Charm nonleptonic decays and final state interactions

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ABSTRACT

A global previous analysis of two-body nonleptonic decays of $D$ mesons has been extended to the decays involving light scalar mesons. The allowance for final state interaction also in nonresonant channels provides a fit of much improved quality and with less symmetry breaking in the axial charges. We give predictions for about 50 decay branching ratios yet to be measured. We also discuss long distance contributions to the difference $\Delta\Gamma$ between the $D_S$ and $D_L$ widths.

(*) partially supported by the European Community under the Human Capital and Mobility Programme, contract CHRX-CT93-0132.
A theoretical description of exclusive nonleptonic decays of charmed hadrons based on general principles is not yet possible. Even if the short distance effects due to hard gluon exchange can be resummed and the effective hamiltonian has been constructed at next-to-leading order \[1\], the evaluation of its matrix elements requires nonperturbative techniques. A classic analysis based on QCD sum rules has been presented in three papers by Blok and Shifman \[2\], but only the general trends were reproduced: the agreement with present data is poor at a quantitative level. Waiting for future progress in lattice QCD calculations one has to rely on approximate methods and on models.

We recently presented \[3\] one such model, based on the factorized approximation, with annihilation terms and rescattering effects due to resonances coupled to the final states, that has been rather successful in describing the bulk of the experimental data. Its main shortcoming was, in our opinion, the large flavour SU(3) breaking in the axial charges forced by the fitting of the data on decay rates to final states with one pseudoscalar and one vector meson (PV).

In this letter we modify the previous approach, inserting rescattering corrections also in nonresonant channels. Moreover, we include the decays to final states containing one of the lowest mass scalar mesons (S), \(f_0(980)\) and \(a_0(980)\), that are connected through rescattering effects to the previously considered PV final states. In this way, we are able to obtain a much better fit of the experimental data, while keeping the SU(3) breaking in the axial charges at a smaller and more acceptable level.

The scattering phase shifts in nonresonant channels were neglected in \[3\]. For decays to PP final states this is essentially correct, given that only one nonresonating phase, corresponding to the 27 representation, is involved and of course the final rates only depend on the phase shift differences. In the case of PV final states, on the other hand, many different SU(3) representations are present: to minimize the number of parameters, we only include a nonzero phase shift for the 27 (besides the resonant 8_F) and keep the others to zero. The 27 phase shift is most welcome, especially to obtain a better fit for \(D^+ \rightarrow PV\) Cabibbo-allowed decays. We admit two different values for the phase shift at the different energies, corresponding to the masses of \(D\) and of \(D_s\): the fitted values are \(\delta_{27}(m_D) = 47.4^\circ\) and \(\delta_{27}(m_{D_s}) = 59^\circ\), reasonably similar to each other, although maybe larger than expected.

The nature of the scalar resonances, \(f_0(980)\) and \(a_0(980)\), has been discussed for quite a long time. They do not look like the members of a normal nonet, in that the \(f_0\) is strongly coupled to \(K\bar{K}\), and could be for this reason identified with an \(s\bar{s}\) state, but it is degenerate in mass with the isovector \(a_0\). Moreover, the strange scalar states lie quite a bit higher. For these reasons, it has been suggested that the \(f_0\) and \(a_0\) are essentially \(K\bar{K}\)
molecules and are made therefore of two quarks and two antiquarks, \textit{i.e.} an \( s\bar{s} \) pair plus a light \( q\bar{q} \) pair \cite{4,5}. In charmed meson decays, both \( D^+_s \rightarrow f_0 \pi^+ \) and \( D^0 \rightarrow f_0 K_S \) have been observed experimentally \cite{6}, \cite{7}, \cite{8}. In the factorized approximation, one would have for the decay amplitudes prior to rescattering corrections:

\[
A_w(D^+_s \rightarrow f_0 \pi^+) = -\frac{G_F}{\sqrt{2}} U_{ud} U^*_{cs} (C_2 + \xi C_1) < f_0|\langle A^c_s \rangle_\mu|D^+_s > < \pi^+|\langle A^d_u \rangle_\mu|0 >,
\]

\[
A_w(D^0 \rightarrow f_0 K^0) = -\frac{G_F}{\sqrt{2}} U_{ud} U^*_{cs} (C_1 + \xi C_2) < f_0|\langle A^c_u \rangle_\mu|D^0 > < K^0|\langle A^d_s \rangle_\mu|0 >,
\]

(1)

In \cite{1} \( C_i \) are the Wilson coefficients in the effective hamiltonian, \( \xi \) is the color screening parameter (that should be equal to \( 1/N_c \) if the factorization approach were exact), the axial currents are denoted by \( \langle A^q \rangle_\mu \equiv \bar{q} \gamma^\mu \gamma_5 q \) and we neglected possible annihilation contributions. The observation of the decay \( D^+_s \rightarrow f_0 \pi^+ \) would imply the \( s\bar{s} \) nature for \( f_0 \), while \( D^0 \rightarrow f_0 K^0 \) points to a nonstrange composition.

