Current, Pion and Photon Transitions between Heavy Baryons

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ABSTRACT

I discuss the structure of current-induced bottom baryon to charm baryon transitions, and the structure of pion and photon transitions between heavy charm or bottom baryons in the Heavy Quark Symmetry limit as $m_Q \to \infty$. The emphasis is on the structural similarity of the Heavy Quark Symmetry predictions for the three types of transitions. The discussion involves the ground state $s$-wave heavy baryons as well as the excited $p$-wave heavy baryon states. Using a constituent quark model picture for the light diquark system with an underlying $SU(2N_f) \otimes O(3)$ symmetry one arrives at a number of new predictions that go beyond the Heavy Quark Symmetry predictions.

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1 Introduction

Because of the initial abundance of data on heavy charm and bottom mesons the attention of experimentalists and theoreticians had initially been directed towards applications of the Heavy Quark Effective Theory (HQET) to the meson sector. In the meantime the situation has considerably changed and data on heavy baryons and their decay properties are starting to become available in impressive amounts. Since theoretical results on the description of semileptonic, one-pion and photon decays of heavy baryons are widely dispersed in the literature it is worthwhile to review the necessary theoretical HQET framework to describe these three types of decays in a comprehensive manner. The emphasis is on the structural similarity of the HQS description of these decays. In this review I am only concerned with the leading order contributions to these decays, leading in terms of the inverse heavy quark mass expansion provided by HQET. At the end of my presentation I discuss possibilities to further constrain the Heavy Quark Symmetry (HQS) structure of the respective decays by resolving the light diquark transitions in terms of a constituent quark model description of the light diquark transitions with an underlying $SU(2N_f) \otimes O(3)$ symmetry.

2 Heavy Baryon Spin Wave Functions

Let us begin by constructing the heavy baryon spin wave functions that enter into the descriptions of heavy baryon decays. A heavy baryon is made up of a light diquark system $(qq)$ and a heavy quark $Q$. The light diquark system has bosonic quantum numbers $j^P$ with total angular momentum $j = 0, 1, 2 \ldots$ and parity $P = \pm 1$. To each diquark system with spin-parity $j^P$ there is a degenerate heavy baryon doublet with $J^P = (j \pm \frac{1}{2})^P$ ($j = 0$ is an exception). It is important to realize that the HQS structure of the heavy baryon states is entirely determinated by the spin-parity $j^P$ of the light diquark system. The requisite angular momentum coupling factors can be read off from the coupling scheme

$$j^P \otimes \frac{1}{2}^+ \Rightarrow J^P. \quad (1)$$
Apart from the angular momentum coupling factors the dynamics of the light system is completely decoupled from the heavy quarks.

Let us cast these statements into a covariant framework in which the heavy baryon wave function \( \Psi \) describes the amplitude of finding the light diquark system and the heavy quark in the heavy baryon. The covariant equivalent of the coupling scheme Eq. (1) is then given by

\[
\Psi = \hat{\phi}_{\mu_1 \cdots \mu_j} \psi^{\mu_1 \cdots \mu_j},
\]

where \( \hat{\phi}_{\mu_1 \cdots \mu_j} \) stands for the tensor representation of the spin-parity \( j^P \) diquark state and \( \psi^{\mu_1 \cdots \mu_j} \) represents the heavy-side baryon spin wave function (in short: heavy baryon wave function) coupling the heavy quark to the heavy baryon. Let us be more specific. If

\[
|J^P, m_j\rangle = \sum_{m_j+m_Q=m_J} \langle j^P, m_j; \frac{1}{2}^+, m_Q|J^P, m_J\rangle|j^P, m_j\rangle|\frac{1}{2}^+, m_Q\rangle
\]

defines the static quark model wave function, the C.G. coefficients determining the heavy quark - light diquark content of the heavy baryon can be obtained in covariant fashion from the heavy baryon spin wave function by the covariant projection

\[
\langle j^P, m_j; \frac{1}{2}^+, m_Q|J^P, m_J\rangle = \varepsilon_{\mu_1 \cdots \mu_j}^{*}(m_j)\bar{u}(m_Q)\psi^{\mu_1 \cdots \mu_j}(m_J).
\]

