A study of relativistic charged particle identification by primary cluster counting

G. Malamud*, A. Breskin and R. Chechik

Departement of Particle Physics
The Weizmann Institute of Science
76100, Rehovot, ISRAEL

ABSTRACT

The possibility to identify charged particles in the relativistic rise region by primary cluster counting in low-pressure gaseous detectors is discussed. Based on experimental parameters measured with a single detector module and with minimum ionizing electrons, the response of a full detection system to relativistic particles was simulated. Particle separation with a realistic device, comprising 20 consecutive detector modules, was evaluated in terms of total system length and momentum range. The estimated performance is compared to that of some existing dE/dx detectors.

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*Corresponding author. Current address: CERN/PPE, CH-1211 Geneve 23, Switzerland;
Fax: 41-22-7829415; E-mail: gmalamud@cernvm.cern.ch
1 Introduction

The most commonly used method to identify charged particles with known momentum, in the relativistic rise region ($4 \leq \beta \gamma \leq 500$), is to measure the velocity-dependent total ionization induced by the particle in gaseous media [1] (usually referred to as the dE/dx technique). In this article we analyse in detail the possibility to identify particles by counting the number of primary ionizing collisions induced along the track of the particle in a low density gas medium [2].

The total number of ionization electrons induced by a particle, $N_e$, and the number of primary ionizing collisions, $N_p$, are related to each other via

$$N_e = \sum_{i=1}^{N_p} N_{pi}$$

(1)

$N_{pi}$ being the number of electrons induced in the $i^{th}$ primary collision [1, 3]. The average number of primary collisions, $\langle N_p \rangle$, and the average total number of ionization electrons, $\langle N_e \rangle$, have both a similar dependence on the particle charge and velocity [3, 4]. However, the fluctuations in $N_p$ are poissonian in nature, with a standard deviation $\sigma(N_p) = \sqrt{\langle N_p \rangle}$, while the fluctuations in $N_e$ are characterised by the much broader "Landau distribution" [5] due to the production of energetic $\delta$ electrons. This implies that, in principle, a better particle resolution per detector length could be achieved by counting the $N_p$ primary ionizing collisions rather than by collecting and measuring the integral number of electrons $N_e$. Only a few previous attempts have been made to identify particles via the Primary Cluster Counting (PCC) method. Different techniques were tested: streamer chambers [6, 7], DC operated spark chambers [8] and a Time Expansion Chamber (TEC) [9]. Yet, the primary cluster counting efficiency was too low. The main reasons were the high cluster density at atmospheric or higher gas pressures and a significant loss of primary clusters due to a low electron detection efficiency. A more detailed discussion on these methods and their related problems can be found elsewhere [10, 11, 12]. A comprehensive approach to the PCC method was made by F. Lapique and F. Piuz [13].

The deposition of the ionization clusters in a low-pressure gas volume could solve
some of the main drawbacks of the PCC technique. The main advantages of this detection method are:

- An increased relativistic rise at lower gas pressures.
- High gaseous gain ($>10^6$).
- Good separation between primary clusters due to their intrinsic low-density deposition in the gas.
- A low probability of producing $\delta$-electrons per detector length.
- Low sensitivity to background ionizing radiation.
- Imaging capabilities.
- Low detector ageing.

A schematic of a cluster counting detector module is shown in fig.1 and is described in detail in refs. [11, 12]. The operation is performed as follows: a track of primary clusters is formed along the path of an incident particle. The clusters drift towards an amplification region. The electric drift field in the conversion region is low so that the arrival rate of individual clusters is "time-expanded" in order to have good separation between primary clusters. The number of primary clusters is obtained by counting the cluster-induced avalanches using fast discriminators or applying peak-finding analysis to the digitised analogue pulse trails. The electron amplification element is a Multistep Avalanche Chamber (MSAC) coupled to the conversion volume. In some cases a Single Wire Proportional Chamber (SWPC) configuration was used, providing shorter signals. This allows to drift the electrons under higher fields, namely at higher drift velocities.

In this article we report on a Monte-Carlo simulation study of the possibility to implement the PCC method to particle identification in the relativistic rise region. The study is based on our experimental data, with a single detector module and minimum ionizing $\beta$ electrons [10, 11, 12] and on the simulation of the detector response. On the basis of the
agreement between experimental and simulated results at various operating conditions, the evaluation of several parameters, like gas choice, gas pressure, drift velocity of electrons in the conversion volume, conversion length and pulse-width, was made [11]. Further calculations such as the relativistic rise slope and saturation due to the density effect at low gas pressures were made according to existing models. The particle identification performance of a realistic detection system combining 20 consecutive modules could thus be predicted. The mass resolution of the PCC detector as function of the momentum was investigated and compared to that of existing dE/dx sampling detectors.

