are formulated for the non-equilibrium system with pairing correlations. The Green functions and self-energies carry two different matrix structures, one due to the Schwinger-Keldysh structure (G_c , $G_<$, $G_>$ and G_a) and the other due to Gorkov structure (G , F , $F^\dagger$ and $\bar{G}$). However the retarded and advanced Green functions obey a equation containing only retarded respectively advanced functions. That enables to find a solution for the equilibrium case in which the spectral functions corresponding to the Green functions G , F , $F^\dagger$ and $\bar{G}$ are determined by the Gorkov structure. The Schwinger-Keldysh structure of all Green functions is described by the use of only one distribution function. That means that the Schwinger-Keldysh and the Gorkov structure decouple formally in the equilibrium case. We say "formally" since both structures get coupled in the calculation of the self-energies. The nucleon spectral function gets two peaks due to the two energy-poles in the presence of pairing correlations. The medium effects lead to a width of both parts of the spectral

function. The resulting spectral function is similar to the one from a T-matrix calculation in which also a doubling of the poles appears.

The derivation of the kinetic equation is based on the ansatz which is the simplest being possible that all Green functions G , F , $F^\dagger$ and $\bar{G}$ are described by one distribution function. For this distribution function a kinetic equation is derived. The kinetic equation undergoes changes because of two facts. First we get a second pole in the Green functions due to the pairing correlations. That is why the distribution function becomes simultaneously sensible to two points in the phase-space corresponding to the two energy-poles. That means a correction to the drift term as well as an opening of a new channel in the collision term. Secondly we get explicitly terms from the anomalous sector: besides the force proportional to the real part of the retarded normal self-energy Σ_R we get a term proportional to the real part of the retarded anomalous self-energy σ_R which contributes mainly at the smeared Fermi surface. This term can be interpreted as a correction of the force due to particles participating in pairs. A similar contribution appears in the collision term caused by the anomalous self-energies $\sigma_<$ and $\sigma_>$. The interpretation of this term is supported by the explicit form of the self-energies where in addition to the particle-particle scattering terms from particle-pair and pair-pair scattering appear.

Since we have chosen only the simplest form of the non-equilibrium Green functions in terms of the spectral and distribution functions, the above considerations can be refined and enlarged which is postponed to future publications. Nevertheless a rigorous derivation of a kinetic equation was presented which already allows insights in essential features and changes of kinetic equations for systems with pairing correlations.

9 Appendix A

The contents of this Appendix is a brief introduction of cumulants. Cumulants are the irreducible correlations functions of a product of observables. If we have n decidable observables x_i , $i = 1, 2, \dots, n$. The moment of these n observables can be decomposed into a sum of all possible products of arbitrary cumulants

$$\langle x_1 \dots x_n \rangle = \sum_{a, p} \sum_{r=1}^n \kappa\{x_1, \dots, x_n\}_r. \quad (110)$$

$\kappa\{x_1, \dots, x_n\}_r$ denotes a product of r cumulants whose arguments are a partition of the n arguments $x_1, \dots, x_n$ into r disjunct sets. $\sum_{a, p}$ is the sum over all possible partitions. For illustration we present the cumulant expansion of $\langle x_1 x_2 x_3 x_4 \rangle$

$$\begin{aligned} \langle x_1 x_2 x_3 x_4 \rangle = & \kappa(1, 2, 3, 4) + \kappa(1)\kappa(2, 3, 4) + \dots + \kappa(1, 2)\kappa(3, 4) + \dots \\ & + \kappa(1)\kappa(2)\kappa(3, 4) + \dots + \kappa(1)\kappa(2)\kappa(3)\kappa(4). \end{aligned} \quad (111)$$

