Color Screening and Dissociation of Charmonium at Finite Temperature and Finite Density *

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Abstract

The Debye screening mass is studied at finite temperature and finite density. The critical temperature and density for the dissociation of the charmonium in deconfined medium are obtained.

PACS numbers: 12.38.M, 14.40.G, 52.25.K

Key words: Debye screening, J/Ψ suppression, QGP, thermo-field dynamics

5W9607

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* Work partly supported by the National Natural Science Foundation of China

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I. INTRODUCTION

It has been proposed by Matsui and Satz[1] that the color screening is related to J/Ψ suppression in nuclear collisions. J/Ψ suppression in relativistic heavy ion collisions as one of the signals for the formation of the quark-gluon plasma(QGP) has attracted much attention from theoretical and experimental physicists. J/Ψ suppression in high energy nuclear collisions has been observed experimentally[2] after the theoretical prediction. In order to have a good understanding of J/Ψ suppression, it is necessary to study the dissociation of the charmonium in the hot and dense deconfined medium. Matsui[3] analysed J/Ψ dissociation condition at finite temperature. However, the effect of finite baryon density was not included in his discussion. We study the dissociation for the charmonium at finite temperature and also finite density in this paper.

In the non-relativistic quark model, the charmonium resonances are described as the $c\bar{c}$ bound states. It is well known that the binding potential at zero temperature is given by the one-gluon-exchange contribution plus the linear confining part

$$V(r, 0) = -\frac{\alpha}{r} + \sigma r. \quad (1)$$

where α is the coupling constant and σ is the string tension coefficient. In the QGP environment, the string tension vanishes due to the ruin of confinement, because of the plasma screening effect, the Coulomb-type part of quark binding potential becomes

$$V(r, r_D(T)) = \frac{\alpha}{r} e^{-r/r_D(T)}, \quad (2)$$

where r_D is the Debye screening length and the Debye screening mass m_D is the inverse of the screening length, $m_D = 1/r_D$.

The potential model in a thermodynamic environment has been suggested by Karsch, Mehr and Satz[4] to study the binding and deconfinement of heavy quark resonances, which gives the potential

$$V(r, m_D(T)) = \sigma r_D(T)(1 - e^{-r/r_D(T)}) - \frac{\alpha}{r} e^{-r/r_D(T)}, \quad (3)$$

where the screening mass m_D (also the screening length) is assumed to be a function of the temperature T . They calculated the screening mass (and the screening length) for the dissociation of heavy quark resonances using a non-relativistic approximation. The critical values of the screening mass (or screening length) for the dissociation of J/Ψ and Ψ' are respectively, $m_D^{dis} = 700, 360$ MeV ($r_D^{dis} = 0.28, 0.548$ fm).

In this paper, the dissociation of the charmonium at finite temperature and finite baryon density is analysed. Using the thermo-field dynamics (TFD), we calculate the static electric screening mass of the gluon which is related to the time-time component of the gluon self-energy at finite temperature and density, and the critical temperature and critical baryon density for charmonium dissociation are obtained.

II. ELECTRIC SCREENING MASS OF THE GLUON AT FINITE TEMPERATURE AND DENSITY

The properties of QCD at finite temperature and density have been extensively studied[5-6]. The electric screening mass for QCD can be obtained by the static infrared limit of the time-time component of the gluon self-energy $\Pi^{00}(q_0 = 0, \vec{q} \rightarrow 0)$. At finite temperature and zero baryon density, let the dynamical quark mass $m_q = 0$, the electric screening mass of the gluon can be calculated based on the perturbative theory, which yields the lowest-order result in the high temperature limit[6]

$$M_{scr}^2 = -\Pi^{00}(q_0 = 0, \vec{q} \rightarrow 0) = \left(\frac{N}{3} + \frac{N_f}{6}\right)g^2T^2, \quad (4)$$

for gauge group $SU(N)$ and N_f fermions.

In order to introduce the baryon density, we would take into account the presence of the dynamical quark with the chemical potential μ , and calculate the Debye screening mass at finite temperature and finite baryon density. Using the Feynman rules in thermo-field dynamics[7-8], it can be proved that the real part of the gluon self-energy in the one-loop approximation is determined by the 1-1 component of the Feynman propagator in TFD[7-9], therefore the self-energy of the gluon given by fig.1 is (in Feynman gauge)

$$\begin{aligned}
\Pi^{\mu\nu}(q) = & -\frac{i}{2}g^2N_f \int \frac{d^4k}{(2\pi)^4} Tr[\gamma^\mu S^{11}(k)\gamma^\nu S^{11}(k+q)] \\
& + 3ig^2Ng^{\mu\nu} \int \frac{d^4k}{(2\pi)^4} \Delta^{11}(k) - \frac{i}{2}g^2N \int \frac{d^4k}{(2\pi)^4} T^{\mu\nu} \Delta^{11}(k)\Delta^{11}(k+q) \\
& + ig^2N \int \frac{d^4k}{(2\pi)^4} (k+q)^\mu \Delta^{11}(k) k^\nu \Delta(k+q) ,
\end{aligned} \tag{5}$$

where

$$T^{\mu\nu} = -g^{\mu\nu}(5q^2 + 2k \cdot q + 2k^2) + 2q^\mu q^\nu - 5q^\mu k^\nu - 5k^\mu q^\nu - 10k^\mu k^\nu .$$