2 Computer simulation procedure

Our simulation of particle identification by the PCC method was based, as much as possible, on our own experimental set-up and results. This includes parameters such as ionization densities, electron losses due to attachment and recombination, single electron counting efficiencies, electron drift velocities and electron amplification distributions. References are given where data from other sources were used. Care was taken to use in simulated results same data analysis protocols as were applied in the measurements with minimum ionizing electrons. Effects, like occasional discharges, detector occupancy and double-track resolution, which may cause a deterioration of the detector performance, were not taken into account in the simulation here. They are discussed in sections 4 and 5.

2.1 Generation of a single particle track

An example of a cluster-track produced by a minimum ionizing electron, at a pressure of 40 Torr and a drift velocity of 1.7 cm/μs, and a corresponding simulated event are shown in fig.2 [11]. Notice the difference between the number of primary collisions and the total number of counted collisions (NOC). The main discrepancies between N_p and NOC are due to cluster-pulse overlap and multiple-electron cluster dissociation by diffusion as is
discussed in more details in sections 3 and 5.

The simulation procedure is discussed below:

- The number of collisions ($N_p$, primary clusters) and their loci along the particle track is generated; the input parameters are the track length, $L$, and the specific ionization, $n_p$. These two parameters completely determine the Poisson distribution of $N_p$. The “track” is represented by a one-dimensional array, every element corresponding to one sampled “point” in a digitising oscilloscope of the cluster-track analogue output [11, 12]. The sampling time interval was $R_t = 5\, ns$, equivalent to a track sub-interval length of $\Delta X_1 = w \cdot R_t$ where $w$ is the drift velocity of electrons in the conversion volume.

- The number of secondary electrons produced in each primary collision is generated; we have used data calculated for Argon by Lapique & Piuza[13]. An experimental determination of the cluster size distribution was recently carried out by Fischle et.al [14] for several gases (He, CO$_2$, Ar, CH$_4$, C$_2$H$_6$, C$_3$H$_8$ and C$_4$H$_{10}$). Table 1 shows the distribution of cluster size (number of electrons) for minimum ionizing and ultrarelativistic particles crossing 10 mm of Argon at NTP. These measured results are significantly different from the calculated ones. However, the calculated cluster size distribution for Argon [13] and the measured distributions for hydrocarbons [14] (not shown here) were very similar and only negligible differences were measured between different hydrocarbons. Furthermore, the effect of small changes in the cluster size distribution was found to be of very little influence on the number of counted clusters.

- The electrons are individually drifted towards the amplification region; the relative arrival time of each electron is calculated, taking into account the drift velocity and the longitudinal diffusion characterising the drift process. Data for diffusion parameters was obtained from ref.[15].

- The relative amplification factor of each electron is generated; in the case of the MSAC multiplier the Alkhazov formula [16] was used. For the SWPC the Polya
distribution [17] was used. Input parameters were chosen so as to fit experimental results obtained with UV-induced single-electron pulses with the same mean amplification factor. A detailed formulation is given in appendix A.

- The individual electron pulses, with shapes identical to the experimental ones are added; the final result is an array which represents the digitised ionization electron track induced by one relativistic particle.

- An off-line correlation technique [11, 18] for identifying and counting the primary clusters is applied to each track. To find the cluster peaks a correlation spectrum, $C(\tau)$, is calculated:

$$C(\tau) = \sum_{i=1}^{N_T} S(t) \cdot [Y(\tau + t) - A_\tau]$$

(2)

where $S(t)$ is a search-spectrum array similar in shape to a typical pulse created by a single cluster. $N_T$ is the number of sampling points of the search spectrum and $A_\tau$ is the average of the data spectrum, $Y(i)$, calculated over the summation points:

$$A_\tau = \left( \frac{1}{N_T} \right) \cdot \sum_{i=\tau}^{\tau+N_T-1} Y(i)$$

(3)

Peaks above an appropriate threshold in the correlated spectrum $C(\tau)$ are detected in a differential manner and counted. The search spectrum shape and the threshold level for pulse recognition in the correlated spectrum are identical to those used for experimental data analysis.

2.2 Study of the operational characteristics of a single PCC detector module

A statistical analysis of parameters which characterise the operation of a single PCC detector module is performed. The analysis includes the statistical distributions (mean and fluctuations) of the number of counted clusters (NOC) over the full track length and over selected time-slices along the particle track. The impacts of several physical parameters which determine the detector cluster counting performance are studied. Parameters of
interest are the electron drift velocity and diffusion in the conversion region, pulse shape, the density of primary ionization, amplification factor and statistics, signal-to-noise ratio etc. The discrepancy between the distribution of NOC and the (intrinsic) Poisson statistics of $N_p$ is evaluated using the following definition for the $\chi^2$ distribution:

$$\chi^2 \equiv \frac{S^2(\text{NOC})}{\langle \text{NOC} \rangle}$$

(4)

where $S^2$ is the estimated variance of the NOC distribution. In the case of a perfect Poisson distribution the value of $\chi^2$ is 1, while for distributions narrower or broader than expected for a Poisson with the same mean, the value of $\chi^2$ is smaller or greater than 1 correspondingly. An illustration of these distributions can be found later in sect.3.1.