Following the suggestion of \cite{5} we consider \( f_0 \) and \( a_0 \) as cryptoexotic two-quark plus two-antiquark states and attribute them to (incomplete) \( 8 \) and \( 1 \) SU(3) representations \( |a_0\rangle \in |8\rangle, \ |f_0\rangle \in \sqrt{1/3} |8\rangle + \sqrt{2/3} |1\rangle \). We then define

\[
\langle f_0|\partial^\mu|A^c_u \rangle_\mu|D^0 > = (M^2_D - M^2_{f_0})\frac{a_S}{\sqrt{2}(1 - q^2/M^2_D)}.
\]

(2)

The axial charge \( a_S \) is a parameter to be fitted. The result is \( a_S \simeq 0.39 \), smaller – as expected – than the corresponding axial charges for \( D \) transitions to vector mesons.

Our model also predicts charmed meson decays to states including the \( a_0(980) \) meson and Cabibbo suppressed decays to \( PS \), not yet observed. The amplitudes in factorized approximation are easily obtained, and the relevant form factors are all expressed in terms of the parameter \( a_S \): as an example,

\[
\langle a_0^0|\partial^\mu|A^d_u \rangle_\mu|D^+ > = - (M^2_D - M^2_{a_0})\frac{a_S}{\sqrt{2}(1 - q^2/M^2_D)}.
\]

(2')
We describe now in more detail the procedure followed to include final state interactions. Defining as $\mathcal{B}$ the decay amplitude including the phase space factor, i.e. $\mathcal{B}_w = A_w \sqrt{p/(8\pi m_D^2)}$ where $p$ is the momentum of the final particles in the $D$ rest frame, we have for $D \to PV (PS)$ decays

$$
\mathcal{B}(D \to V_h P_k) = \mathcal{B}_w(D \to V_h P_k) + c_{hk}[\exp(i\delta_8) - 1]A_T^8 + d_{hk}[\exp(i\delta_{27}) - 1]A_T^{27},
$$

$$
\mathcal{B}(D \to S_h P_k) = \mathcal{B}_w(D \to S_h P_k) + x_{PS} \tilde{c}_{hk}[\exp(i\delta_8) - 1]A_T^8 + y_{PS} \tilde{d}_{hk}[\exp(i\delta_{27}) - 1]A_T^{27},
$$

(3)

where

$$
A_T^8 = \sum_{h'k'} c_{h'k'} B_w(D \to V_{h'} P_{k'}) + x_{PS} \sum_{h''k''} \tilde{c}_{h''k''} B_w(D \to S_{h''} P_{k''}) \left( \frac{\sum_{h'k'} |c_{h'k'}|^2 + x_{PS}^2 \sum_{h''k''} |\tilde{c}_{h''k''}|^2}{\sum_{h'k'} |d_{h'k'}|^2 + y_{PS}^2 \sum_{h''k''} |\tilde{d}_{h''k''}|^2} \right),
$$

$$
A_T^{27} = \sum_{h'k'} d_{h'k'} B_w(D \to V_{h'} P_{k'}) + y_{PS} \sum_{h''k''} \tilde{d}_{h''k''} B_w(D \to S_{h''} P_{k''}) \left( \frac{\sum_{h'k'} |d_{h'k'}|^2 + y_{PS}^2 \sum_{h''k''} |\tilde{d}_{h''k''}|^2}{\sum_{h'k'} |c_{h'k'}|^2 + x_{PS}^2 \sum_{h''k''} |\tilde{c}_{h''k''}|^2} \right).
$$

(4)

In (3) and (4) $c_{hk}$ ($d_{hk}$) are the $PV$ couplings to $8_F$ ($27$), multiplied by a $(p/M_p)^{\frac{3}{2}}$ kinematical factor. This $p$ dependence must be present in the $\mathcal{B}$ amplitudes and, as in $\bar{8}$, we include it in the coefficients in order to automatically decouple the channels below threshold. The $PS$ couplings $\bar{8}$ to $8_D$ ($27$), multiplied by their kinematical factor (in this case $(p/M_p)^{\frac{3}{2}}$), are denoted $\tilde{c}_{hk}$ ($\tilde{d}_{hk}$).

We note that the phase shift $\delta_8$ is determined by the parameters of the resonance $\tilde{P}$ appropriate to the decay channel considered ($\tilde{P} = K(1830)$ or $\pi(1770)$), as follows

$$
\sin \delta_8 \exp(i\delta_8) = \frac{\Gamma(\tilde{P})}{2(M_{\tilde{P}} - M_D) - i\Gamma(\tilde{P})}.
$$

(5)

In the isoscalar case, $\delta_8^I = 0$ is a free parameter instead.