The r.h.s. of Eq. (4) can be evaluated for any velocity \( v_\mu \) of the heavy baryon which, at leading order, equals the velocity of the heavy quark and the diquark system. Details including questions of normalization can be found in [1,5]. Differing from [1] we have normalized the spinors appearing in Eq. (4) to 1 and not to \( 2M \) and \( 2M_Q \) as in [1]. It is not difficult to construct the appropriate heavy baryon spin wave functions using the heavy quark on-shell constraint \( \psi^{\mu_1 \cdots \mu_j} = \psi^{\mu_1 \cdots \mu_j} \) and the appropriate normalization condition. In Table 1 (fourth column) we have listed a set of correctly normalized heavy baryon spin wave functions that are associated with the diquark states \( j^P = 0^+, 1^+, 0^-, 1^-, 2^- \).

Next we turn our attention to the question of which low-lying heavy baryon states can be expected to exist. From our experience with light baryons and light mesons we know that one can get a reasonable description of the light particle spectrum in the constituent quark model picture. This is particularly true for the enumeration of states, their spins and their parities. As much as we know up to now, gluon degrees of freedom
do not seem to contribute to the particle spectrum. It is thus quite natural to try the
same constituent approach to enumerate the light diquark states, their spins and their
parities.

From the spin degrees of freedom of the two light quarks one obtains a spin 0
and a spin 1 state. The total orbital state of the diquark system is characterized by two
angular degrees of freedom which we take to be the two independent relative momenta \( k = \frac{1}{2}(p_1-p_2) \) and \( K = \frac{1}{2}(p_1+p_2-2p_3) \) that can be formed from the two light quark momenta \( p_1 \)
and \( p_2 \) and the heavy quark momentum \( p_3 \). The \( k \)-orbital momentum describes relative
orbital excitations of the two quarks, and the \( K \)-orbital momentum describes orbital
excitations of the center of mass of the two light quarks relative to the heavy quark. The
\( (k,K) \)-basis is quite convenient in as much as it allows one to classify the diquark states
in terms of \( SU(2N_f) \otimes O(3) \) representations as will be discussed later on. Table 1 lists
all ground state \( s \)-wave and excited \( p \)-wave heavy baryon wave functions as they occur in
the constituent approach to the light diquark excitations. They are grouped together in
terms of \( SU(2N_f) \otimes O(3) \) representations with \( N_f = 2(u,d) \). The \( s \)-wave states are in the
\( 10 \otimes 1 \) representation, and the \( p \)-wave states are in the \( 10 \otimes 3 \) and \( 6 \otimes 3 \) representation
of \( SU(4) \otimes O(3) \) for the \( K \)- and \( k \)-multiplets, respectively. Apart from the ground state
\( s \)-wave baryons one thus has altogether seven \( \Lambda \)-type \( p \)-wave states and seven \( \Sigma \)-type \( p \)-
wave states. The analysis can easily be extended to the case \( SU(6) \otimes 0(3) \) bringing in the
strangeness quark in addition.

Let us mention that, in the charm sector the states \( \Lambda_c(2285) \) and \( \Sigma_c(2453) \) are
well established while there is first evidence for the \( \Sigma_c^*(2510) \) state. Two excited states
\( \Lambda_c^{**}(2593) \) and \( \Lambda_c^{**}(2627) \) have been seen which very likely correspond to the two \( p \)-wave
states making up the \( \Lambda_{cK_1}^{**} \) HQS doublet. The charm-strangeness states \( \Xi_c(2470) \) and
\( \Omega_c(2720) \) as well as the \( \Xi_c^*(2643) \) have been seen. First evidence was presented for the
\( J^P = \frac{1}{2}^+ \) state \( \Xi_c^*(2570) \) with the flavour configuration \( c\{sq\} \). Thus almost all ground
state charm baryons have been seen including two \( p \)-wave states. In the bottom sector
the \( \Lambda_b(5640) \) has been identified as well as the \( \Sigma_b(5713) \) and the \( \Sigma_b^*(5869) \). Some indirect
evidence has been presented for the \( \Xi_b(5800) \).
3 Generic Picture of Current, Pion and Photon Transitions