In order to establish the validity and interpretation of our simulation a detailed study which cross-checked experimental and simulated results under a wide range of conditions was carried out. A complete description of this study can be found in refs. [11, 12, 19]. We shortly summarise that a very good agreement under all tested situations was found between simulated and experimental results; it gave us a high confidence in our method of evaluating the particle resolution of a PCC detector system in the relativistic rise region.

### 2.3 Simulation of particle identification in the relativistic rise region

It is shown that the fluctuations in NOC vary with ionization density, so that two particles with different velocities may have quite a different NOC distribution width, depending on the difference between their expected primary ionizations. We have therefore defined the resolution $R$ between two particles, $i$ and $j$, as the difference between the corresponding mean values of counted clusters, divided by the mean standard deviation:

$$R \equiv \frac{\Delta_{ij}\langle \text{NOC} \rangle}{\sigma_{ij}}$$

(5)

where:

$$\Delta_{ij}\langle \text{NOC} \rangle = |\langle \text{NOC} \rangle_i - \langle \text{NOC} \rangle_j|$$

$$\sigma_{ij} = [\sigma_i(\text{NOC}) + \sigma_j(\text{NOC})]/2$$
The relativistic rise of the specific primary ionization was calculated using Bethe's theory [3]. Following measurements and formulation used by Rieke et al., [20] the cross section $\sigma_{np}$ for a fast electron to excite an atomic or a molecular bound electron into the vacuum level (a primary ionizing collision) is given by:

$$\sigma_{np} = \frac{\hbar^2}{\pi (m_c \kappa)^2 \beta^2} \times \{ M^2 \cdot (\ln[\beta^2/(1 - \beta^2)] - \beta^2) + C \}. \quad (6)$$

$M^2$ and $C$ being empirical constants fitted to experimental data.

The saturation in the relativistic rise due to the density effect at high particle velocities was treated as follows: an end-point which defines the highest $\beta \gamma$ for which the PCC detector is still efficient was calculated using Landau's [5] and Sternheimer's formulae [21, 22] for the most probable energy loss, $\Delta E_{mp}$. In his calculations Sternheimer defined two approximated limiting values for the particle velocity, as illustrated in fig.3: a low one, $X_0$, and a higher one, $X_1$ (a third intermediate value, $X_2$, is of no interest for the discussion). $X_0$ defines the beginning of the density effect: particles with lower velocity would not be affected. $X_1$ defines where the density effect stops any further relativistic rise and a full plateau is reached. Sternheimer's formulae is given in appen.B.

As there is no similar analytic formulation which fits our interest in the density effect on the formation of primary clusters, $\delta dN_p/dx(X)$, the following approximation have been used:

$$\delta dN_p/dx(X) = 0 \quad \text{for} \quad X \leq X_0$$

$$\frac{d(dN_p/dx)}{dX} = 0 \quad \text{for} \quad X > X_0 \quad (7)$$

where $X$ and $X_0$ are the parameters calculated by Sternheimer for the saturation in the relativistic rise of the energy loss. The meaning of using eq.7 is that we assume that the PCC method is fully effective up to the lower velocity limit, $X_0$, as there is no apparent density effect in this region. and that above this velocity the primary ionizing density is not increasing any more, i.e. a full plateau has been reached.

The justification for these approximations is as follows: a) most of the primary collisions are "distant collisions", with a relativistic rise similar to that of $dE/dx$; b) the Fermi plateau is reached, for this type of collisions, at a velocity only slightly higher than that
corresponding to \( X_0 \) [13, 23].

It should be stressed that using this approach we fix the usable PCC momentum range at a restricting low value. The useful range might be slightly larger than that.

3 Results

Our present study is based on the operational characteristics of a PCC detector module (shown in fig.1) filled with 40 Torr of pure isobutane, an effective conversion length of 13.6 cm, and a drift velocity of electrons of 1.7 \( cm/\mu s \) [11]. The electron amplifier is a MSAC with the parallel grids in each amplification element 3.2 mm apart. A 14 mm transfer region separates the two amplification stages. This configuration was found to be a reasonable compromise between several factors like detector gain and stability, cluster diffusion and dissociation, drift velocity and consequently time occupancy per event, primary ionization density and others.