The parameters $x_{PS}$ and $y_{PS}$ are connected with the mixing between $PV$ and $PS$ channels. The representations $\bar{8}_F$ (for $PV$) and $\bar{8}_D$ (for $PS$) have the same parity and charge conjugation and may therefore naturally mix, $x_{PS} \neq 0$. The two $27$ representations have opposite charge conjugation; the zero hypercharge sectors cannot mix if isospin is a good symmetry, while the $Y=\pm 1$ terms may be mixed with opposite mixing angles $y_{PS}$ (this is an SU(3) violating effect: SU(3) symmetry requires equal mixing angles for any $Y$ value). We required in the fit $|y_{PS}| \leq |x_{PS}|$.

We have to face the problem of enforcing orthogonality between the resonant $\bar{8}$ and the non–resonant $27$ channels. These would be orthogonal in the SU(3) symmetric limit,

1 For the couplings of the singlet parts of $f_0$, $\eta$ and $\eta'$ we adopt nonet symmetry.
but they are not. For the $PS$ channels, the scalar multiplet is incomplete and therefore the orthogonality is badly broken. Even for the $PV$ channels the cancellations that would give orthogonality do not actually take place, since we included in the rescattering coefficients the kinematical factors. For $D^0$ Cabibbo allowed decays one has $\sum_{h''k''} c_{h''k''} d_{h''k''} \simeq 0.16$ and $\sum_{h'k'} c_{h'k'} d_{h'k'} \simeq 0.008$.

The difficulty may be nicely overcome taking advantage of the mixing between $PV$ and $PS$ final states. The orthogonality requirement

$$\sum_{h'k'} c_{h'k'} d_{h'k'} + x_{PS} y_{PS} \sum_{h''k''} \tilde{c}_{h''k''} \tilde{d}_{h''k''} = 0 \quad (6)$$

establishes a relation between $x_{PS}$ and $y_{PS}$, so that only one of them remains as a free parameter. The best fit values are $x_{PS} \simeq 0.25$ and

$$y_{PS} = \begin{cases} 
\mp 0.20, & \text{for } D \text{ Cabibbo allowed (doubly–forbidden) decays}; \\
0.00, & \text{for } D_s \text{ Cabibbo allowed and } D \text{ first–forbidden decays}; \\
+0.19, & \text{for } D_s \text{ first–forbidden decays}.
\end{cases}$$

We briefly recall now the aspects of $[3]$ that are not modified in the present approach. For the evaluation of the weak decay amplitudes $A_w$ we use the factorization approximation and a pole model for the form factors, as in eqs. (1), (2). The weak vector charges are assumed SU(3) symmetric: their value, 0.79, is taken from the experimental results for $D \rightarrow K e \nu$. For the axial charges we allow some SU(3) breaking, and let them vary in the range $0.8 \div 0.9$ independently. The decays to final states including $\eta$ or $\eta'$ mesons have been treated following the approach of D’yakonov and Eides $[9]$: the $\eta-\eta'$ mixing angle is therefore fixed to $-10^\circ$. For the decays to $PP$ and $PV$ channels we also consider the contribution from annihilation (or $W$–exchange) diagrams: the relevant matrix elements of the divergences of weak currents are given in terms of two parameters to be fitted, $W_{PP}$ and $W_{PV}$, with $[3]$

$$< K^- \pi^+ | \partial^\mu (V_s^d) | 0 > = i (m_s - m_d) \frac{M_D^2}{f_D} W_{PP},$$

$$< K^- \rho^+ | \partial^\mu (A_s^d) | 0 > = - (m_s + m_d) \frac{2 M_\rho}{f_D} e^* \cdot p_K W_{PV}. \quad (7)$$

The final state interactions for the $PP$ channels are dominated by the scalar resonances. Only one of them, the strange $K_0^*(1950)$, has been observed $[10]$ in the interesting mass region. In $[3]$ we assumed the existence of a nearby isovector resonance $a_0$ and we estimated its mass from the equispacing formula

$$M_{a_0}^2 = M_K^2 - M_K^2 + M_\pi^2. \quad (8)$$
In the fit we allowed the mass, width and branching ratio in the $K\pi$ channel of the $K^*_0$ resonance to vary within the experimental bounds. From their best fit values (1930 MeV, 300 MeV and 63.5%, respectively) we get $M_{a_0} = 1870$ MeV and $\Gamma_{a_0} = 299.4$ MeV.

In the nonstrange isoscalar case, only relevant for $D^0$ first–forbidden decays, the situation is complicated by the possibility of singlet–octet mixing of not yet established resonances. The number of parameters (mixing angles, masses and coupling constants) is a priori quite large. We imposed the decoupling of the higher mass resonance from the $\pi\pi$ channel, which together with the requirement of orthogonality reduces the number of new parameters to two: the mixing angle $\phi$ and the difference $\Delta^2 = m^2_{f'_0} - m^2_{f_0}$ of the mass squared (see [3] for details). Using the fitted parameters, the masses and widths of the two scalar isoscalar resonances are $(M_{f_0}, \Gamma_{f_0}) = (1789, 354)$ MeV and $(M_{f_0'}, \Gamma_{f_0'}) = (2127, 328)$ MeV.