In Fig. 1 we have drawn the generic diagrams that describe $b \to c$ current transitions, and $c \to c$ pion and photon transitions between heavy baryons in the HQS limit. The heavy-side and light-side transitions occur completely independent of each other (they “factorize”) except for the requirement that the heavy side and the light side have the same velocity in the initial and final state, respectively, which are also the velocities of the initial and final heavy baryons. The $b \to c$ current transition induced by the flavour-spinor matrix $\Gamma$ is hard and accordingly there is a change of velocities $v_1 \to v_2$, whereas there is no velocity change in the pion and photon transitions. The heavy-side transitions are completely specified whereas the light-side transitions $j_1^{P_1} \to j_2^{P_2}$, $j_1^{P_1} \to j_2^{P_2} + \pi$ and $j_1^{P_1} \to j_2^{P_2} + \gamma$ are described by a number of form factors or coupling factors which parametrize the light-side transitions. The pion and the photon couple only to the light side. In the case of the pion this is due to its flavour content. In the case of the photon the coupling of the photon to the heavy side involves a spin flip which is down by $1/m_Q$ and thus the photon couples only to the light side in the Heavy Quark Symmetry limit.

Referring to Fig. 1 we are now in the position to write down the generic expressions for the current, pion and photon transitions according to the spin-flavour flow depicted in Fig. 1. One has ($\omega = v_1 \cdot v_2$)

**current transitions:**

$$\bar{\psi}_2^{\nu_1 \cdots \nu_2} \Gamma_{\psi_1^{\mu_1 \cdots \mu_1}} \left( \sum_{i=1}^{N} f_i(\omega) t^{\nu_1 \cdots \nu_2;\mu_1 \cdots \mu_1}_i \right)$$

$$n_1 \cdot n_2 = 1 \quad N = j_{\text{min}} + 1$$

$$n_1 \cdot n_2 = -1 \quad N = j_{\text{min}}$$

**pion transitions:**

$$\bar{\psi}_2^{\nu_1 \cdots \nu_2} \psi_1^{\mu_1 \cdots \mu_1} \left( \sum_{i=1}^{N} f_i^{\pi} t^{\nu_1 \cdots \nu_2;\mu_1 \cdots \mu_1}_i \right)$$

$$n_1 \cdot n_2 = 1 \quad N = j_{\text{min}}$$

$$n_1 \cdot n_2 = -1 \quad N = j_{\text{min}} + 1$$
photon transitions:
\[ \bar{\psi}_{2}^{\nu_{1}\cdots \nu_{j_{2}}} \psi_{1}^{\mu_{1}\cdots \mu_{j_{1}}} \left( \sum_{i=1}^{N} f_{i}^{\gamma} T_{\nu_{1}\cdots \nu_{j_{2}}; \mu_{1}\cdots \mu_{j_{1}}}^{i} \right) \]  

(7)

\[ j_{1} = j_{2} \quad N = 2j_{1} \]

\[ j_{1} \neq j_{2} \quad N = 2j_{\text{min}} + 1 \]

where the \( \psi^{\mu_{1}\cdots \mu_{j}} \) are the heavy baryon spin wave functions introduced in Sec. 2.

In each of the above cases we have also given the result of counting the number \( N \) of independent form factors or coupling factors. These are easy to count by using either helicity amplitude counting or \( LS \) partial wave amplitude counting. In the case of current and pion transitions the counting involves the normalities of the light-side diquarks which is defined by \( n = (-1)^{j_{P}} \).

