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TESTING VECTOR BOSON DOMINANCE
OF ELECTROWEAK INTERACTIONS
VIA e^+e^- ANNIHILATION
INTO BOSON PAIRS

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Abstract

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In the framework of the unified compositeness of leptons, quarks and Higgs bosons, supplemented with the hypothesis of the vector boson dominance of the SM gauge interactions, restrictions on the universal residual fermion-boson and boson-boson interactions are investigated. Manifestations of the universal interactions in the W^+W^- and ZH boson pair production at the 2 TeV e^+e^- linear collider are studied. Definite signatures of these interactions are exposed, and their relations to the non-standard triple gauge interactions are considered. It is shown that the unified substructure could be tested at the collider in these processes up to the compositeness scale $\mathcal{O}(25 \text{ TeV})$, which lies in the naturally preferred deca-TeV region. Importance of the optimal angular cutoffs and the polarized beams is found.

Аннотация

Кабаченко В. В., Пирогов Ю. Ф. Проверка векторно-бозонной доминантности электро-слабых взаимодействий в e^+e^- -аннигиляции в бозонные пары: Препринт ИФВЭ 95-92. – Протвино, 1995. – 18 с., 4 рис., библиогр.: 10.

В рамках единой составленности лептонов, кварков и хиггсовских бозонов, дополненной гипотезой векторно-бозонной доминантности электрослабых взаимодействий, исследованы ограничения на универсальные остаточные фермион-бозонные и бозон-бозонные взаимодействия. Изучены проявления таких взаимодействий в процессах рождения бозонных пар W^+W^- и ZH на 2-ТэВ линейном e^+e^- -коллайдере. Рассмотрены определяющие сигнатуры этих взаимодействий и их связь с нестандартными тройными калибровочными взаимодействиями. Показано, что единая субструктура может быть исследована в этих процессах на таком коллайдере вплоть до масштаба составленности $\mathcal{O}(25 \text{ ТэВ})$, что находится в предположимой из соображений натуральности декатэвной области. Подчеркнута важность оптимальных угловых обрезаний и поляризованных пучков.

Introduction

The Vector Boson Dominance (VBD) of gauge interactions in the Standard Model (SM) [1] is expected to be a universal manifestation of the unified compositeness of leptons, quarks, and Higgs bosons as Goldstone ones [2]. In this framework, the SM is considered to be just a part of a “low energy” effective theory. The latter corresponds to an unknown strongly coupled underlying theory responsible for the tight binding of the SM particles, now being composite, as well as for the appearance of the new heavy ones. In particular, the heavy composite vector bosons are expected to emerge and to mix with the SM elementary gauge bosons, resulting in the VBD.

Arguments had been put forward that the VBD phenomenon should be one of the most natural, model independent consequences in the scheme. In essence, the phenomenon is quite similar to the VBD of electromagnetic interactions of hadrons (modulo symmetry properties), with the SM gauge bosons being the counterpart of the photon, and the heavy composite vector bosons that of the ρ, ω, φ , etc., mesons.

In a previous paper [3], we have investigated the manifestations of the VBD of electroweak interactions at the e^+e^- linear collider of 2 TeV energy in the universal fermion-fermion interactions via $e^+e^- \rightarrow \bar{f}f$. The present paper elaborates on the VBD manifestations in the universal fermion-boson and boson-boson interactions via the ZH and W^+W^- pair production at the collider.

The paper is organized as follows. In the next section we shall discuss the general properties of the VBD effective Lagrangian and illuminate its parts relevant to the given processes. In the second section, we study the VBD corrections to the SM Higgs bremsstrahl process $e^+e^- \rightarrow Z^* \rightarrow ZH$. The third section is devoted to the VBD corrections to the W^+W^- production, as well as to studying the question to what extent they could mimic (or rather be mimicked by) the triple gauge boson non-standard vertices. The reach for the compositeness scale $\mathcal{F}$ via the boson pair production in the 2 TeV e^+e^- collisions is found and discussed. Importance of the optimal angular cutoffs and the right-handed polarized electron beams is found.

1. Effective Lagrangian

The unified compositeness effective Lagrangian for fermion-boson and boson-boson interactions is as follows [1]:

$$\begin{aligned}\mathcal{L}_{eff}(\psi, \phi) = & -\frac{1}{\mathcal{F}^2} \left(J_\mu^i(\psi) J_\mu^i(\phi) + \eta_1 J_\mu^0(\psi) J_\mu^0(\phi) \right) \\ & -\frac{1}{\mathcal{F}^2} \left(\frac{1}{3} J_\mu^i(\phi) J_\mu^i(\phi) + J_\mu^0(\phi) J_\mu^0(\phi) \right),\end{aligned}\quad (1)$$

where $\mathcal{F}$ is the overall compositeness mass scale and $\eta_1 > 0$ is the relative strength of the weak isoscalar and isovector contact interactions. Here the SM weak isovector and hypercharge isoscalar currents of fermions ψ are given by

$$\begin{aligned}J_\mu^i(\psi) &= \sum \bar{\psi}_L T_i \gamma_\mu \psi_L, \\ J_\mu^0(\psi) &= \sum \left(\bar{\psi}_L Y_L \gamma_\mu \psi_L + \bar{\psi}_R Y_R \gamma_\mu \psi_R \right)\end{aligned}\quad (2)$$

and these of the Higgs doublet ϕ by

$$\begin{aligned}J_\mu^i(\phi) &= \phi^\dagger \frac{\tau_i}{2} i \overleftrightarrow{D}_\mu \phi, \\ J_\mu^0(\phi) &= \phi^\dagger \frac{\tau_0}{2} i \overleftrightarrow{D}_\mu \phi,\end{aligned}\quad (3)$$

where $\phi^\dagger \overleftrightarrow{D}_\mu \phi \equiv \phi^\dagger D_\mu \phi - (D_\mu \phi)^\dagger \phi$, etc., and D_μ is the SM covariant derivative.