We performed a least square fit with 15 parameters to the 49 data points or experimental bounds for the branching ratios. The results are presented in Tables 1 to 4, together with predictions for the channels not yet measured. The values of the eleven parameters already used in the previous fits are now: $\xi = 0.015$, $a_{cu} = a_{cd} = 0.9$, $a_{cs} = 0.8$, $W_{PP} = -0.269$, $W_{PV} = 0.270$, $M_{K^*_0} = 1930$ MeV, $\Gamma_{K^*_0} = 300$ MeV, $r = -0.86$, $\phi = 47.7^\circ$, $\Delta = 1149.4$ MeV and $\delta^I_{8=0} = 236.5^\circ$. In [3] the axial charges were $a_{cu} = a_{cd} = 1.0$ and $a_{cs} = 0.59$, while the other parameters are not changed much. We list again the four “new” parameter values: $\delta_{27}(m_D) = 47.4^\circ$, $\delta_{27}(m_{D_s}) = 59^\circ$, $a_S = 0.390$ and $x_{PS} = 0.249$. The values of decay constants, quark masses and resonance parameters not explicitly mentioned are identical to the values given in [3].

The total $\chi^2$ is 70.3 (of which 6.2 from two Cabibbo doubly–forbidden decays and two decays to $PS$ final states, not included in the previous fits). In ref. [3], $\chi^2$ was 90 for 45 data points and 11 parameters. A more detailed comparison of the two fits is shown in Table 5. We note that the most remarkable improvement occurs for the $D^+ \to PV$ decays: it is mainly due to rescattering in the exotic $I = \frac{3}{2}$ channel, that is the only rescattering effect present in the Cabibbo–allowed $D^+$ decay amplitudes. The worst single point in the fit of ref. [3], the branching ratio $B(D^+ \to \overline{K}^*\pi^+)$ (that was $B_{th} = 0.64 \%$ versus

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2 Actually, the parameter to be fitted is the ratio $r = g_{818}/g_{888}$, where $g_{818}$ is the SU(3) invariant coupling of the octet of scalar resonances to a singlet and an octet of pseudoscalar mesons and $g_{888}$ is the coupling to two pseudoscalar octets [3]. Nonet symmetry corresponds to $r = 1$. The branching ratio is a quadratic function of $r$.

3 Denoting by $|f_0\rangle$ the lower mass state, we define $|f_0\rangle = \sin \phi |f_8\rangle + \cos \phi |f_1\rangle$, $|f_0'\rangle = -\cos \phi |f_8\rangle + \sin \phi |f_1\rangle$. 

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$B_{\exp} = 2.2 \pm 0.4 \%$, is now fitted quite well, $B_{\th} = 2.47 \%$. This overcompensates the slightly worse fit for the decay $D^+ \to K_S \rho^0$: $B_{\th} = 5.60 \%$ now (5.28 \% in [3]) versus $B_{\exp} = 3.3 \pm 1.25 \%$. The greater freedom provided by the presence of the new parameters $\delta_{27}$ allows the reduction of the SU(3) breaking in the axial constants $a_{cu} = a_{cd}$ and $a_{cs}$, that we imposed not to differ by more than 0.1 in this work. It also allows an apparently minor change in the annihilation parameters and in the parameter $\xi$, which now happens to be small and positive: this has the effect of improving considerably the success of the fit also for the decay $D^+ \to K_S \rho$: $B_{\th} = 5.60 \%$ now (5.28 \% in [3]) versus $B_{\exp} = 3.3 \pm 1.25 \%$. The greater freedom provided by the presence of the new parameters $\delta_{27}$ allows the reduction of the SU(3) breaking in the axial constants $a_{cu} = a_{cd}$ and $a_{cs}$.

Concerning the decay rates of $D_s^+$ and $D^0$, the quality of the present fit is comparable to the fit in ref. [3]. In particular, for $D^0 \to K^* \eta$ and $D_s^+ \to \rho^+ \eta'$ the results are still unsatisfactory (more than three standard deviations lower than the data points). Neither annihilation contributions, nor final state interactions were present for channels with positive $G$-parity and $I = 1$, like $\rho^+ \eta'$, in [3]. In this fit the exotic rescattering affects these channels, giving for instance a nonzero branching ratio for the decay $D_s^+ \to \omega \pi^+$; however, it only slightly lowers (going in the wrong direction) the theoretical prediction for $D_s^+ \to \rho^+ \eta'$. It might be possible to attribute the discrepancy to an annihilation contribution, not taken into account here, through the glue components in $\eta'$ and $\eta$ [12].