$\kappa(1, 2, 3, 4)$ is the irreducible 4-point correlation or cumulant of the observables x_1, x_2, x_3 and x_4 . In an inverse relation each cumulant can be calculated from the moments

$$\kappa(x_1, \dots, x_n) = \sum_{a \in \mathcal{P}} \sum_{r=1}^n (-1)^{r-1} (r-1)! \langle \{x_1, \dots, x_n\}_r \rangle, \quad (112)$$

where $\langle \{x_1, \dots, x_n\}_r \rangle$ denotes a product of r moments whose arguments are a partition of the n arguments $x_1, \dots, x_n$ into r disjunct sets. So the above cumulant $\kappa(1, 2, 3, 4)$ is in terms of the corresponding moments

$$\begin{aligned} \kappa(1, 2, 3, 4) = & \langle x_1 x_2 x_3 x_4 \rangle - \langle x_1 \rangle \langle x_2 x_3 x_4 \rangle - \dots - \langle x_1 x_2 \rangle \langle x_3 x_4 \rangle - \dots \\ & + 2! \langle x_1 \rangle \langle x_2 \rangle \langle x_3 x_4 \rangle + \dots - 3! \langle x_1 \rangle \langle x_2 \rangle \langle x_3 \rangle \langle x_4 \rangle. \end{aligned} \quad (113)$$

These identities can be formulated in a more formal manner by the help of the generating functions of the moments and the cumulants. Essential is that the generating function of the cumulants is the logarithm of the generating function of the moments. For details we refer to Ref. [17] and references therein. We note furthermore that if we apply the cumulant expansion to fermion field operators, we have to add a factor $(-1)^P$ on the r.h.s. of Eqs. (111) and (113) for the permutations necessary to get the correct ordering of the operators. Since the cumulants should be complex functions in this context, contributions with moments from an odd number of fermion field operators must be excluded. Therefore the sum over all possible partitions in Eqs. (111) and (113) is restricted to sets with an even number of fermion field operators.

If we consider contour ordered functions i.e. we want to factorize a n -particle Green function, contour ordered cumulants must be used

$$x_1, \dots, x_n = \sum_{a \in \mathcal{P}} \sum_{r=1}^n (-1)^{r-1} \kappa_C \{x_1, \dots, x_n\}_r, \quad (114)$$

where the index C indicates contour ordering. The contour ordered cumulants are defined through the corresponding inverse relation

$$\kappa_C(x_1, \dots, x_n) = \sum_{a \in \mathcal{P}} \sum_{r=1}^n (-1)^P (-1)^{r-1} (r-1)! \langle O_C \{x_1, \dots, x_n\}_r \rangle, \quad (115)$$

Eqs. (115) and (116) are also correct for fermion field operators.

10 Appendix B

The contents of this Appendix is the derivation of a Wick theorem which allows to factorize a contour ordered product of field operators in the perturbation

series (see Eq. (22)) for an arbitrary non-equilibrium system with pairing correlations. For a non-equilibrium system initial correlations must be considered in the Wick theorem [17,22,23]. Here this non-equilibrium Wick theorem is generalized for the presence of pairing correlations. The argumentation follows Refs. [17,23].

We start with introducing some abbreviations. First we do not distinguish between field operators and their adjunct operators. So, all fermion field operators are written as $\chi := \{\psi, \psi^\dagger\}$. The product, the normal ordered and the contour ordered product of n field operators are abbreviated as

$$P_n := \chi_1 \cdots \chi_n, \quad (116)$$

$$N_n := :\chi_1 \cdots \chi_n:, \quad (117)$$

$$O_n := \text{Ord}(\chi_1 \cdots \chi_n). \quad (118)$$

The Wick theorem is an operator identity which reorders an ordinary or a contour ordered product of field operators as a function of normal ordered products and pairings respectively contractions. In the following we have to evaluate this Wick theorem for systems with pairing correlations. If we take the expectation value of the Wick theorem we get

$$\langle O_n \rangle = \sum_{a,p} \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} (-1)^p \langle N_{n-2k} \rangle (\langle O_2 \rangle - \langle N_2 \rangle)^k, \quad (119)$$

$$\langle P_n \rangle = \sum_{a,p} \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} (-1)^p \langle N_{n-2k} \rangle (\langle P_2 \rangle - \langle N_2 \rangle)^k. \quad (120)$$

The aim is now to reformulate the expectation values of the normal ordered products by physically more meaningful functions. The first step is to substitute the normal ordered products in Eq. (120) by the expectation values of the ordinary products $\langle P_n \rangle$ in Eq. (121). For explicit steps of this calculation we refer to Ref. [23]. Since in these products all kind of correlations are included, we want to subtract order by order all irreducible correlations from the products. This is done using irreducible correlations or cumulants [17].