The first term in Eq. 1 is due to the supplementary hypothesis of the VBD of electroweak interactions, whereas the second one is independent of it and is just determined by the Goldstone origin of the Higgs doublet in the scheme. Note, that dependence on the unknown coupling constant $\hat{g}$ of the heavy vector bosons is cancelled out in the first term: $\hat{g}^2/\hat{M}^2 = 1/\mathcal{F}^2$, where $\hat{M} = \hat{g}\mathcal{F}$ is the vector boson mass. Eq. 1 is valid only at total energy squared $s \ll \mathcal{F}^2$. The case of $\eta_1 = 0$ corresponds to the pure isovector contact interactions, while that of $\eta_1 \gg 1$ (at $\eta_1/\mathcal{F}^2$ fixed) does to the pure isoscalar ones. Both these limiting cases are unnatural, for *a priori* one expects $\eta_1 = \mathcal{O}(1)$. Nevertheless, this parameter well may lie, say, in the range from 1/10 to 10 which we will adopt in what follows.

Thus the unified compositeness plus VBD prescribes the two-parameter set of the universal contact interactions, with their space-time and internal structure being fixed, the phase including. It is of paramount importance here that, in the very spirit of the VBD, the couplings of the SM elementary gauge bosons with composite fermions (and Higgses) are mediated entirely by the intermediate composite vector bosons. Hence, the first term in $\mathcal{L}_{eff}$ is flavor aligned with the proper term coming from the SM gauge boson exchanges. In particular, as for the neutral currents, both standard and non-standard ones are flavor conserving [3]. This eliminates otherwise potentially strong limits on $\mathcal{F}$ from the flavor violation processes.

One has in an arbitrary gauge

$$\phi = \begin{pmatrix} iw^+ \\ (v + h + iz)/\sqrt{2} \end{pmatrix}, \quad (4)$$

where $w^\pm$ and z are the would-be Goldstone bosons corresponding to the longitudinal components of $W^\pm$ and Z , respectively, and h is the physical Higgs boson, with v being the vacuum expectation value of the Higgs doublet.

In the unitary gauge ($w^\pm = z = 0$, $h \equiv H$), keeping only those terms that are operative at the processes of interest, the effective Lagrangian can be written as

$$\begin{aligned} \mathcal{L}_{eff} = & -\frac{1}{\mathcal{F}^2}(v + H)^2 \left(\frac{\bar{g}}{4\bar{c}} \left(\frac{\eta_1 - 1}{2} \bar{e}_L \gamma_\mu e_L + \eta_1 \bar{e}_R \gamma_\mu e_R \right) Z_\mu \right. \\ & + \frac{\bar{g}}{4\sqrt{2}} \left(\bar{e}_L \gamma_\mu \nu_L W_\mu + \bar{\nu}_L \gamma_\mu e_L W_\mu^\dagger \right) \\ & \left. - \frac{1}{3\mathcal{F}^2}(v + H)^4 \left(\frac{\bar{g}^2}{4\bar{c}^2} Z_\mu Z_\mu + \frac{\bar{g}^2}{8} W_\mu^\dagger W_\mu \right) + \dots \right), \end{aligned} \quad (5)$$

where $\bar{g}$ is the effective weak isospin coupling, $\bar{c} \equiv \cos \bar{\theta}_W$, $\bar{s} \equiv \sin \bar{\theta}_W$, with $\bar{\theta}_W$ being the effective weak mixing angle and W_μ being the W^- boson field.

For the processes chosen, this Lagrangian leads to the modification of some of the SM vertices, as well as to the appearance of the new ones. Namely, $V_{e\nu W}$ and V_{HZZ} are multiplied by the factors $(1 - v^2/4\mathcal{F}^2)$ and $(1 - 4v^2/3\mathcal{F}^2)$, respectively, $g_{L(R)} \equiv T_{L(R)}^3 - \bar{s}^2 Q$ in V_{eeZ} are replaced by $g_L - (\eta_1 - 1)/2 \cdot (v^2/4\mathcal{F}^2)$ or $g_R - \eta_1 v^2/4\mathcal{F}^2$, accordingly. Besides, the new vertex $V_{eeHZ} = (gv/2\bar{c}\mathcal{F}^2)[(1 - \eta_1)/2 \cdot \bar{e}_L \gamma_\mu e_L - \eta_1 \bar{e}_R \gamma_\mu e_R]$ appears.

Owing to destroying the unitarity cancellations for production of the longitudinal gauge bosons $e^+e^- \rightarrow W_L^+ W_L^-$, as well as to the appearance of the contact terms in amplitude in the case of $e^+e^- \rightarrow Z_L H$, the sensitivity to these new interactions grows with energy. Note that there are also two processes, $e^+e^- \rightarrow \gamma Z$ and ZZ , to which new interactions make contribution, too. Unfortunately, these contributions result only in a rescaling of the fermion- Z vertices and do not grow with energy. So, they could be best studied just at the Z pole. That is why we will restrict our consideration to the e^+e^- annihilation into the W^+W^- and ZH only¹.

Note finally, that all the results will be obtained further in the effective Born approximation. Under the latter we understand retaining only the universal corrections in the SM low energy effective Lagrangian through the use of the effective parameters, but neglecting the nonuniversal vertex and box corrections².