In the case of the current transitions the tensors \( T_{\nu_{1}\cdots \nu_{j_{2}}; \mu_{1}\cdots \mu_{j_{1}}}^{i} \) appearing in Eq. (5) have to be build from the vectors \( v_{1}^{\nu_{1}} \) and \( v_{2}^{\mu_{1}} \), the metric tensors \( g_{\mu_{i}\nu_{k}} \) and, depending on parity, from the Levi-Civita object \( \epsilon(\mu_{i}\nu_{k} v_{1} v_{2}) := \epsilon_{\mu_{i}\nu_{k}\alpha\beta} v_{1}^{\alpha} v_{2}^{\beta} \). For the pion transitions in Eq. (6) the \( (j_{1} + j_{2}) \)-rank tensors \( T_{\nu_{1}\cdots \nu_{j_{2}}; \mu_{1}\cdots \mu_{j_{1}}}^{i} \) describing the light-side transitions \( j_{1}^{P_{1}} \rightarrow j_{2}^{P_{2}} + \pi \) have to be composed from the building blocks \( g_{\perp \mu\nu} = g_{\mu\nu} - v_{\mu} v_{\nu} \), the pion momentum \( p_{\perp \mu} = p_{\mu} - v \cdot v_{\mu} \) and, depending on parity, from the Levi-Civita tensor \( \epsilon(\mu_{i}\nu_{k} p v) \). Finally, in the photon transition case \( j_{1}^{P_{1}} \rightarrow j_{2}^{P_{2}} + \gamma \) Eq. (7) one has to use the field strength tensor \( F_{\alpha\beta} = k_{\alpha} \epsilon_{\beta} - k_{\beta} \epsilon_{\alpha} \) or, depending on parity, its dual \( \tilde{F}_{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\beta\gamma\delta} F^{\gamma\delta} \) in order to guarantee a gauge invariant coupling of the photon to the light side. As in the current and pion transition case further building blocks for the diquark transition tensor are the metric tensor, the velocity \( v_{\alpha} \) and the photon momentum \( k_{\mu} \). The number of independent tensors that can be written down in each of the three cases is necessarily identical to the numbers listed in Eqs. (5), (6) and (7). Lack of space prevents us from giving the explicit forms of these tensors. They can be found in [1].

The generic expressions Eq. (5), Eq. (6) and Eq. (7) completely determine the HQS structure of the current, pion and photon transition amplitudes. It is not difficult to work out relations between rates, angular decay distributions etc. from these expressions.

It is well worth mentioning that all three covariant coupling expressions can also be written down in terms of Wigner’s 6-\( j \) symbol calculus [1,2] as can be appreciated from

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the discussion in Sec. 2 (see Eqs. (2) and (3)). For example, looking at the pion transition in Fig. 1 one sees that one has to perform altogether three angular couplings. They are

(i) \( j_1^1P_1 \otimes \frac{1}{2}^+ \Rightarrow J_1^P \)
(ii) \( j_2^2P_2 \otimes \frac{1}{2}^+ \Rightarrow J_2^P \)
(iii) \( J_2^2P_2 \otimes L_\pi \Rightarrow J_1^1P_1 \)

where \( L_\pi = l_\pi \) is the orbital momentum of the pion and \( J_1^P \) and \( J_2^P \) denote the \( J^P \) quantum numbers of the initial and final baryons. This is a coupling problem well-known from atomic and nuclear physics and the problem is solved by Wigner’s 6-\( j \) symbol calculus. One finds \([1,2]\)

\[
M_\pi^0(J_1^z J_1^z \rightarrow J_2^z J_2^z + L_\pi m) = M_{L_\pi} (-1)^{L_\pi+j_2+J_2^z} (2j_1+1)^{1/2} (2J_2+1)^{1/2} \left\{ \begin{array}{ccc} j_2 & j_1 & L_\pi \\ J & J_2 & \frac{1}{2} \end{array} \right\} \langle Lm J_2^z | J_1^z \rangle,
\]

where the expression in curly brackets is Wigner’s 6-\( j \) symbol and \( \langle L_\pi M J_2^z | J_1^z \rangle \) is the Clebsch-Gordan coefficient coupling \( L_\pi \) and \( J_2 \) to \( J_1 \). \( M_{L_\pi} \) is the reduced amplitude of the one-pion transition. It is proportional to the invariant coupling \( f_\pi \) occurring in the covariant expansion in Eq. (6).

Let us, for example, calculate the doublet to doublet transition rates for e.g. \{\Lambda^{**}_{Qk2}\} \rightarrow \{\Sigma_Q\} + \pi. The rates are in the ratios 4 : 14 : 9 : 9 as represented in Fig. 2 \([1,3]\). This result can readily be calculated using the 6-\( j \) formula Eq. (9) and some standard orthogonality relations for the 6-\( j \) symbols. The corresponding calculation in the covariant approach involves considerably more labour. Also, the result “4 + 14 = 9 + 9” for doublet to doublet one-pion transitions is a general result which again can easily be derived using the 6-\( j \) approach\([1]\).
4 \( SU(2N_f) \otimes O(3) \) Structure of the Light-Side Transitions

Interest in the constituent quark model has recently been rekindled by the discovery (or rediscovery) that two-body spin-spin interactions between quarks are non-leading in \( 1/N_C \), at least in the baryon sector [4]. Thus, to leading order in \( 1/N_C \), light quarks behave as if they were heavy and a classification of a light quark system in terms of \( SU(2N_f) \otimes O(3) \) symmetry multiplets makes sense. Transitions between light quark systems are parametrized in terms of a set of one-body operators whose matrix elements are then evaluated between the \( SU(2N_f) \otimes O(3) \) multiplets.