The 1-1 component propagator in TFD is [7-8]

$$S^{11}(k) = (\not{k} + m) \left[\frac{1}{k^2 - m^2 + i\epsilon} + 2\pi i \delta(k^2 - m^2) N_F(k) \right] \tag{6}$$

for fermion with $N_F(k) = \theta(k_0)n_F(k) + \theta(-k_0)\bar{n}_F(k)$, and $\theta(k_0)$ is the step function, $n_F(k)$ and $\bar{n}_F(k)$ are the fermion and antifermion distribution function, respectively. $\Delta^{11}(k)$ in eq.(5) can be written as

$$\Delta^{11}(k) = \frac{1}{k^2 - m_B^2 + i\epsilon} - 2\pi i \delta(k^2 - m_B^2) n_B(k) \tag{7}$$

for boson with mass m_B , where $n_B(k)$ is the boson distribution function. For the gluon field, it is needed to put a prefactor $[-g_{\mu\nu} + \frac{1-\xi}{\xi}k_\mu k_\nu \partial/\partial k^2]$ in front of $\Delta^{11}(k)$, here ξ is a gauge parameter. For the ghost field, the prefactor is -1 .

The self-energy of the gluon at finite temperature and density can be obtained from eqs.(5)-(7) as

$$\begin{aligned}
\Pi^{\mu\nu}(q, T, \rho) = & \frac{1}{2}g^2N_f \int \frac{d^4k}{(2\pi)^3} \tau^{\mu\nu} \left\{ \frac{\delta(k^2 - m_q^2)}{(k+q)^2 - m_q^2} N_F(k) + \frac{\delta[(k+q)^2 - m_q^2]}{k^2 - m_q^2} N_F(k+q) \right\} \\
& - \frac{1}{2}g^2N \int \frac{d^4k}{(2\pi)^3} T^{\mu\nu} \left\{ \frac{\delta(k^2)}{(k+q)^2} n_B(k) + \frac{\delta[(k+q)^2]}{k^2} n_B(k+q) \right\} \\
& - g^2N \int \frac{d^4k}{(2\pi)^3} (k+q)^\mu k^\nu \left\{ \frac{\delta(k^2)}{(k+q)^2} n_B(k) + \frac{\delta[(k+q)^2]}{k^2} n_B(k+q) \right\} \\
& - 3g^2Ng^{\mu\nu} \int \frac{d^4k}{(2\pi)^3} \delta(k^2) n_B(k) ,
\end{aligned} \tag{8}$$

where $\tau^{\mu\nu} = Tr[\gamma^\mu(\not{k} + m_q)\gamma^\nu(\not{k} + \not{q} + m_q)]$. The time-time component of the gluon self-energy Π^{00} in the limit of $q_0 = 0$ and $\vec{q} \rightarrow 0$ yields the screening mass. From eq. (8) and in the limit $q_0 = 0, \vec{q} \rightarrow 0$, one obtains

$$\Pi^{00}(0, \vec{q} \rightarrow 0, T, \rho) = -\frac{1}{3}Ng^2T^2 - \frac{1}{\pi^2}N_f g^2 I_F, \quad (9)$$

where

$$\begin{aligned} I_F &= \int \frac{d|\vec{k}||\vec{k}|^2}{\sqrt{|\vec{k}|^2 + m_q^2}} [n_F(k) + \bar{n}_F(k)] \\ &= \int_0^\infty \frac{x^2 dx}{\sqrt{x^2 + m_q^2}} \left\{ \frac{1}{\exp[(\sqrt{x^2 + m_q^2} - \mu)/T] + 1} + \frac{1}{\exp[(\sqrt{x^2 + m_q^2} + \mu)/T] + 1} \right\}. \end{aligned} \quad (10)$$

The chemical potential μ of quarks is determined by the baryon density ρ as

$$\rho = \frac{\gamma}{3} \int \frac{d^3k}{(2\pi)^3} [n_F(k) - \bar{n}_F(k)], \quad (11)$$

where $\gamma = \text{flavor} \times \text{spin} \times \text{color}$ is the degeneracy factor.