Two out of four data points not included in the fit of [3] are very well fitted, but the predictions for the other two are not equally satisfactory. The amplitude for the decay $D^0 \to f_0 K_S$ is colour suppressed and is further decreased by the rescattering effects in our model: the theoretical value is therefore smaller than the experimental datum. The doubly–forbidden decay $D^+ \to K^+ \phi$ can only proceed through annihilation or rescattering: also in this case, the theoretical value is considerably lower than experiment. It should be noted, however, that recent data from E791 collaboration [13] do not observe a signal in this channel and establish an upper bound slightly less than the central value of E691 [14], reported in Table 1.

As to the predictions for not yet measured decay branching ratios, the largest among them refers to the Cabibbo first–forbidden decay $D^+ \to \overline{K}^0 K^+$. The decay amplitude is colour favoured in this case, and it has a small interfering annihilation contribution instead of the larger, although colour suppressed, contribution present in Cabibbo allowed $D^+$ decays. The same is true for the process $D^+ \to K^0 K^+$. The rescattering effects

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4 The large branching ratio for $D_s^+ \to \rho^+ \eta'$ is difficult to reproduce in many a model, see also [11].
decrease the decay rate for $K^0 K^+$ (which is in very good agreement with experiment) and increase instead the rate for $K^0 K^{*+}$. The next bigger prediction, for $\text{B}(D_s^+ \rightarrow K^0 \rho^+)$, deserves a similar comment: it is also increased ($\sim 20\%$) by rescattering effects. Among Cabibbo doubly–forbidden decays, we predict the largest branching fractions ($\sim 5 \times 10^{-4}$) for the decays $D^+ \rightarrow K^{++(\ast)}\pi^0$. A check for the assumption we made on the scalar particles will be the observation of decays with $a_0(980)$ production. The largest prediction for not yet observed PS decay channels is $\text{B}(D^+ \rightarrow a_0^+ K_S) = 0.32\%$.

We will not present here the predictions for CP violating decay asymmetries, that depend strongly on the rescattering phases: therefore, they remain similar to those previously published for the $PP$ final states, and differ appreciably in some cases for the $PV$ channels. The largest asymmetries ($\sim -3 \times 10^{-3}$) are now predicted in the decays $D^+ \rightarrow \rho^+\eta$ and $D^0 \rightarrow \omega\eta'$: they are entirely due to exotic rescattering, and were therefore zero in [3]. The branching ratios of these decays are however small, so that the best candidate should be given by the decays $D^+ \rightarrow \rho^0\pi^+$ and $D^- \rightarrow \rho^0\pi^-$, the predicted asymmetry being approximately $-2 \times 10^{-3}$.

A considerable interest has been recently devoted to the interplay of $D^0 - \bar{D}^0$ mixing and doubly Cabibbo forbidden amplitudes in the time evolution for $D^0$ decays [14]. Particular attention has been given to a term proportional to $\Delta M$ and providing linear correction to the exponential decay, present as a consequence of $CP$ violation and/or final state interactions, as a possible source of information on “new physics”. A term proportional to $\Delta \Gamma$ is also present. The short distance contributions predicted by the standard model are very small for both $\Delta M$ and $\Delta \Gamma$ [15]. It was suggested that the mixing may be dominated by long distance (hadronic) contributions [17] that could result in mixing parameters $x = \Delta M/\Gamma$ and $y = \Delta \Gamma/(2 \Gamma)$ as large as $10^{-2}$, although this was later criticized [18].

In our model, we can make an estimate of the long distance contribution to $\Delta \Gamma$ coming from the two–body states that we included in our fit. This quantity should vanish in the SU(3) limit, through an exact cancellation of the contribution of Cabibbo allowed and doubly–forbidden transitions with the contribution of once–forbidden decays [17]. In the presence of SU(3) breaking the cancellation is however not complete. As a consequence, our prediction for $\Delta \Gamma$ is subject to a large uncertainty; on the other hand, it is to be noted that the prediction is independent on the rescattering, provided that, as we impose, the sum of the branching ratios remains the same before and after rescattering corrections.

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5 We remark that all the asymmetries reported in Table V of ref. [3] in correspondence to $D^0$ decays have a wrong sign. The signs for the charged $D$ decay asymmetries are correct.
We have \((|\bar{D}^0⟩ = CP |D^0⟩)\)

\[
\Gamma_{12} = \sum_{|f>|} B^*(D^0 \to f) B(\bar{D}^0 \to f) \simeq (1.5 + i0.0014) 10^{-3} \Gamma_{D^0} \tag{9}
\]

In (9) the sum has been approximated including only the contributions of \(PP\) \((2/3)\), \(PV\) \((1/3)\) and \(PS\) \((\sim 0)\) final states. Note that the contribution to \(\Gamma_{12}/\Gamma_{D^0}\) coming from Cabibbo first–forbidden decays alone is \(35.2 \times 10^{-3}\), showing that the SU(3) cancellation is still rather effective. Although larger than the short–distance prediction, our estimate is much smaller than the present experimental bound \(|y| = |\Gamma_{12}| / \Gamma_{D^0} \leq 0.08\). The positive sign of the real part of \(\Gamma_{12}\) means (if taken seriously) that the shorter lifetime state, \(D^0_S\), decays dominantly into \(CP\)-even final states, similarly to the neutral \(K\) mesons.