¹Other potentially relevant process $e^+e^- \rightarrow e^+(\bar{\nu})e^-(\nu)H$ with the $ZZ(W^+W^-)$ fusion, though do not fall with energy, are of no use here too, due to all vertex subenergies being finite.

²Qualitative influence of the soft electromagnetic radiative correction will be discussed later on.

2. $e^+e^- \rightarrow ZH$

Consider now the processes $e_\chi^+ e_{\bar{\chi}}^- \rightarrow ZH$, where χ and $\bar{\chi}$ label the appropriate chiralities: $\chi, \bar{\chi} = L, R$. The total initial helicity is $J_3 = \chi - \bar{\chi}$, with $L \equiv -1/2$, $R \equiv 1/2$. For the differential cross-sections one obtains straightforwardly the following result:

$$\frac{d\sigma_{J_3}}{d\cos\theta} = \sigma_0 |M_{J_3}|^2 \frac{E_Z^2}{s} \left((1 - \cos^2\theta) + \frac{m_Z^2}{E_Z^2} (1 + \cos^2\theta) \right). \quad (6)$$

The cross-sections for all the remaining helicities are negligible in the limit $m_e^2/s \ll 1$. Here $\cos\theta$ is the cosine of the scattering angle between Z and e^- , E_Z is the Z boson energy, $\sigma_0 \equiv (\pi\bar{\alpha}^2/s)\sqrt{(E_Z^2 - m_Z^2)/s}$ is normalizing cross-section and

$$\begin{aligned} M_{-1} &= \frac{1}{\bar{s}^2 \bar{c}^2} \left(1 - \frac{4v^2}{3\mathcal{F}^2} \right) \left(g_L - \frac{v^2}{4\mathcal{F}^2} \frac{\eta_1 - 1}{2} \right) - \frac{s}{\bar{e}^2 \mathcal{F}^2} \frac{\eta_1 - 1}{2}, \\ M_{+1} &= \frac{1}{\bar{s}^2 \bar{c}^2} \left(1 - \frac{4v^2}{3\mathcal{F}^2} \right) \left(g_R - \frac{v^2}{4\mathcal{F}^2} \eta_1 \right) - \frac{s}{\bar{e}^2 \mathcal{F}^2} \eta_1. \end{aligned} \quad (7)$$

The last expressions are valid only at $m_Z^2 \ll s \ll \mathcal{F}^2$ in neglect of the Z boson mass in propagator. Here one has $\bar{s} \equiv \sin\bar{\theta}_W$, $\bar{c} \equiv \cos\bar{\theta}_W$, with $\bar{g} = \bar{e}/\bar{s}$, $\bar{g}_1 = \bar{e}/\bar{c}$ and $\bar{e}$ being the appropriate effective couplings and electric charge constant, $g_L = -1/2 + \bar{s}^2$ and $g_R = \bar{s}^2$.

In Eq. 6, the dominant first term describes the production of the longitudinal boson Z_L , while the small second term does for the transverse boson Z_T . This is to be expected from the equivalence theorem [4]. According to the theorem, the production of the longitudinal vector bosons at high energies is dominated by that of the would-be Goldstone bosons. The latter ones have to be produced here in P -wave and, hence, have to be distributed like $\sin^2\theta$. Direct application of the equivalence theorem technique confirms the result of Eq. 6 up to the relative corrections $\mathcal{O}(m_Z^2/s)$ to the leading terms.

The differential cross-section is the most sensitive observable (in absence of the large systematic errors) for detecting any kind of contact interactions in the unpolarized beams. Still, it is seen that the parameter dependence of the angular distribution is quite involved. To unravel it without producing a lot of angular distributions for the various values of parameters we choose to expose as more illuminating a set of integral characteristics.

Let P_e denotes the polarization of the electron beam (the positron beam is taken to be unpolarized), and $\sigma(P_e) \equiv (1 - P_e)/2 \cdot \sigma(e_L^-) + (1 + P_e)/2 \cdot \sigma(e_R^-)$ is the polarized cross-section. Then the observables are: relative deviation in the total cross-section from the SM cross-section

$$\Delta(P_e) = \left(\sigma(P_e) - \sigma_{SM}(P_e) \right) / \sigma_{SM}(P_e) \quad (8)$$

and the left-right polarization asymmetry

$$A_{LR}(P_e) \equiv \frac{\sigma(P_e) - \sigma(-P_e)}{\sigma(P_e) + \sigma(-P_e)}, \quad (9)$$

which is

$$A_{LR}(P_e) = P_e \frac{\sigma(e_R^-) - \sigma(e_L^-)}{\sigma(e_R^-) + \sigma(e_L^-)} = P_e \frac{|M_{+1}|^2 - |M_{-1}|^2}{|M_{+1}|^2 + |M_{-1}|^2}. \quad (10)$$

Note that due to the CP invariance the differential cross-section of the process $e^+e^- \rightarrow ZH$ contains just the quadratic in $\cos\theta$ terms but not the linear ones. Hence one has here $A_{FB} \equiv 0$.

The results of the calculations of these observables at $\sqrt{s} = 2 \text{ TeV}$ as the functions of the parameter η_1 are plotted in Fig.1. We use with sufficient accuracy for $\bar{\alpha}$ its value $\bar{\alpha} = 1/129$ on the Z pole, and for the effective mixing angle the corresponding value $\bar{s} = 0.23$. In the spirit of the unified compositeness, with the Higgs doublet being Goldstone boson, it is natural to expect the light or intermediate physical Higgs boson [2]. So, all the results plotted in the figures correspond to the Higgs mass $m_H = 200 \text{ GeV}$, dependence on m_H being negligible in the range from 100 to 300 GeV. Due to vanishing of the leading term of Eq. 6 in the forward and backward directions, taking into account a real angular cutoff is of no principle importance.