3.1 Simulated operation of a single detector module in the relativistic rise region

In figs. 4a-b we present the results for the number of deposited and counted clusters for Pions with a momentum of 55.8 GeV/c (\( \beta \gamma = 400 \)). The simulation is made for a detector equipped with a MSAC electron multiplier, under conditions similar to the ones used for minimum ionizing \( \beta \) electrons (see figure caption). The only difference is in the increased specific primary ionization: \( n_p = 0.204 \) clusters/cm·Torr, compared to the value \( n_p = 0.12 \) for minimum ionizing electrons [19] (at \( \beta \gamma = 400 \) \( n_p/n_{p0} \approx 1.70 \)). In figure 4-a the statistical distributions of the number of collisions (left) and of the number of counted clusters (right) are shown. The collision distribution \( (N_p) \) is Poisson \( (\chi^2 = 1) \), while for NOC it is narrower \( (\chi^2 = 0.84) \). Notice the high inefficiency for cluster counting, of 32% \( (\langle NOC \rangle/N_p = 0.68) \), mainly due to the high ionization density. This is to be compared with the lower inefficiency \( (18\%) \), simulated and measured for a lower (minimum) ionization density [11]. The counting inefficiency is well reflected in fig.4-b, where the number of collisions and the number of counted clusters within 0.5 \( \mu s \) time
slices vs. drift time are presented. Both the narrowing of the NOC distribution and the inefficiency in cluster counting are due to the overlapping between pulses of neighbouring clusters [11],[12, chapter 5.1].

fig. 5 shows the simulated relative deposited and counted number of clusters (\( \langle N_p \rangle \) and \( \langle \text{NOC} \rangle \) respectively) as a function of the particle velocity (expressed in \( \beta \gamma \) units). The end-point in the relativistic rise occurs at \( \beta \gamma \approx 430 \). All simulated experimental conditions are the same as in fig.4. It is noticed that the inefficiency in the number of counted clusters exists over the full \( \beta \gamma \) range. The difference between the number of collisions and the number of counted clusters increases with \( \beta \gamma \) along the relativistic rise. The reason is the increasing ionization densities. This effect tends to reduce particle resolution, but on the other hand is accompanied by a narrowing of the statistical distribution of NOC, which partially compensates for this effect.

3.2 Particle resolution of a multi-module detector system

Particle resolution was simulated for a hypothetical PCC detector system with a total effective conversion length of 2.72 m. It consists of 20 consecutive modules mounted in series. The total NOC of a single event is calculated by summing up over the 20 independent modules.

The first simulation was performed assuming a MSAC multiplication structure identical to that described in section 3.1. It was run for three values of particle momenta: 3.4, 9.9 and 55.8 GeV/c. The experimental conditions are similar to those described in fig.4. In table 2 we present the results of the simulation for a drift velocity of 1.7 cm/\( \mu \)s. One can remark that the statistical relative width (\( \sigma(\text{NOC})/(\text{NOC}) \)), describing the detector energy resolution, decreases with increasing particle momentum, which may imply an improved particle resolution. However, looking back at figs.4,5 one realises that this improvement is accompanied by a lower cluster counting efficiency at higher particle momenta. These two competing effects are taken into account in the definition of particle resolution \( R \) (eq.5).

Based on our experience with detectors having a SWPC electron multiplier configu-

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ration [24, 25] we also simulated particle identification of a similar, 20 module detector, having such amplification elements. The pulse width is 11 ns FWHM and the drift velocity in the conversion region is 3.8 cm/\mu s. A similar pulse width is expected for a multiplier incorporating series of single proportional wires. The longitudinal diffusion, \sigma_l, is 0.056 cm^{-1/2}. Results for this configuration are presented in table 3. Here again the width of the NOC distribution (\sigma_{\text{NOC}}/(\text{NOC})) is narrower at higher momenta. For example, the widths of the NOC distributions for minimum ionizing particles (\beta\gamma \approx 3.5) and for particles with \beta\gamma=400 are 2.8\% and 1.8\% respectively. However, as will be shown below (Table 4), this does not imply a better particle separation at higher velocities.

The distributions of the number of generated collisions and the number of counted collisions for protons, kaons and pions at the two different drift velocities and pulse widths (see caption) are shown in fig.6. The particle resolution appears to be better in the case of shorter pulses (11 vs. 40 ns FWHM) and higher drift velocity (3.8 vs. 1.7 cm/\mu s), being, for example, 3.6 and 3.2 respectively for \( K - \pi \) separation. These distributions are particular cases in tables 4(a,b), where we present calculated particle resolution as function of the momentum and of the drift velocity in the conversion volume, for the two types of electron multipliers. The calculations are based on tables 2,3 and on Eq. 5. Considering for example \( p - \pi \) resolution with the MSAC multiplier, it is found that at 3.4 GeV/c the best result (R = 4.9) is obtained at the highest drift velocity (1.8 cm/\mu s). However, at higher particle velocities the best resolutions would be reached at other drift velocities: 1.7 cm/\mu s at 9.9 GeV/c, with R = 5.8 and 1.4 cm/\mu s for 55.8 GeV/c, with R = 5.5. The reason for these variations is that at higher specific ionization values a lower drift velocity is needed to separate the denser primary clusters. On the other hand it is apparent that at low ionization densities (lower momenta) the resolution may deteriorate at lower drift velocities, a result of the increasing cluster dissociation by diffusion. Further discussion on that phenomena can be found in refs.[11, 12].

