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Hamiltonian Formulation of FRW Equations of Cosmology

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Abstract

We give a general Hamiltonian formulation of the FRW fundamental equations of an homogeneous and isotropic Universe in the Standard Cosmology Model. It gives some interesting insight on the 3-curvature of the Universe and leads to an easy determination of a Schrödinger equation for the $\psi(R)$ wave function of the Universe, in the radiative era.

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In a recent publication [1] M. Novello and his collaborators at C.B.P.F. have shown that a particular choice of variables could lead to a Hamiltonian formulation of the fundamental equation of the Standard Cosmology in the Friedman Robertson Walker (FRW) comoving coordinates for the radiation era of the cosmic fluid. Such a model deals with an homogeneous isotropic geometry characterized by a single $R(t)$ function, the scale factor (also called "radius of the Universe") and the parameter ϵ which defines the 3-curvature constant. When one introduces the energy-density ρ and the pressure P of the cosmic fluid considered as a perfect gas obeying the state equation for the radiation pressure

$$P = \frac{1}{3} \rho + (\gamma - 1) P \quad (1)$$

where

$$\begin{aligned} \gamma &= \frac{4}{3} \text{ for the radiation era} \\ \gamma &= 1 \text{ for the matter era} \\ \gamma &= 2 \text{ for the stiff matter} \\ \gamma &= 0 \text{ for the vacuum} \end{aligned} \quad (2)$$

and the expansion parameter θ given by :

$$\theta = 3 \frac{\dot{R}}{R} = \frac{d}{dt} (\log R^3) \quad (3)$$

the fundamental equations of Standard Cosmology are [2] :

i) the conservation law for the stress tensor of the cosmic fluid

$$\dot{\rho} + \gamma \rho \theta = 0 \quad (4)$$

ii) the Raychaudhuri equation of a perfect fluid :

$$\dot{\theta} + \frac{1}{3} \theta^2 + \frac{1}{2} (3\gamma - 2) \rho = 0 \quad (5)$$

iii) the Friedmann equations of the spacial curvature :

$$\frac{\theta^2}{3} = \rho + \frac{3\epsilon}{R^2} \quad (6)$$

where natural units ($\hbar = c = G = 1$) have been introduced for simplicity sake.

Let us introduce the $\mathcal{H}$ hamiltonian discribing the cosmic fluid during its evolution after the Big-bang in the following form

$$\mathcal{H} = \frac{1}{2} p^2 + A q^n \quad (7)$$

where A and n are constants to be determined according to the use of the following conjugate canonical variables

$$\begin{cases} p = a \theta^\alpha \rho^\beta \\ q = b \rho^\sigma \end{cases} \quad (8)$$

The constants a, b, α, β and σ are defined by comparison with the fundamental equations (4), (5) and (6) .

The first Hamilton equation :

$$\dot{q} = \frac{\partial \mathcal{H}}{\partial p} = p \quad (9)$$

gives with the above definition of the (q, p) conjugate variables :

$$\dot{\rho} = \frac{a}{b\sigma} \theta^\alpha \rho^{\beta-\sigma+1} \quad (10)$$

A comparison with the conservation law (3) determines easely :

$$\alpha = 1 \quad \beta = \sigma \quad \frac{a}{b\sigma} = -\gamma \quad (11)$$

The second Hamilton equation:

$$\dot{p} = -\frac{\partial \mathcal{H}}{\partial q} = -n A q^{n-1} \quad (12)$$