To evaluate the statistical significance of the observed deviations let us consider the total cross-sections. For definiteness we take into consideration only statistical errors neglecting the systematic ones. Introduce the quantity $n_\sigma = |\Delta N|/\sqrt{N_{SM}} = (|\Delta\sigma|/\sigma_{SM})\sqrt{\sigma_{SM} \int \mathcal{L} dt}$, showing the number of standard deviations from the SM predictions. Thus, taking the integrated luminosity $\int \mathcal{L} dt$ to be 20 fb^{-1} and having $\sigma_{SM}(P_e = 0) = 3.03 \text{ fb}$ and $\sigma_{SM}(P_e = +0.8) = 2.70 \text{ fb}$ one gets that the 2σ deviation (95% C.L.) in the total cross-section corresponds to $|\Delta(0)| = 0.257$ and $|\Delta(+0.8)| = 0.272$. The reach for $\mathcal{F}$ at 95% C.L. via the total cross sections is shown in Fig.2. For definiteness, luminosities are taken to be equal in both cases, though some deficiency in luminosity for polarized beams may worsen the reach. The reach scales with $\sqrt{s}$ roughly as $\mathcal{F} \sim \sqrt[4]{s}$ at $\int \mathcal{L} dt$ fixed, and as $\mathcal{F} \sim \sqrt{s}$ at $\int \mathcal{L} dt \sim \sqrt{s}$ [3].

Let us explain the structure of the figures in more detail. There are two contributions in the total cross-section from the contact interactions: the interference term $\mathcal{O}(s/\mathcal{F}^2)$ and the square term $\mathcal{O}((s/\mathcal{F}^2)^2)$, the first one dominating at not too high values of η_1 . With η_1 growing, the second one grows faster (as η_1^2). So, at some η_1 the discrepancy $|\Delta|$ achieves its largest value, where the two terms are of equal importance. With further growing of η_1 , at some its particular value, both terms are mutually compensated. It is clear that in general one can not consider much larger values of η_1 (at $\mathcal{F}$ given), for in this case the disposed terms $\mathcal{O}(s^2/\mathcal{F}^4)$ in the effective Lagrangian should be taken into account, too. But we consider these terms to be of the radiative correction origin and hence additionally suppressed as $\mathcal{O}(1/(4\pi)^2)$. So, retaining the terms quadratic in the tree level contact interactions could still be meaningful.

In Fig.2 for $\Delta(0)$, the region labeled by I corresponds to those values of η_1 and $\mathcal{F}$ where the linear term dominates, whereas only the quadratic term is essential in the region II. For $\Delta(+0.8)$ the first domain becomes larger and is separated from the second one by a narrow “black out” region that is not drawn on the plot. This region can be ultimately studied under other conditions.

To understand such a behaviour let us rewrite Eq. 7 in a simpler form, neglecting the term $\mathcal{O}(v^2/\mathcal{F}^2)$ with respect to $\mathcal{O}(s/\bar{e}^2\mathcal{F}^2)$,

$$M_{J_3} = k_{J_3} M_{J_3}^{SM}, \quad (11)$$

where the rescaling factors are

$$\begin{aligned} k_{-1} &= 1 - \frac{\eta_1 - 1}{2} \frac{\bar{s}^2 \bar{c}^2}{g_L \bar{e}^2} \frac{s}{\mathcal{F}^2}, \\ k_{+1} &= 1 - \eta_1 \frac{\bar{s}^2 \bar{c}^2}{g_R \bar{e}^2} \frac{s}{\mathcal{F}^2}. \end{aligned} \quad (12)$$

For the totally polarized electrons ($P_e = \pm 1$), the discrepancy $\Delta(\pm 1)$ gains its extreme value $|\Delta(\pm 1)| = 1$ when the corresponding factors $k_{\pm 1}$ become zero (cross-section zeros). Because $g_L < 0$, the value of η_1 realizing the extremum in the case of the left-handed electrons (due to $k_{-1} = 0$) is negative, i.e., “unphysical” in the scheme. This is not the case for the right-handed polarization, because $k_{+1} = 0$ is always possible. So, one may conclude that it is preferable to work with the purely right-handed electrons to observe in the polarized total cross-section as large deviation from the SM as possible. But we take $P_e = +0.8$ as more realistic.

As for the asymmetry A_{LR} , there is some peculiar value of $\eta_1 = \bar{s}^2/\bar{c}^2 \simeq 0.3$ for which $k_{-1} = k_{+1} = k$ and hence $A_{LR} = A_{LR}^{SM}$. Such a behaviour for all the asymmetries at $\eta_1 \simeq 0.3$ has been observed previously [3] for all the processes $e^+e^- \rightarrow \bar{f}f$. This situation presents the universal worst case study for the deviation from the SM. Note that as it is seen from Fig.2, the reach for $\mathcal{F}$ in the given process is independent of P_e at this particular value of η_1 , and according to Eq. 8 is, in fact, determined by $\Delta(P_e) \simeq k - 1$ (luminosities being equal).

It is worth mentioning finally that the contact term $\mathcal{O}(s/\mathcal{F}^2)$ in the amplitude, originating from the fermion-boson term in $\mathcal{L}_{eff}$, is dominant, while the influence of the Higgs doublet self-interactions is negligible. Thus, actually, one may test in the process at hand only manifestations of the unified compositeness supplemented with the VBD, but not of the unified compositeness alone.