An important outcoming conclusion of these examples is that one can optimise the drift velocity in the conversion region so as to get the best particle separation for a given energy range. This is a trade-off between the higher cluster dissociation by diffusion at
lower drift velocities and the increasing inefficiency for cluster separation and counting at higher drift velocities.

4 Comparison with some dE/dx sampling detectors

A complete comparison between the PCC detector and other existing dE/dx sampling detectors should take into account many parameters, like the radiation flux and luminosity, the energy range in which particles should be resolved etc. Also more technical aspects such as the mechanical and electronics complexity, price and the required computer analysis-time and storage capacity, should be considered. Such a detailed comparison is not yet possible, as our PCC detector was a preliminary laboratory prototype, and was not designed and optimised for large scale operation.

However, it is instructive and desirable to compare some basic parameters, mainly the particle separation and momentum range: Table 5 shows some dE/dx sampling detector measured resolutions [26]. The results are for 15 GeV/c particles measured by 64 samples, 4 cm long each; in fig. 7 the expected particle separation in Ne and in Ar, at 2 Atm, is shown [27]. The results are the truncated means of 128×2 cm samples. Other examples of the performance of dE/dx sampling detectors can be found elsewhere (see for example refs. [1, 28, 29, 30, 31]. From the comparison of table 5 and fig.7 with our simulated results the following conclusions can be made (all the comparisons assume similar active detector lengths):

1. The relativistic rise amplitude in the number of collisions is higher for the low-pressure PCC detector ($N_p/N_{p_{min}} \approx 1.7$ for 40 Torr of $i-C_4H_{10}$) than for most dE/dx detectors (only Xenon operated dE/dx detectors reach this value [1]). It should be stressed that other gases, with a higher specific primary ionization (for example: triethylamine [19]) may allow the operation at pressures lower than 40 Torr with an even higher relativistic rise.

2. The relativistic rise in the number of counted clusters ($\langle NOC \rangle$) is of the same order as for dE/dx detectors ($\approx 1.3 - 1.5$).
3. The particle separation of both detector types is comparable, of the order 4-6 for $\pi/p$ at 10-15 GeV/c.

5 Summary and Discussion

In this work we studied the possibility to resolve relativistic charged particles by counting the number of deposited primary ionization clusters rather than measuring the total ionization charge ($dE/dx$). The study was based on our own experimental results and on a computer simulation.

Several factors should be considered for the design of a PCC detector for relativistic particle identification:

- **Gas pressure**: A higher gas pressure (higher cluster density) should lead to a shorter detector length for a given resolution. On the other hand, the demand for an enhanced relativistic rise favours the operation at the lowest possible gas pressures [13, 21].

- **Drift velocity**: The finite width of single electron pulses sets limits on the cluster counting efficiency. To overcome this problem low drift velocities (1-4 cm/$\mu$s) should be maintained in the conversion region. The value of the drift velocity is subject to fine optimisation since a low drift field also results in large diffusion and higher probability for electron losses. Operation at a drift velocity where the over counting of primary clusters due to diffusion is somewhat compensated by the under counting caused by cluster overlapping could be suggested [11].

- **Diffusion**: Multi-electron primary clusters are dissociated by diffusion along their drift path. Therefore, a gas with low diffusion coefficient should be chosen. It will also narrow the single-electron pulse width and improve pulse separation. A good candidate could be DME (dimethylether).

- **Gain**: A gas which allows a stable operation at high detector gain is needed for efficient single electron detection. We note here that the large fluctuations in the
amplification of single electrons also affect cluster separation, as it is less likely to distinguish between adjacent clusters with very different pulse heights. It would be therefore advantageous to operate the detector out of proportional amplification, with as much as possible "saturated" gain. Possible gases are isobutane, propane, ethane and DME [32].

- **Electron losses**: Electron losses due to recombination or attachment by gas molecules or by electronegative impurities should be as small as possible. Hydrocarbons and noble gases have low electronegativity.

- **Conversion length**: The dissociation of electron clusters by diffusion and the counting rate requirements set limits on the length of the detector elements. In long detectors one expects large statistical fluctuations in the number of counted clusters [11], resulting in a diminished resolution. Thus, many short consecutive detectors should be used. However, the length of each amplifying element remains constant, and therefore limits the useful fraction of the total detector length. We estimate that the optimal conversion length of a single module, which best optimizes the ratio between useful conversion and total detector (including the amplification region) lengths is of the order of 15–20 cm.

From the list above and from some conclusions in previous sections it is apparent that a careful compromise between various parameters, of contradicting effects, should be made. The proper choice of detector configuration, filling gas and operation conditions are the key to the successful application of the PCC method. We did not discuss here the readout and pulse counting method, but rather used a correlation technique which was found in our previous studies as the most accurate procedure [11, 12]. However, for practical applications one should consider the use of fast electronics, i.e., fast differential discriminators (see also ref. [10],). In this case a good signal-to-noise ratio will be of crucial importance for efficient cluster recognition.