M.L. acknowledges an interesting discussion with L. Maiani, S. Malvezzi and D. Menasce.


References


     L. Moroni (E687), contribution to LISHEP-95 International School.


[13] see M.V. Purohit in Proceedings of the XXVII International HEP Conference, Glas-
     gow, july 1994, p. 479.


     147.

     179.

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
f_i & B_{\text{exp}}(D^+ \to f_i) & B_{\text{th}} & f_i & B_{\text{exp}}(D^+ \to f_i) & B_{\text{th}} \\
\hline
K_S \pi^+ & 1.37 \pm 0.15 & 1.35 & \pi^+ \pi^0 & 0.25 \pm 0.07 & 0.19 \\
K_L \pi^+ & - & 1.70 & \pi^+ \eta & 0.75 \pm 0.25 & 0.34 \\
\overline{K}^0 \pi^+ & 2.2 \pm 0.4 & 2.47 & \pi^+ \eta' & < 0.9 & 0.73 \\
K_S \rho^+ & 3.30 \pm 1.25 & 5.60 & \overline{K}^0 K^+ & 0.78 \pm 0.17 & 0.81 \\
K_L \rho^+ & - & 6.30 & \rho^0 \pi^+ & < 0.14 & 0.13 \\
a_0^+ K_S & - & 0.32 & \rho^+ \pi^0 & - & 0.44 \\
a_0^+ K_L & - & 0.24 & \rho^+ \eta & < 1.2 & 0.013 \\
K^+ \pi^0 & - & 0.056 & \rho^+ \eta' & < 1.5 & 0.12 \\
K^+ \eta & - & 0.018 & \omega \pi^+ & < 0.7 & 0.019 \\
K^+ \eta' & - & 0.031 & \phi \pi^+ & 0.67 \pm 0.08 & 0.61 \\
K^{*0} \pi^+ & - & 0.019 & \overline{K}^0 K^{*+} & - & 1.71 \\
K^{*+} \pi^0 & - & 0.048 & \overline{K}^0 K^+ & 0.51 \pm 0.10 & 0.38 \\
K^{*+} \eta & - & 0.030 & f_0 \pi^+ & - & 0.028 \\
K^{*+} \eta' & - & 0.0002 & a_0^+ \pi^+ & - & 0.059 \\
K^+ \rho^0 & - & 0.030 & a_0^+ \pi^0 & - & 0.012 \\
K^+ \omega & - & 0.021 & a_0^+ \eta & - & 0.074 \\
K^+ \phi & 0.039 \pm 0.022 & 0.0051 & K^+ f_0 & - & 0.0023 \\
K^+ a_0^0 & - & - & K^+ a_0^0 & - & 0.0062 \\
\hline
\end{array}
\]

**TABLE 1**

Branching ratios for $D^+$ nonleptonic decays.

[Experimental data and 90% c.l. upper bounds from ref. [3]]
<table>
<thead>
<tr>
<th>$f_i$</th>
<th>$B_{\text{exp}}(D_s^+ \to f_i)$</th>
<th>$B_{\text{th}}$</th>
<th>$f_i$</th>
<th>$B_{\text{exp}}(D_s^+ \to f_i)$</th>
<th>$B_{\text{th}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_S K^+$</td>
<td>1.75 ± 0.35</td>
<td>2.37</td>
<td>$K^+ \pi^0$</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>$K_L K^+$</td>
<td>2.09</td>
<td>2.09</td>
<td>$K^+ \eta$</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>$\pi^+ \eta$</td>
<td>1.90 ± 0.40</td>
<td>1.23</td>
<td>$K^+ \eta'$</td>
<td>0.44</td>
<td></td>
</tr>
<tr>
<td>$\pi^+ \eta'$</td>
<td>4.7 ± 1.4</td>
<td>5.39</td>
<td>$K^0 \pi^+$</td>
<td>$&lt; 0.7$</td>
<td>0.40</td>
</tr>
<tr>
<td>$\rho^+ \eta$</td>
<td>10.0 ± 2.2</td>
<td>7.49</td>
<td>$K^{*+} \pi^0$</td>
<td>0.044</td>
<td></td>
</tr>
<tr>
<td>$\rho^+ \eta'$</td>
<td>12.0 ± 3.0</td>
<td>2.41</td>
<td>$K^+ \rho^0$</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>$K^{*0} K^+$</td>
<td>3.3 ± 0.5</td>
<td>3.96</td>
<td>$K^{*+} \eta$</td>
<td>0.18</td>
<td></td>
</tr>
<tr>
<td>$K_S K^{*+}$</td>
<td>2.1 ± 0.5</td>
<td>1.87</td>
<td>$K^{*+} \eta'$</td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td>$K_L K^{*+}$</td>
<td>2.13</td>
<td>2.13</td>
<td>$K^+ \omega$</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>$\phi \pi^+$</td>
<td>3.5 ± 0.4</td>
<td>4.08</td>
<td>$K^+ \phi$</td>
<td>$&lt; 0.25$</td>
<td>0.018</td>
</tr>
<tr>
<td>$\omega \pi^+$</td>
<td>&lt; 1.7</td>
<td>0.26</td>
<td>$K^{*0} \pi^+$</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>$\rho^0 \pi^+$</td>
<td>&lt; 0.28</td>
<td>0.24</td>
<td>$K^0 \rho^+$</td>
<td>1.39</td>
<td></td>
</tr>
<tr>
<td>$\rho^+ \pi^0$</td>
<td>0.24</td>
<td>0.24</td>
<td>$f_0 K^+$</td>
<td>0.069</td>
<td></td>
</tr>
<tr>
<td>$f_0 \pi^+$</td>
<td>1.0 ± 0.4</td>
<td>1.06</td>
<td>$a_0^+ K^0$</td>
<td>0.003</td>
<td></td>
</tr>
<tr>
<td>$a_0^+ \eta$</td>
<td>0.007</td>
<td>0.007</td>
<td>$a_0^0 K^+$</td>
<td>0.007</td>
<td></td>
</tr>
<tr>
<td>$a_0^+ \eta'$</td>
<td>0.002</td>
<td>0.002</td>
<td>$K^{*0} K^+$</td>
<td>0.008</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 2**