3. $e^+e^- \rightarrow W^+W^-$

Vector boson dominance. As in the previous case, in the limit of the massless electrons, the differential cross-sections for W^+W^- pair production is nonzero only for two total initial helicities $J_3 = \pm 1$. In the effective Born approximation, these differential cross-sections are given in the high-energy limit by

$$\begin{aligned} \frac{d\sigma_{-1}}{d\cos\theta} &= \sigma_0 \left(\left(\frac{1}{4\bar{c}^2\bar{s}^2} + \Delta M_{-1} \right)^2 + \frac{1}{4\bar{s}^4} \frac{u^2 + t^2}{t^2} \right) (1 - \cos^2\theta), \\ \frac{d\sigma_{+1}}{d\cos\theta} &= \sigma_0 \left(-\frac{1}{2\bar{c}^2} + \Delta M_{+1} \right)^2 (1 - \cos^2\theta). \end{aligned} \quad (13)$$

Here θ is the scattering angle between W^- and e^- , $\sigma_0 \equiv \pi \bar{\alpha}^2/2s$ is the normalizing cross section, and $\Delta M_{\pm 1}$ are given by

$$\begin{aligned}\Delta M_{-1} &= -\frac{\eta_1 + 1}{4} \frac{s}{\bar{e}^2 \mathcal{F}^2}, \\ \Delta M_{+1} &= \frac{\eta_1}{2} \frac{s}{\bar{e}^2 \mathcal{F}^2}.\end{aligned}\tag{14}$$

Eqs. 13 and 14 take into account only the dominant contributions and are valid in the kinematical region $s, |t| \gg m_W^2$. In Eq. 13, σ_{+1} and the first term in σ_{-1} correspond to the longitudinally polarized $W_L^+ W_L^-$ pair. The sharp forward peak in the first part of this equation is due to the t -channel neutrino exchange for the $e_R^+ e_L^- \rightarrow W_T^+ W_T^-$ process. We have neglected the terms $\mathcal{O}(m_W^2/s)$ and $\mathcal{O}(v^2/\mathcal{F}^2)$ in the amplitudes for $W_L W_L$ and $W_T W_T$ production. We have also omitted all the terms describing $W_L W_T$ pair production, their amplitudes being $\mathcal{O}(m_W/\sqrt{s})$ relative to that of the $W_L W_L$ production³.

We analyse the same set of integral characteristics as in the case of $e^+ e^- \rightarrow ZH$ and additionally the forward-backward charge asymmetry

$$A_{FB}(P_e) \equiv \frac{\sigma_F(P_e) - \sigma_B(P_e)}{\sigma_F(P_e) + \sigma_B(P_e)}.\tag{15}$$

These observables are plotted in Figs.3 and 4 as the functions of the parameter η_1 ($\mathcal{F}$ being fixed). Due to the sharp forward peak, some inevitable instrumental cutoff should be taken into account. For definiteness, we take it to be $|\cos \theta| \leq 0.8$. Note, that all the numerical calculations have been performed using the exact effective Born expression. Nevertheless, Eq. 13 is still sufficient for qualitative discussion. Moreover, it serves as a good approximation even for the observables which include the integration over the whole allowed angular region $-0.8 \leq \cos \theta \leq 0.8$, in spite of the formal inapplicability of this expression in the near forward direction.

It is easy to see, that one has $\sigma_{SM}(-1) \gg \sigma_{SM}(+1)$ due to the strong forward peak. At the same time, the differences $\sigma(-1) - \sigma_{SM}(-1)$ and $\sigma(+1) - \sigma_{SM}(+1)$ are of the same order because contribution of the t -channel (for $P_e = -1$) is cancelled out. Thus, one has $|\Delta(-1)| \ll |\Delta(+1)|$ and, as in the case of $e^+ e^- \rightarrow ZH$, one should operate with the maximal right-handed polarization of the electron beam. The relative deviations from the SM in the total cross-section, integrated over $-0.8 \leq \cos \theta \leq 0.8$, are presented in Figs.3(a) and 4(a) for $P_e = 0$ and $P_e = +0.8$, respectively. Unfortunately, even in the case of P_e close to 1, the $(1 - P_e)/2$ admixture of the left-handed electrons restricts sensitivity to Δ . Using the left-handed polarized positron beams would suppress this admixture further as $(1 + P_{e+})/2$ and improve the sensitivity.

Another way to improve the reach for $\mathcal{F}$ via the total cross-section, is to introduce an additional cutoff in the forward direction for the W^- around the beam axis (cutoff in the backward direction for the W^- is unnecessary). This cutoff reduces mainly the number of transverse $W^\pm$ bosons. The influence upon them of the contact interactions (namely,

³Remarks of the preceding section concerning the equivalence theorem are applicable here too, in particular, what concerns the angular distribution.

terms $\mathcal{O}(v^2/\mathcal{F}^2)$ omitted in Eq. 13) is much smaller than that on the number of the longitudinal bosons (terms $\mathcal{O}(s/\bar{e}^2\mathcal{F}^2)$). So, the cutoff affects mainly the SM contribution but not that of the contact interactions. From the requirement of the maximal reach for $\mathcal{F}$ at 95% C.L. via the total cross-section we found that the optimal cutoff for this observable is equal to $\cos\theta \leq 0.3$ (Fig.2). Here $\sigma_{SM}^{0.3}(0) = 23.59 \text{ fb}$, $\sigma_{SM}^{0.3}(+0.8) = 6.18 \text{ fb}$ and 2σ deviation corresponds to $|\Delta^{0.3}(0)| = 0.092$ and $|\Delta^{0.3}(+0.8)| = 0.180$, respectively. The reach is maximal owing to the maximal suppression of the t -channel peak at the statistics still high enough. This optimal cutoff is near the same for the values of $P_e = 0$ and ± 0.8 . The statistical deviations are not large. So, the account for the systematic errors of the same order may, in fact, be obligatory here, too. This would effectively reduce the C.L. from 2σ to, say, 1σ .