Considering the measured density of primary collisions and the statistical fluctuations in NOC [11] we conclude that isobutane, propane and DME may be good gas candidates
for detectors used for particle identification by the PCC method. Yet, other gases which could not be studied in details within the framework of this work should be investigated as well.

Our Monte-Carlo simulation shows that 20 consecutive detectors, each with 13.6 cm active conversion length (the total length of \( \approx 3 \) m is of the same order as typical dE/dx detectors operating at atmospheric pressure), filled with isobutane gas at 40 Torr should have a \( K-p \) resolution \( R = \Delta p_{K}(\langle NOC \rangle)/\sigma p \) in the relativistic rise region \( (4 \leq \beta \gamma \leq 400) \) of the order of 3.5 for the MSAC multiplier, and 4 for the SWPC. This particle resolution is competitive with existing dE/dx detectors with similar lengths [27, 31, 33], but not, at this stage, significantly better. Nevertheless, the low pressure operation of the PCC detector has some advantages which were not taken into account in this comparison: a lower detector ageing, a lower probability for energetic \( \delta \)-rays and the possibility to recognise them due to their longer range in the low-pressure gas, a lower multiple-scattering and Bremsstrahlung radiation production etc.

We would like to emphasise that all the measurements in the present study were made with a read-out from the full surface of the last anode mesh (80 or 50 cm\(^2\)) of the detector. In a practical detector, the use of other techniques, in which the capacitance of the readout element is much lower (pad read-out for example), will significantly improve the signal-to-noise ratio, thus allowing operation at lower detector gain. The application of other electron multipliers like the SWPC or recently developed MSGC's (microstrip gas chambers) should be of great benefit due to the narrower pulses and therefore the more efficient cluster separation, implying narrower statistical distributions, and consequently a better particle resolution at a given detector length. Several other multi-stage multipliers could be considered as well, preferably with a saturated gain response to reduce the sparking probability of highly ionizing events [34]. It should be noted that it was recently shown [35] that single electron tracks could be resolved, and single electrons were efficiently counted after long drift distances at high pressure Helium TPC detector with proper readout and electronics.

A drawback of the PCC-particle identification detector is its relatively poor tracking
(and double-track resolution), due to the long cluster paths, at low drift velocities in the low pressure gas, in the conversion volume. This problem may be more severe for particles which enter the detector in an inclined direction with respect to the detector axis (defined by the direction of the electric field in the conversion volume), or in the presence of high magnetic fields. A possible solution might be the addition of tracking elements between the PCC detector modules.

The primary cluster counting technique is not limited to the identification of relativistic particles only, but may also be used for the fields of nuclear and atomic physics, for identification of slow light particles ($\beta\gamma \leq 4$). The possibility to efficiently count single electrons opens many applications in various fields of research. It is used for basic studies of ionization phenomena in gases, providing ways of measuring the specific primary ionization [19], the mean energy for electron-ion pair creation and the Fano factor [24], the relativistic rise etc. It is employed for very soft X-ray spectroscopy [24] and for studies of radiation-induced ionization distribution in tissue equivalent gases. The latter aims at the understanding of radiation damage to living tissue at the level of the DNA [36, 37].

Another application could be in the search for rare elements, using TPC-like detectors. The method could be a unique technique for identifying short-range low-energy recoils in experiments searching for dark matter [38] or solar neutrinos [39].

Appendix

A Statistics of electron multiplication

A mathematical model for the amplification statistics in a uniform electric field was developed by Alkhazov [16]. It is assumed by this model that the main process of avalanche growth is an impact ionization by electrons and that recombination, electron attachment to impurities, space charge and photoelectric effects are negligible. To a first approximation the distribution can be described by the function

$$P(Z) \approx Z^{m-1} \cdot \Gamma \{ \nu, Z(m + \nu)/(m + 1) \}$$

(8)
where $Z \equiv M/(M)$, $M$ being the number of electrons in an avalanche initiated by a single primary electron; $m$, $\nu > 1$ are parameters to be fitted to experimental data and $\Gamma(a, x)$ is the incomplete Gamma function:

$$\Gamma(a, x) = \int_x^\infty e^{-t}t^{a-1}dt$$  \hspace{1cm} (9)

A typical amplification curve is shown in Fig. 8.

The amplification statistics of cylindrical field configurations is given by the Polya distribution [17]:

$$P(Z) = \frac{\theta^Z}{\Gamma(\theta)} \cdot Z^{\theta-1}e^{-\theta Z}$$  \hspace{1cm} (10)

where $\theta$ is a parameter to be fitted to experimental data.