$$\langle P_n \rangle = \sum_{r=1}^n \sum_{a,p} (-1)^p \kappa\{n\}_r, \quad (121)$$

where $\kappa\{n\}_r$ is the abbreviation for the product of r cumulants or irreducible correlations whose arguments are a partition of the n arguments of W_n into r sets. $\sum_{a,p}$ is the sum over all possible partitions. The number of fermion field operators in each set is even, since we want to handle with complex functions. The factor $(-1)^p$ takes into account the necessary permutations of fermion field operators to get the correct ordering. If this expansion is inserted, we get the

following expression for the expectation value of a contour ordered product

$$\langle O_n \rangle = \sum_a \sum_p \sum_{s=0}^{\lfloor \frac{n}{2} \rfloor} \sum_{r=1}^{n-2s} (-1)^P \kappa\{n-2s\}'_r \langle O_2 \rangle^s. \quad (122)$$

Here in $\kappa\{n-2s\}'_r$ contains irreducible correlations of four and more fermion field operators. The contour ordered 2-point cumulants are explicitly collected in $\langle O_2 \rangle$ which stands for all normal and anomalous Green functions since O_2 represents the following contour ordered products: $O_C(\psi\psi^\dagger)$, $O_C(\psi\psi)$ and $O_C(\psi^\dagger\psi^\dagger)$. Eq. (123) is the generalized Wick theorem for systems with pairing correlations and arbitrary initial correlations. It therefore generalizes the Gorkov factorization to arbitrary systems. Specifying explicitly which of the normal and anomalous Green functions appear, Eq. (123) can be finally formulated as

$$\langle O_n \rangle = \sum_a \sum_p \sum_{s=0}^{\lfloor \frac{n}{2} \rfloor} \sum_{r=1}^{n-2s} \sum_{i+j+k=s} (-1)^P \kappa\{n-2s\}'_r G^i F^j F^{\dagger k}. \quad (123)$$

This expression is used above

11 Appendix C

The retarded and advanced Green function are defined as

$$G_+(x-y) = -i\theta(x_0-y_0)\langle\{v(x), \psi^\dagger(y)\}\rangle, \quad (124)$$

$$G_-(x-y) = -i\theta(y_0-x_0)\langle\{\psi(x), \psi^\dagger(y)\}\rangle. \quad (125)$$

From the definition of the different Green functions in the Schwinger-Keldysh formalism follows immediately

$$G_c^*(x, y) = -G_a(y, x), \quad (126)$$

$$G_<^*(x, y) = -G_<(y, x), \quad (127)$$

$$G_+^*(x, y) = -G_-(y, x). \quad (128)$$

If we consider the complex conjugate of the Wigner transformed Green functions, we get from Eqs. (127)-(129)

$$G_c^*(p, R) = -G_a(p, R), \quad (129)$$

$$G_<^*(p, R) = -G_<(p, R), \quad (130)$$

$$G_+^*(p, R) = -G_-(p, R). \quad (131)$$

This implies

$$\text{Re}G_c(p, R) = -\text{Re}G_a(p, R), \quad \text{Im}G_c(p, R) = \text{Im}G_a(p, R), \quad (132)$$

$$\text{Re}G_<(p, R) = 0 = \text{Re}G_>(p, R), \quad (133)$$

$$\text{Re}G_+(p, R) = -\text{Re}G_-(p, R), \quad \text{Im}G_+(p, R) = -\text{Im}G_-(p, R). \quad (134)$$

For the anomalous Green functions subsequent relations are valid

$$F_c(x, y) = -F_c(y, x), \quad (135)$$

$$F_c^*(x, y) = F_a^\dagger(x, y). \quad (136)$$

$$F_>(x, y) = -F_<(x, y), \quad (137)$$

$$F_>^*(x, y) = F_<^\dagger(x, y), \quad (138)$$

$$F_+(y, x) = -F_-(x, y), \quad (139)$$

$$F_+^*(x, y) = -F_+^\dagger(x, y). \quad (140)$$

The Wigner transformation leads to

$$F_c^*(p, R) = -F_a^\dagger(p, R), \quad (141)$$

$$F_>^*(p, R) = -F_>^\dagger(p, R), \quad (142)$$

$$F_+^*(p, R) = +F_-^\dagger(p, R). \quad (143)$$

Analogous relations are valid for the corresponding self-energies.