The relative deviations from the SM in the total cross-section integrated over $-0.8 \leq \cos\theta \leq 0.3$ are plotted in the Figs.3(b) and 4(b). Fig. 4(d) presents the left-right asymmetry $A_{LR}(+0.8)$ calculated with the same cutoff. The asymmetry gains its maximal value equal to -0.8 at $\eta_1 = \bar{g}_1^2 \mathcal{F}^2/s \simeq 3.2(\mathcal{F}/10 \text{ TeV})^2$, when σ_{+1} becomes zero. Note, that this value of η_1 realizes the extremum in the asymmetries both in the case of $e^+e^- \rightarrow ZH$, as it follows from Eq. 12, and for the processes $e^+e^- \rightarrow \bar{f}f$, $f \neq e^-$ [3]. This situation presents the best case study for deviation from the SM, which is process insensitive but energy dependent.

As for the forward-backward asymmetry $A_{FB}(P_e)$ calculated within the whole angular range $-1 \leq \cos\theta \leq 1$, its deviation from that in the SM is rather small. It is easy to understand this using the Eq. 13. Here the leading terms from the contact interactions are symmetric in $\cos\theta$, and so the numerator $\sigma_F - \sigma_B$ is approximately the same as in the SM. On the other hand, the denominator $\sigma_F + \sigma_B$ is dominated by SM contribution from the t -channel peak. Taking into account just the instrumental cutoff $|\cos\theta| \leq 0.8$ improves the situation considerably. This observable is depicted in Figs.3(c) and 4(c).

Triple gauge interactions (TGI). In addition to the previously considered residual interactions described by the electroweak VBD effective Lagrangian Eq. 1, a lot of other “low energy” interactions is allowed in the scheme of the unified compositeness. In particular, the exotic triple gauge boson interactions [5], now being popular, are conceivable too, and can contribute to the W^+W^- pair production. The question arises as to what extent the two types of new interactions could imitate each other. Let us elucidate the question somewhat.

Phenomenologically, the most general, explicitly $U(1)_{em}$ gauge invariant effective Lagrangian in the gauge sector, with two and three gauge bosons, can be put in the form:

$$\begin{aligned} \mathcal{L}_{eff}^{(gauge)} = & -\frac{1}{2}W_{\mu\nu}^\dagger W_{\mu\nu} - \frac{\kappa_{ZZ}}{4}Z_{\mu\nu}Z_{\mu\nu} - \frac{\kappa_{\gamma\gamma}}{4}A_{\mu\nu}A_{\mu\nu} - \frac{\kappa_{Z\gamma}}{2}Z_{\mu\nu}A_{\mu\nu} \\ & + i\bar{g}_Z\bar{g}_{1Z}\left(W_{\mu\nu}^\dagger W_\mu Z_\nu - W_{\mu\nu}W_\mu^\dagger Z_\nu\right) \\ & + i\sum_V \bar{g}_V\left(\bar{\kappa}_V W_\mu^\dagger W_\nu V_{\mu\nu} + \frac{\bar{\lambda}_V}{m_W^2}W_{\mu\rho}^\dagger W_{\rho\nu}V_{\mu\nu}\right) + \dots \end{aligned} \quad (16)$$

In Eq. 16, W_μ is the W^- boson field, and V_μ stands for either $V_\mu = A_\mu$ field in the case of the photon $V = \gamma$, or for $V_\mu = Z_\mu$ in the case of the Z boson⁴. The gauge strengths are as follows. For the charged fields this is $W_{\mu\nu} = D_\mu W_\nu - D_\nu W_\mu$ with $D_\mu = \partial_\mu - i\bar{e}A_\mu$ (and similarly for $W_{\mu\nu}^\dagger$) and for the neutral fields, respectively, $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$. When written explicitly in term of A_μ , the W kinetic term in the first line of Eq. 16 results in the $WW\gamma$ interactions, similar to these of the Z boson in the second line. Here Z_μ should be substituted by A_μ , and the proper constant $\bar{g}_\gamma \bar{g}_{1\gamma} = \bar{e}$ is fixed by the $U(1)_{em}$ gauge invariance⁵.

All the terms in the effective Lagrangian Eq. 16 possess the parities $C = P = CP = +1$. One more term with $CP = +1$ and $C = P = -1$ is possible too, but is not shown for reasons stated later on. With this exception, the dots in Eq. 16 represent the omitted CP violating interactions. They are supposed to be much smaller than the CP conserving ones, because the underlying strong interactions responsible for the unified substructure are considered to be CP conserving.