Branching ratios for $D_s^+$ nonleptonic decays.
[Experimental data and 90% c.l. upper bounds from ref. [3]]
<table>
<thead>
<tr>
<th>$f_i$</th>
<th>$B_{\exp}(D^0 \to f_i)$</th>
<th>$B_{\text{th}}$</th>
<th>$f_i$</th>
<th>$B_{\exp}(D^0 \to f_i)$</th>
<th>$B_{\text{th}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^- \pi^+$</td>
<td>4.01 ± 0.14</td>
<td>4.04</td>
<td>$\pi^0 \eta$</td>
<td>–</td>
<td>0.052</td>
</tr>
<tr>
<td>$K_S \pi^0$</td>
<td>1.02 ± 0.13</td>
<td>0.72</td>
<td>$\pi^0 \eta'$</td>
<td>–</td>
<td>0.16</td>
</tr>
<tr>
<td>$K_L \pi^0$</td>
<td>–</td>
<td>0.53</td>
<td>$\eta \eta$</td>
<td>–</td>
<td>0.088</td>
</tr>
<tr>
<td>$K_S \eta$</td>
<td>0.34 ± 0.06</td>
<td>0.42</td>
<td>$\eta \eta'$</td>
<td>–</td>
<td>0.18</td>
</tr>
<tr>
<td>$K_L \eta$</td>
<td>–</td>
<td>0.31</td>
<td>$\pi^0 \pi^0$</td>
<td>0.088 ± 0.023</td>
<td>0.110</td>
</tr>
<tr>
<td>$K_S \eta'$</td>
<td>0.83 ± 0.15</td>
<td>0.78</td>
<td>$\pi^+ \pi^-$</td>
<td>0.159 ± 0.012</td>
<td>0.159</td>
</tr>
<tr>
<td>$K_L \eta'$</td>
<td>–</td>
<td>0.61</td>
<td>$K^+ K^-$</td>
<td>0.454 ± 0.029</td>
<td>0.446</td>
</tr>
<tr>
<td>$\overline{K}^{*0} \pi^0$</td>
<td>3.0 ± 0.4</td>
<td>3.49</td>
<td>$K^0 \overline{K}^{0}$</td>
<td>0.11 ± 0.04</td>
<td>0.098</td>
</tr>
<tr>
<td>$K_S \rho^0$</td>
<td>0.55 ± 0.09</td>
<td>0.47</td>
<td>$\omega \pi^0$</td>
<td>–</td>
<td>0.014</td>
</tr>
<tr>
<td>$K_L \rho^0$</td>
<td>–</td>
<td>0.33</td>
<td>$\rho^0 \eta$</td>
<td>–</td>
<td>0.020</td>
</tr>
<tr>
<td>$\overline{K}^{*0} \pi^0$</td>
<td>4.9 ± 0.6</td>
<td>4.85</td>
<td>$\rho^0 \eta'$</td>
<td>–</td>
<td>0.008</td>
</tr>
<tr>
<td>$K^- \rho^+$</td>
<td>10.4 ± 1.3</td>
<td>11.02</td>
<td>$\omega \eta$</td>
<td>–</td>
<td>0.20</td>
</tr>
<tr>
<td>$\overline{K}^{*0} \eta$</td>
<td>1.9 ± 0.5</td>
<td>0.37</td>
<td>$\omega \eta'$</td>
<td>–</td>
<td>0.0001</td>
</tr>
<tr>
<td>$\overline{K}^{*0} \eta'$</td>
<td>&lt; 0.11</td>
<td>0.004</td>
<td>$\phi \pi^0$</td>
<td>–</td>
<td>0.11</td>
</tr>
<tr>
<td>$K_S \omega$</td>
<td>1.0 ± 0.2</td>
<td>0.88</td>
<td>$\phi \eta$</td>
<td>–</td>
<td>0.090</td>
</tr>
<tr>
<td>$K_L \omega$</td>
<td>–</td>
<td>0.80</td>
<td>$K^{*0} \overline{K}^{0}$</td>
<td>&lt; 0.08</td>
<td>0.064</td>
</tr>
<tr>
<td>$K_S \phi$</td>
<td>0.415 ± 0.060</td>
<td>0.40</td>
<td>$\overline{K}^{*0} \overline{K}^{0}$</td>
<td>&lt; 0.15</td>
<td>0.062</td>
</tr>
<tr>
<td>$K_L \phi$</td>
<td>–</td>
<td>0.42</td>
<td>$K^{*+} K^-$</td>
<td>0.34 ± 0.08</td>
<td>0.43</td>
</tr>
<tr>
<td>$f_0 \ K_S$</td>
<td>0.23 ± 0.10</td>
<td>0.037</td>
<td>$K^{*0} K^+$</td>
<td>0.18 ± 0.10</td>
<td>0.30</td>
</tr>
<tr>
<td>$f_0 \ K_L$</td>
<td>–</td>
<td>0.031</td>
<td>$\rho^+ \pi^-$</td>
<td>–</td>
<td>0.69</td>
</tr>
<tr>
<td>$a_0^0 \ K_S$</td>
<td>–</td>
<td>0.109</td>
<td>$\rho^- \pi^+$</td>
<td>–</td>
<td>0.57</td>
</tr>
<tr>
<td>$a_0^0 \ K_L$</td>
<td>–</td>
<td>0.083</td>
<td>$\rho^0 \pi^0$</td>
<td>–</td>
<td>0.12</td>
</tr>
<tr>
<td>$a_0^+ \ K^-$</td>
<td>–</td>
<td>0.078</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 3**