Thus the eclectic screening mass (Debye screening mass) of the gluon is obtained from eq. (9) as

$$M_{scr} = g \left(\frac{1}{3}NT^2 + \frac{1}{\pi^2}N_f I_F \right)^{1/2}. \quad (12)$$

The result of eq.(12) for zero density recovers that given by Landsman et al. based on the real time method[7] and that by Kapusta using the Matsubara formalism[10]. In the zero temperature limit, $n_F(k) = \theta(k_F - k)$, $\bar{n}_F(k) = 0$, eq.(12) becomes

$$M_{scr} = \left(\frac{g^2}{\pi^2} N_f \int_0^{k_F} \frac{dx x^2}{\sqrt{x^2 + m_q^2}} \right)^{1/2}, \quad (13)$$

where $k_F = (\frac{6\pi^2}{\gamma}\rho)^{1/3}$. The Debye screening mass M_{scr} can be calculated numerically using eqs.(12) and (13). In our calculations, we take the coupling constant $\alpha = g^2/4\pi = 0.39$ [3], the quark mass $m_q = 10$ MeV and $\rho_0 = 0.17 \text{ fm}^{-3}$, here ρ_0 is the saturation density of the nuclear matter. Therefore the Debye screening mass in eq.(12) is a function of temperature T for a fixed baryon density. The numerical results of the Debye screening mass with $N_f = 2$ changing with temperature for the different baryon densities are shown in fig. 2. We use the baryon density $\rho = C_x \rho_0$ as an input in our calculations, the number 1, 3, 6, 9 in fig. 2 are different C_x values which represent different baryon densities, the

dotted line is the result of $\rho = 0$. But the solid line in fig. 2 is the result of $N_f = 0$. It is interesting to see in fig. 2 that the density effect on the screening mass is significant at low temperature and increases with increasing ρ . However, the screening mass is not sensitive to the change of the baryon density in the high temperature region.

III. CRITICAL TEMPERATURE AND DENSITY FOR CHARMONIUM DISSOCIATION

The analysis of the binding and deconfinement of heavy quark resonances can provide information about the color screening in the QGP. The authors in ref.[4] have applied a non-relativistic potential model to study the dissolution and obtained the critical values of the screening mass M_c for heavy quark resonances, beyond which there are no bound states. We reproduced their results by solving the schrödinger equation based on the potential of eq.(3), and obtained $M_c = 700$ MeV for J/Ψ and $M_c = 360$ MeV for Ψ' . As stated in ref[4], $M_c = 700$ MeV for J/Ψ would lead to a diverging radius, which means that $M_c = 700$ MeV is the upper limit value for J/Ψ dissociation. The authors of ref.[4] also obtained $M_c = 500$ MeV for J/Ψ dissociation based on the semi-classical approximation and claimed that beyond this value of M_c all $c\bar{c}$ binding would be dissolved.

We are interested in the critical temperature T_c of the medium, to which the obtained M_c corresponds. As shown in our results, the screening mass is related to both temperature and density. In order to see how an critical temperature T_c changes with the critical baryon density ρ_c at a fixed M_c , we give the relation between the critical temperature T_c and baryon density ρ_c for the charmonium at several fixed M_c which are chosen based on the result of ref.[4]. Our results are shown in fig. 3. The solid curves are results for $N_f = 2$ and the dashed curves for $N_f = 3$. The crosses on T axis are the results for $N_f = 0$ corresponding to the critical temperature values indicated at the nearby lower curves for $N_f \neq 0$.

Our results in fig. 3 show that the critical baryon density at low temperature seems

too large, especially for $M_c = 700$ MeV, where ρ_c/ρ_0 reaches 50. On the other hand, the critical temperature for $M_c = 700$ MeV at low baryon density is also too large. And the critical temperature for $N_f = 2$ is larger than the one for $N_f = 3$. Looking at the result of $N_f = 2$ for $M_c = 500$ MeV, the critical density of ρ_c/ρ_0 is about 18 which is acceptable, while the critical temperature at low baryon density region is about 200 MeV which is consistent with the result of Lattice QCD[11]. In the deconfined QGP, the contribution to the screening mass from quarks is dominant and the contribution from gluon is very small at low temperature. However, the contributions to the screening mass are from both quarks and gluon at high temperature. Therefore, the color screening effect is much stronger at high temperature, however, is not sensitive to the change of the baryon density.

IV. CONCLUSIONS

We have studied the effects of temperature and density on the screening mass in the deconfined QGP. The critical temperature and density for the charmonium dissociation are calculated based on the charmonium potential model and thermo-field dynamics. Our calculations show that the critical density at low temperature turns out to be very large. This indicates that the density does not play the same role [12] as the temperature in the deconfined QGP. The color screening effects in the QGP are remarkable at high temperature and low baryon density regions.

Therefore, if J/Ψ suppression in relativistic heavy-ion collisions is mainly due to the color screening, it is likely to be observed in the regions of high temperature and low baryon density, i.e. in the central rapidity region. Detailed calculations of the screening mass for the dissociation of heavy quark resonances are expected.

ACKNOWLEDGMENTS

The authors would like to thank H. C. Chiang for useful discussions.

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Figure Captions

Fig. 1. Feynman diagrams which contribute to the self-energy of the gluon. The full line denotes quark, the curl and dotted lines denote gluon and ghost, respectively.

Fig. 2 The screening mass M_{scr} varies with temperature T for different baryon densities with $N_f = 2$, the dotted line is the contribution of $\rho = 0$, the number 1, 3, 6, 9 denote different densities. The solid line is the result of $N_f = 0$.

Fig. 3 The relation between critical temperature and critical density for the dissociation of J/Ψ and Ψ' . The solid curves are results of $N_f = 2$ and dashed curves are that of $N_f = 3$.

fig. 1

fig. 2