The overall coupling constants $\bar{g}_\gamma$ and $\bar{g}_Z$ being chosen as $\bar{g}_\gamma = \bar{e}$ and $\bar{g}_Z = \bar{e}\bar{c}/\bar{s}$, the free parameters $\bar{g}_{1V}$, $\bar{\kappa}_V$ and $\bar{\lambda}_V$ take in the SM tree-level limit the values $\bar{g}_{1V} = \bar{\kappa}_V = 1$ and $\bar{\lambda}_V = 0$. All other terms, omitted in Eq. 16, are zero in the SM, too⁶. The physical meaning of these free parameters is that they are related to the magnetic dipole moments $\mu_W^{(V)}$ and the electric quadrupole moments $Q_W^{(V)}$ of the $W^\pm$ for interactions with the V as [5]

$$\begin{aligned}\mu_{W^\pm}^{(V)} &= \pm \frac{\bar{g}_V}{2m_W} (\bar{g}_{1V} + \bar{\kappa}_V + \bar{\lambda}_V), \\ Q_{W^\pm}^{(V)} &= \mp \frac{\bar{g}_V}{m_W^2} (\bar{\kappa}_V - \bar{\lambda}_V).\end{aligned}\tag{17}$$

Convenient as Eq. 16 is for phenomenological purposes as a reference frame, it sheds no light on the magnitude of the free parameters. To get an idea of this, let us address ourselves to some more sophisticated considerations. To this end, Eq. 16 should be viewed as an unitary gauge expression of the electroweak gauge invariant effective Lagrangian for a SM extension. Now, the whole SM symmetry $SU(2)_L \times U(1)_Y$ could be realized here either linearly or non-linearly.

In the case of a non-linear realization (being still linear on the $U(1)_{em}$ subgroup), the nonlinearity scale Λ is related to the W mass ($m_W = \mathcal{O}(g\Lambda)$) and hence this scale is just the SM v.e.v. v . Thus, this kind of extensions has nothing to do with the unified compositeness we consider.

⁴The SM equations of motion are used here to eliminate the terms $D_\mu W_{\mu\nu}$ and $\partial_\mu V_{\mu\nu}$. Besides, the terms with derivatives, which give zero when contracted with the light fermion currents, are omitted, too. Here $\kappa_{Z\gamma} \equiv \kappa_{Z\gamma}^{SM} + \Delta\kappa_{Z\gamma}$ describes both the SM radiative mixing and the new interactions. Via the gauge field redefinition, the normalization of the diagonal kinetic terms could be chosen in the canonical form equal to unity, but this would result in a shift of the boson-fermion vertices.

⁵This fixes the *a priori* possible parameter κ_{WW} in Eq. 16 to be equal to unity.

⁶(One-loop radiative corrections shift these effective parameters from their tree-level values generically as $\mathcal{O}(\alpha/\pi)$ [6].

On the other hand, for a linear SM realization the scale Λ is not directly related with m_W and could be as high as desired. In the framework of the unified compositeness, we chose it to be the nonlinearity scale $\mathcal{F}$ of the low energy effective field theory produced by the underlying strong binding gauge interactions: $\mathcal{F} = \mathcal{O}(10\text{-}100 \text{ TeV})$ [2].

All the conceivable linearly realized residual interactions are described by the most general invariant effective Lagrangian built of the SM fields [7]. It consists of the $SU(2)_L \times U(1)_Y$ invariant operators of the dimension $d = 6$, scaled with the dimensionful parameter $\mathcal{F}^2$. All these operators which are relevant to the triple gauge boson vertices are naturally expected to be $\mathcal{O}(g^3)$ in the gauge couplings, but one caveat. This is the operator $\mathcal{O}_{WS} = \phi^\dagger \tau_i / 2 \phi W_{\mu\nu}^i S_{\mu\nu}$. It stems from the non-linear generalization of the field strengths, and is quadratic in the latter ones. The similar gauge kinetic terms of the isotriplet W and isosinglet S bosons are enhanced in the gauge couplings and are $\mathcal{O}(1)$. The same must naturally happen for $\mathcal{O}_{WS}$, for its origin is of the same nature. This is the unique case when the residual TGI due to the unified compositeness could compete with the VBD. Are the former still $\mathcal{O}(g^3)$, the VBD will be unambiguously dominant.

Thus, we retain the $\mathcal{O}_{WS}$ operator alone and choose the proper effective Lagrangian as

$$\mathcal{L}_{eff} = \frac{C}{2} \frac{1}{\mathcal{F}^2} \phi^\dagger \frac{\tau_i}{2} \phi W_{\mu\nu}^i S_{\mu\nu}, \quad (18)$$

with $C = \mathcal{O}(1)$. Hence the triple gauge boson vertices are $\mathcal{O}(g)$. Eq. 18 results in the following one-parameter sets for Eq. 16 ($\bar{\kappa}_V \equiv 1 + \Delta\bar{\kappa}_V$)

$$\begin{aligned} \Delta\bar{\kappa}_Z &= -\frac{\bar{s}^2}{\bar{c}^2} \Delta\bar{\kappa}_\gamma, \\ \Delta\bar{\kappa}_\gamma \equiv \Delta\bar{\kappa} &= \frac{\bar{c}}{\bar{s}} \frac{C}{4} \frac{v^2}{\mathcal{F}^2} \end{aligned} \quad (19)$$

and

$$\begin{aligned} \kappa_{ZZ} &= 1 - \bar{c}\bar{s} \frac{C}{16} \frac{v^2}{\mathcal{F}^2}, \\ \kappa_{\gamma\gamma} &= 1 + \bar{c}\bar{s} \frac{C}{16} \frac{v^2}{\mathcal{F}^2}, \\ \kappa_{\gamma Z} &= (\bar{s}^2 - \bar{c}^2) \frac{C}{4} \frac{v^2}{\mathcal{F}^2}, \end{aligned} \quad (20)$$

with $\bar{g}_{1V} = 1$, $\bar{\lambda}_V = 0$, all the remaining parameters in Eq. 16 being zero, too.