Branching ratios for $D^0$ Cabibbo allowed and first–forbidden decays.

[Experimental data and 90% c.l. upper bounds from ref. [3]]
\( f_0 \pi^0 \quad \) -- \quad 0.0006 \quad \( K^+\pi^- \quad 0.031 \pm 0.014 \quad 0.033 \)

\( f_0 \eta \quad \) -- \quad 0.004 \quad \( K^{*0}\pi^0 \quad \) -- \quad 0.0039

\( a_0^0 \pi^0 \quad \) -- \quad 0.011 \quad \( K^{*+}\pi^- \quad \) -- \quad 0.035

\( a_0^0 \eta \quad \) -- \quad 0.015 \quad \( K^+\rho^- \quad \) -- \quad 0.025

\( a_0^\pi^- \quad \) -- \quad 0.003 \quad \( K^{*0}\eta \quad \) -- \quad 0.009

\( a_0^- \pi^+ \quad \) -- \quad 0.070 \quad \( K^{*0}\eta' \quad \) -- \quad \( \sim 10^{-5} \)

\( a_0^- K^+ \quad \) -- \quad 0.004

**TABLE 4**

Branching ratios for \( D^0 \) Cabibbo first– and doubly–forbidden decays.

[Experimental data from ref. [6]]

<table>
<thead>
<tr>
<th>Branching Ratios</th>
<th># data</th>
<th>( \chi^2 ) (ref. [3])</th>
<th>( \chi^2 ) (This work)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D^+ \rightarrow PP )</td>
<td>5</td>
<td>9.56</td>
<td>5.34</td>
</tr>
<tr>
<td>( D^+ \rightarrow PV )</td>
<td>8</td>
<td>29.55</td>
<td>8.46</td>
</tr>
<tr>
<td>( D^+_s \rightarrow PP )</td>
<td>4</td>
<td>8.79</td>
<td>7.10</td>
</tr>
<tr>
<td>( D^+_s \rightarrow PV )</td>
<td>8</td>
<td>15.35</td>
<td>17.64</td>
</tr>
<tr>
<td>( D^0 \rightarrow PP )</td>
<td>8</td>
<td>8.44</td>
<td>8.43</td>
</tr>
<tr>
<td>( D^0 \rightarrow PV )</td>
<td>12</td>
<td>18.35</td>
<td>17.17</td>
</tr>
</tbody>
</table>

**TABLE 5**

Comparison of our results with the fit of ref. [3].

Only Cabibbo–allowed and first–forbidden decays are included.