These relations allow to follow that the distribution functions of F and $F^\dagger$ must be equal. With the help of the retarded and advanced Green functions and the distribution functions the causal and anticausal Green functions F_c , F_a and $F_c^\dagger$ can be formulated as

$$F_c = (1 - f_F)F_R + f_FF_A, \quad F_a = -f_FF_R - (1 - f_F)F_A, \quad (144)$$

$$F_c^\dagger = (1 - f_{F^\dagger})F_R^\dagger + f_{F^\dagger}F_A^\dagger \quad (145)$$

$$= (1 - f_{F^\dagger})F_A^* + f_{F^\dagger}F_R^* \quad (146)$$

$$= -F_a^* = f_FF_R^* + (1 - f_F)F_A^*. \quad (147)$$

$$\Rightarrow f_{F^\dagger} = f_F \quad (148)$$

The equations of motion of the free Green functions are

$$D_x \mathbf{G}^0(x, y) = -\sigma^3 \delta(x - y) = D_y^\dagger \mathbf{G}^0(x, y), \quad (149)$$

$$D_x \mathbf{F}^0(x, y) = 0 = D_y \mathbf{F}^0(x, y), \quad (150)$$

$$D_x^\dagger \mathbf{F}^{\dagger 0}(x, y) = 0 = D_y^\dagger \mathbf{F}^{\dagger 0}(x, y), \quad (151)$$

where we used the abbreviations D_x and $D_y^\dagger$ of Section 2.

12 Appendix D

The contents of this Appendix is the diagonalization of the contour ordered Green functions. Diagonalization means the fact that the matrix of the Green functions can be transformed by a linear transformation into a diagonal matrix. The transformation matrix contains the distribution function of the system.

The elements of the diagonal matrix are the full retarded and advanced Green function which carry no explicit dependence on the distribution function [20]. To formulate this linear transformation the contour ordered Green functions are first expressed in terms of the retarded and advanced Green function and the distribution function

$$G_r(p) = (1 - f(\omega))G_R(p) + f(\omega)G_A(p), \quad (152)$$

$$G_<(p) = -f(\omega)G_A(p). \quad (153)$$

$$G_>(p) = (1 - f(\omega))(G_R(p) - G_A(p)). \quad (154)$$

$$G_a(p) = -f(\omega)G_R(p) - (1 - f(\omega))G_A(p). \quad (155)$$

The self-energy obeys analogous equations. The diagonalization states that the Green function $\mathbf{G} = \begin{pmatrix} G_r & G_< \\ G_> & G_a \end{pmatrix}$ can be get from $\mathbf{G}_0 = \begin{pmatrix} G_R & 0 \\ 0 & G_A \end{pmatrix}$ by linear transformations

$$\mathbf{G}(p) = \sigma^3 \mathbf{T}(f(\omega)) \mathbf{G}_0(p) \mathbf{T}^{-1}(f(\omega)) \quad (156)$$

Direct verification shows that following choice is possible for $\mathbf{T}$

$$\mathbf{T}(f(\omega)) = \begin{pmatrix} \sqrt{f(\omega)} & \sqrt{f(\omega)} \\ -\sqrt{f(\omega)} & -\frac{1}{\sqrt{f(\omega)}}(1 - f(\omega)) \end{pmatrix}. \quad (157)$$

It should be noted that different choices for $\mathbf{T}$ are possible [20]. For σ^3 the following identities are valid

$$\mathbf{T}(f(\omega))\sigma^3 = \sigma^3 \mathbf{T}^T(f(\omega)), \quad \mathbf{T}^{T^{-1}}(f(\omega))\sigma^3 = \sigma^3 \mathbf{T}^{-1}(f(\omega)). \quad (158)$$

Therefore instead of Eq. (159) can be written

$$\mathbf{G}(p) = \mathbf{T}^T(f(\omega))\sigma^3 \mathbf{G}_0(p) \mathbf{T}^{-1}(f(\omega)). \quad (159)$$

Since the self-energy obeys an analogous equation like $\mathbf{G}$, it is possible to diagonalize the self-energy matrix

$$\Sigma(p) = \sigma^3 \mathbf{T}(f(\omega)) \Sigma_0(p) \mathbf{T}^{-1}(f(\omega)). \quad (160)$$