B Saturation in the relativistic rise of energy loss

The most probable energy loss, $\Delta E_{mp}$, and the saturation in its relativistic rise, $\delta_{\delta E/dx}$, are given by Landau’s [5] and Sternheimer’s formulae [21, 22]:

$$\Delta E_{mp} = \frac{\alpha t}{\beta^2} \cdot \left\{ \ln \left( \frac{m_ec^2\alpha t}{I^2} \right) + 0.891 + \ln(\gamma^2) - \beta^2 - \delta_{\delta E/dx} \right\}$$  \hspace{1cm} (11)

where $\alpha t = 0.153(Z/A)\rho t$ (MeV. for $\rho t$ in g/cm$^2$) and $t$ is the absorbing material thickness. $I$ is the mean excitation potential. Sternheimer and Peierls calculated an approximation to $\delta_{\delta E/dx}$ for various gases using only four parameters ($X_0$, $X_1$, $X_a$ and $m$) given by the following expression:

$$\delta_{\delta E/dx}(X) = \begin{cases} 
0 & \text{for } X < X_0 \\
4.606(X - X_0) + a(X_1 - X)^m & \text{for } X_0 \leq X \leq X_1 \\
4.606(X - X_a) & \text{for } X > X_1
\end{cases}$$  \hspace{1cm} (12)

where $X \equiv \log_{10}(\lambda_\gamma)$ and $a$ is determined by the boundary conditions. The meaning of $X_0$, $X_1$ and $X_a$ is illustrated in fig.3. At $X \leq X_0$ the density effect is zero, thus $X_0$ determines its starting point. At $X \geq X_1$ the density effect completely cancels the relativistic rise. Thus $X_1$ represents the end-point of the relativistic rise.
The best fit of eq.12 to individually calculated $\delta_{dE/4x}$ values for various gases at NTP was obtained by Sternheimer [21] with the following results:

$$m = 3.0$$  \hspace{1cm} (13)

$X_0$ is given by

$$X_0 = \frac{1}{4.606} \cdot \ln \left( \frac{I^2}{(h\nu_p)^2} + 1 \right)$$  \hspace{1cm} (14)

with $h\nu_p = 28.8\sqrt{\rho Z/A}$ (eV for $\rho$ in gr/cm$^3$),

and

$$X_0 = \begin{cases} 
1.6 & \text{for } X_a < 2.17 \\
1.7 & \text{for } 2.17 \leq X_a \leq 2.28 \\
1.8 & \text{for } 2.28 \leq X_a \leq 2.39 \\
1.9 & \text{for } 2.39 \leq X_a \leq 2.50 \\
2.0 & \text{for } 2.50 \leq X_a
\end{cases}$$

$$X_1 = \begin{cases} 
4.0 & \text{for } X_a < 2.66 \\
5.0 & \text{for } X_a \geq 2.66
\end{cases}$$  \hspace{1cm} (15)

For a density other than NTP, $\delta_{dE/4x}$ is given by

$$\delta_{dE/4x} (X \eta) = \delta_{dE/4x}(X + 0.5 \log(\eta))$$

$$X_{0\eta} = X_0 - 0.5 \log(\eta)$$

$$X_{1\eta} = X_1 - 0.5 \log(\eta)$$

$$X_{2\eta} = X_2 - 0.5 \log(\eta)$$

$$X_{3\eta} = X_3 - 0.5 \log(\eta)$$  \hspace{1cm} (16)

where $\eta$ is the density relative to NTP conditions: $\eta \equiv \rho/\rho_{NTP}$. 

17
References


Table Captions

1. Cluster size distribution for a) (two left columns) minimum ionizing and ultrarelativistic particles crossing 10 mm of Argon (NTP): The populations are given in percentage of the total number of primary collisions, which are on the average 26.6 and 34.9 at $\beta_\gamma = 4$ and 1000 (calculated, [13]); b) (right column) minimum ionizing electrons crossing Argon at 100 torr (measured, [14]).

2. Simulation of particle identification by the PCC method at various momenta. The results are for 20 consecutive modules incorporating MSAC electron multipliers, each having an effective conversion region of 13.6 cm long; 40 Torr of isobutane; drift velocity in the conversion region 1.7 cm/$\mu$s; diffusion coefficient 0.09 cm$^{1/2}$; pulse width 40 ns (FWHM); no. of events: 100.

3. Simulation of particle identification at various momenta by the PCC method in a detector system incorporating series of SWPC electron multipliers; drift velocity in the conversion region 3.8 cm/$\mu$s; diffusion coefficient 0.06 cm$^{1/2}$; pulse width is 11 ns (FWHM); Other parameters are as for table 2.

4. Simulation of particle resolution (see definition in text) of 20 consecutive detectors, for various particle momenta and drift velocities in the conversion region. Detector dimensions, gas type and pressure are as for tables 2-3. (a) MSAC: Drift velocities in the conversion volume: 1.4, 1.7 and 1.8 cm/$\mu$s; pulse width: 40 ns (FWHM). (b) SWPC: Drift velocity in the conversion volume: 3.8 cm/$\mu$s; pulse width: 11 ns (FWHM).