Operator of Eq. 18 results in the W_3 - S propagator mixing, which is the main source of the restrictions on the magnitude of the operator. The present-day precision experiments give for it only $|\Delta\bar{\kappa}| \leq \mathcal{O}(1)$ [8], and the future LEP II and FNAL data will improve the sensitivity only several times [8]–[10]. Whereas in the unified compositeness framework with $\mathcal{F} = \mathcal{O}(10 \text{ TeV})$, we expect naturally $|\Delta\bar{\kappa}| \leq \mathcal{O}(10^{-3})$, which lies beyond the limits

of the near-future experiments⁷. Thus, multi-TeV energies are needed to enhance these effects.

Summing over the contributions due to all the terms in Eqs. 19 and 20, we get finally for the terms $\Delta M_{\pm 1}$ in Eq. 13 the values of the order of the neglected terms in the VBD case, i.e. $\mathcal{O}(m_W^2/s)$ and $\mathcal{O}(v^2/\mathcal{F}^2)$. This can readily be inferred from the equivalence theorem. Indeed, the operator of Eq. 18 does not contribute to the production of the LL pair of the would-be Goldstone bosons, but at most only to that of the LT pair. Hence, the dominant anomalous TGI contribution is suppressed in the amplitude at least as $\mathcal{O}(m_W/\sqrt{s})$ relative to the LL pair in the VBD case, and the sensitivity to the anomalous TGI vertices is much lower than that to the VBD. So, in fact, the VBD new interactions are the dominant ones even for the maximally enhanced operator of Eq. 18⁸.

Some final comments on the electroweak radiative corrections are in order. Up to now, we discussed only the genuine electroweak radiative corrections. These are the one-loop corrections. They result in two effects. First, they lead in the effective Born approximation to a shift of the SM parameters (including the TGI ones) from their respective tree-level values. These are the dominant universal corrections, which we took into account via the effective coupling constants. Second, they produce the nonuniversal box and vertex corrections. The latter ones are, in general, smaller and so are neglected.

There is one more class of radiative corrections, physically more transparent but quantitatively very important. These are the corrections due to the photon radiation from the colliding positrons and electrons. They constitute the bulk of all the radiative corrections. Their influence could be understood as follows. Due to the photon bremsstrahlung emission the useful initial energy $\sqrt{s}$ reduces down to some $\sqrt{s} - \Delta\sqrt{s}$. This results in an increase of the falling down SM background cross-section, and v.v., in a decrease of the rising new physics cross-section. Hence, the sensitivity to the new physics is lowered⁹.

To evaluate it one has to smear out the resulting cross-section over the actual initial energy spectrum, which could readily be done if required. Let us estimate the effect qualitatively. If the reach for $\mathcal{F}$ scales as $\sqrt[3]{s}$ (at $\int \mathcal{L} dt$ fixed) than $\mathcal{F}$ effectively shifts down to $(1 - \Delta\sqrt{s}/2\sqrt{s})\mathcal{F}$. This results in no more than $\mathcal{O}(10\%)$ reduction of the achievable $\mathcal{F}$, the accuracy being quite enough for the present paper exploratory purposes. These considerations apply irrespective of the final heavy particle states if they are far above the threshold.

⁷Strictly speaking, these values of $|\Delta\tilde{\kappa}|$ are less than or of the order of those expected from the SM one-loop radiative corrections [6]. So, in fact, the later ones should be properly accounted for, prior to comparing with any SM extension in a more thorough study. In the present exploratory study, we neglect them. By the same token, we consider more refined statistical analysis to be unjustified in the given circumstances, too.

⁸The operator of Eq. 18 contributes also to the ZZH anomalous coupling and, hence, to the $e^+e^- \rightarrow ZH$ process. But according to the equivalence theorem, quite similar to the TGI case, this operator influences at high energies only the HZ_T production which is negligible w.r.t. the dominant HZ_L production due to the VBD.

⁹In fact, the influence is somewhat weaker because the most important interference term is energy independent.

Conclusion

The universal residual interactions due to the VBD of the electroweak interactions could be studied via the W^+W^- and ZH boson pair production at the 2 TeV e^+e^- linear collider up to the compositeness scale $\mathcal{F} = \mathcal{O}(25 \text{ TeV})$. This, along with the similar result for the process $e^+e^- \rightarrow \bar{f}f$ at the same collider, could give the first strong evidences in favour of the unified substructure of leptons, quarks and Higgs boson in the deca-TeV region.

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Fig. 1. Process $e^+e^- \rightarrow ZH$: (a) relative deviation in the total cross-section for $P_e = 0$; (b) relative deviation in the total cross-section for $P_e = 0.8$; (c) left-right asymmetry. Here and in what follows, the labels mean corresponding values of the compositeness scale $\mathcal{F}$ in TeV.

Fig. 2. The reach at 95% C.L. for the compositeness scale $\mathcal{F}$, vs. parameter η_1 , via studying the total cross-sections of the processes $e^+e^- \rightarrow ZH$ and W^+W^- .

Fig. 3. Process $e^+e^- \rightarrow W^+W^-$ for $P_e = 0$: (a) relative deviation in the total cross-section integrated over the whole allowed region $|\cos\theta| \leq 0.8$; (b) relative deviation in the total cross-section calculated with the optimal cutoff $-0.8 \leq \cos\theta \leq 0.3$; (c) forward-backward asymmetry.

Fig. 4. Process $e^+e^- \rightarrow W^+W^-$ for $P_e = 0.8$: (a), (b) and (c) are the same as in Fig. 3; (d) left-right asymmetry calculated with the optimal cutoff $-0.8 \leq \cos \theta \leq 0.3$.

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Проверка векторно-бозонной доминантности электрослабых взаимодействий в e^+e^- -аннигиляции в бозонные пары.

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