Finally we want to examine the consequences on the solution of the equation of motion. We look at first on the free case

$$(\omega - \epsilon_p) \mathbf{G}(p) = \sigma^3. \quad (161)$$

Applying $\mathbf{T}$ from the left and the right, we get

$$(\omega - \epsilon_p) \mathbf{G}_0(p) = 1, \quad (162)$$

as it should be. This can be solved immediately

$$\mathbf{G}_0(p) = \begin{pmatrix} \frac{1}{\omega - \epsilon_p + i\epsilon} & 0 \\ 0 & \frac{1}{\omega - \epsilon_p - i\epsilon} \end{pmatrix}. \quad (163)$$

The solution for $\mathbf{G}$ is then obtained through Eq. (157). In the case with interaction the solution is similar

$$(\omega - \epsilon_p)\mathbf{G}(p) = \sigma^3 + \Sigma(p)\sigma^3\mathbf{G}(p), \quad (164)$$

$$\implies ((\omega - \epsilon_p) - \Sigma_0(p))\mathbf{G}_0(p) = 0. \quad (165)$$

Since all matrices in the last equation are diagonal matrices, the equation can be solved

$$\mathbf{G}_0(p) = \begin{pmatrix} \frac{1}{\omega - \epsilon_p - \Sigma_R(p) + i\epsilon} & 0 \\ 0 & \frac{1}{\omega - \epsilon_p - \Sigma_A(p) - i\epsilon} \end{pmatrix}. \quad (166)$$

The solution for $\mathbf{G}$ is again obtained through Eq. (157). We see that the solution of the equation of motion is simplified by the diagonalization of the matrices. Therefore diagonalization can be used as a formal, straightforward method to derive such a solution. For this purpose it is applied in the above context.

References

- [1] K. G. R. Dole et. al., Phys. Rev. **C37** (1988) 163,
- [2] M. Baldo, J. Dukelsky, F. Gulminelli et. al., Preprint LPCC 95-15,
- [3] G. Röpke, L. Münchow and H. Schulz, Nucl. Phys. **A379** (1982) 536,
- [4] T. Alm, G. Röpke and M. Schmidt, Z. Phys. **A337** (1990) 355, Phys. Lett. **B302** (1993) 170,
- [5] T. Alm, G. Röpke et. al., MPG-VT-UR 65/95, hep-nucl-th/9511039,
- [6] P. Danielewicz and G. F. Bertsch, Nucl. Phys. **A533** (1995) 712,
- [7] M. Baldo, I. Bombaci, U. Lombardo, Phys. Lett. **B283** (1992) 8,
- [8] T. Alm, doctor thesis, Univ. Rostock (1993),
- [9] M. Baldo, U. Lombardo, I. Bombaci, P. Schuck, Phys. Rep. **242** (94) 159,
- [10] L. V. Keldysh and A. N. Kozlov, Sov. Phys. JETP **27** (1968) 521,
P. Nozieres and S. Schmitt-Rink, J. Low Temp. Phys. **59** (1985) 195,
- [11] P. Danielewicz, Ann. Phys. (N.Y.) **152** (1984) 239,
- [12] W. Botermans and R. Malfliet, Phys. Rep. **198** (1990) 115,

- [13] S. Mrowczynski and U. Heinz, Ann. Phys. (N.Y.) **229** (1994) 1,
- [14] A. L. Fetter and J. D. Walecka, Quantum Theory of Many-Particle Systems (McGraw-Hill, New York, 1971).
- [15] H. Kucharek and P. Ring, Z. Phys. **A339** (1991) 23.
- [16] L. P. Gorkov, Sov. Phys. JETP **7** (1958) 505.
- [17] R. Fauser and H. H. Wolter, submitted to Nucl. Phys. **A**, Preprint GSI-95-72 (1995).
- [18] J. Knoll and D. L. Voskresensky, Phys. Lett. **B351** (1995) 43; Preprint GSI-95-63 (1995)
- [19] E. Quack and P. A. Henning, Preprint GSI-95-42 (1995),
- [20] P. A. Henning, Phys. Rep. **253** (1995) 235.
- [21] P. A. Henning, Nucl. Phys. **A546** (1992) 653.
- [22] P. A. Henning, Nucl. Phys. **B337** (1990) 547, Phys. Lett. **A145** (1990) 329.
- [23] R. Fauser and H. H. Wolter, Nucl. Phys. **A584** (1995) 604.