5. Results with “conventional” dE/dx detectors separating 15 GeV/c particles with 64 samples of 4 cm (Lehraus et.al. [26]). The relativistic rise is shown in terms of the ratio of truncated mean peaks of electron and proton distributions. The $\pi/p$ resolution ($D/\sigma$) expresses the ratio of the distance (D) between the peaks
of truncated distribution for the 64 samples to the standard deviation (σ) of the distribution for pions.
Figure Captions

1. A schematic of the low-pressure cluster counting detector.

2. Examples of measured (a) and simulated (b) minimum ionizing $\beta^-$ electron tracks [11]. The results of the cluster identification by the correlation technique are shown (circles). Conditions: $p=40$ Torr isobutane; drift velocity: 1.7 cm/$\mu$s; conversion length: 13.6 cm.

3. Illustration of the three parameters $X_0$, $X_a$ and $X_1$ used in Sternheimer approximation [22]. (a) for the density effect correction, $\delta_{AE/dE}(X)$; (b) for the normalized energy loss, $E/\alpha t$. The calculation is for 10.5 atm P-10 ($Ar/CH_4(90/10)$).

4. Statistics of simulated primary cluster formation and counting at 40 Torr isobutane, for a particle with $\beta \gamma = 400$. Input conditions: drift velocity: 1.7 cm/$\mu$s; effective track length: 13.6 cm; input $n_\gamma$: 0.204 clusters/(cm-Torr); pulse width: 40 ns (FWHM); $\sigma_0 = 0.09 cm^{1/2}$.

   (a): Full-track analysis. (b): Number of collisions and of counted clusters within 0.5 $\mu$s time slices vs drift time in the conversion volume.

5. The simulated relative primary cluster formation and counting, as function of particle velocity, for 40 Torr of isobutane. Pulse width: 40 ns (FWHM); drift velocity: 1.7 cm/$\mu$s; no. of events: 2000.

6. Simulated distributions of the number of collisions (a) and of counted clusters (b, c) for 55.8 GeV/c Protons. Pions and Kaons, using 20 consecutive detectors with 13.6 cm effective conversion length. The pressure is 40 Torr of isobutane.

   (b) MSAC modules: $w = 1.7 cm/\mu$s; pulse width: 40 ns (FWHM); no. of events: 5000.

   (c) SWPC modules: $w = 3.8 cm/\mu$s; pulse width: 11 ns (FWHM); no. of events: 3000.
7. Expected particle separation in Ne and Ar at 2 Atm (Ref. [27]). Results are for the truncated mean of 128×2 cm samples. For the definition of particle separation (D/σ) see table 5.

8. A typical normalized-amplification probability curve [P(Z)] for single electrons. The amplification statistics is calculated using Alkhazov's formula for MSAC's [16], with m=1.1 and ν = 1.6 (see eq.8).
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<tr>
<th>Cluster size (number of electrons)</th>
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<th>Calculated by Lapique &amp; Piuz $\beta\gamma = 1000$</th>
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Table 3
## Table 4

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## Table 4

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Table 5
Figure 1
(a) Experimental cluster track

i-C\textsubscript{3}H\textsubscript{6}

40 Torr

Correlated spectrum
No. of Counted Clusters (NOC): 55

Time (\mu s)

(b) Simulated cluster track

Correlated spectrum

Number of collisions (N\textsubscript{p}): 63
Total number of electrons (N\textsubscript{e}): 128

Time (\mu s)

Figure 2
Figure 3
Gas: i-C₄H₁₀  \( \langle N_p \rangle = 111.0 \)
Pressure: 40 Torr
Conversion: 13.6 cm
\( \chi^2 = 1.0 \)

No. of Primary Collisions (Nₚ)

No. of Events

Primary Collisions (Nₚ)

Gas: i-C₄H₁₀  \( \langle NOC \rangle = 76.4 \)
Pressure: 40 Torr
Conversion: 13.6 cm
\( \chi^2 = 0.84 \)

No. of Events

No. of Counted Clusters (NOC)

No. of Counted Clusters (NOC)

Figure 4-a
Figure 4-b

Gas: i-C$_4$H$_{10}$
Pressure: 40 Torr
Conversion: 13.6 cm

No. of Primary Collisions ($N_p$)

No. of Counted Clusters (NOC)

Drift time (µs)
Gas: i-C$_4$H$_{10}$
Pressure: 40 Torr
Conversion: 13.6 cm

---

$N_p/N_p^0$

- No. of Primary Collisions ($N_p$)
- No. of Counted Clusters (NOC)

Particle velocity (in $\beta\gamma$ units)

Figure 5
Figure 6-a
Counted clusters (NOC) -- MSAC

Figure 6-b
Figure 6-c
Figure 7
Figure 8

\[ P(Z) \]

\[ Z (Z = M / \langle M \rangle) \]

- \[ m = 1.10 \]
- \[ \nu = 1.60 \]
- \[ \langle M \rangle = 1.00 \]