Let us discuss the necessary steps for the implementation of the light-side \( SU(2N_f) \otimes O(3) \) symmetry in the current transition case. The relevant one-body operators are given by [5]

\[ s\text{-wave to } s\text{-wave:} \]
\[ O = A(\omega) \cdot \mathbb{1} \otimes \mathbb{1} \quad (10) \]

\[ s\text{-wave to } p\text{-wave:} \]
\[ (l_K = 1; l_k = 0), \ (l_K = 0; l_k = 1) \text{ resp.)} \]
\[ O_K^\alpha = A_K(\omega)v_1^\alpha \mathbb{1} \otimes \mathbb{1} + B_K(\omega)(\mathbb{1} \otimes \gamma^\alpha + \gamma^\alpha \otimes \mathbb{1}) \]
\[ O_k^\alpha = A_K(\omega)v_1^\alpha \mathbb{1} \otimes \mathbb{1} + B_k(\omega)(\mathbb{1} \otimes \gamma^\alpha - \gamma^\alpha \otimes \mathbb{1}) \]  

(11)

Operators of the type \( \gamma^\mu \otimes \gamma^\mu \) or \( v_1^\alpha \gamma^\mu \otimes \gamma^\mu \) are two-body operators and will therefore not be included in our discussion. The reduced form factors in Eqs. (10) and (11) depend on the velocity transfer variable \( \omega \) and are unknown functions except for the normalization condition \( A(1) = 1 \) in Eq. (10). The operators \( \mathbb{1} \otimes \mathbb{1} \) and \( v_1^\alpha \mathbb{1} \otimes \mathbb{1} \) do not couple angular and spin degrees of freedom. These were the operators used some thirty years ago when the consequences of the collinear symmetry \( SU(6)_W \) or \( \tilde{U}(12) \) were worked out. In the following we shall therefore refer to the \( A \)-type operators as the collinear operators. The operators \( (\mathbb{1} \otimes \gamma^\alpha \pm \gamma^\alpha \otimes \mathbb{1}) \) on the other hand introduce spin-orbit coupling interactions and are called spin-orbit operators.

The matrix elements of the one-body operators in Eqs. (10) and (11) can be readily evaluated using the light-side spin wave functions in Table 1. For the \( s\)-wave to \( s\)-wave
transition the relevant matrix element is given by
\[
A(\omega) \hat{\phi}_{\alpha \beta}^{\mu_1 \cdots \mu_j} \hat{\phi}_{\alpha \beta}^{\nu_1 \cdots \nu_j}.
\] (12)

There are altogether three ground state to ground state form factors or Isgur-Wise functions, one for the \( \Lambda_b \to \Lambda_c \) transition and two for the \( \{\Sigma_b\} \to \{\Sigma_c\} \) transitions. Equation (12) tells us that they can all be expressed in terms of the single form factor \( A(\omega) \), where \( A(1) = 1 \) at zero recoil. In fact the current transition amplitudes are given by [1,5,6]

\[
\Lambda_b \to \Lambda_c : \quad M^\Lambda = \bar{u}_2 \Gamma^\Lambda u_1 \frac{\omega + 1}{2} A(\omega) \tag{13}
\]

\[
\{\Sigma_b\} \to \{\Sigma_c\} : \quad M^\Lambda = \bar{\psi}_2 T^\Lambda \psi_1 \left( -\frac{\omega + 1}{2} g_{\mu \nu} + \frac{1}{2} v_{1 \nu} v_{2 \mu} \right) A(\omega)
\]

where \( \Gamma^\Lambda = \gamma^\Lambda (1 - \gamma_5) \) in the Standard Model. The same result has been obtained by C.K.Chow by analyzing the large \( N_C \) limit of QCD [7].

When doing a partial wave analysis, the \( \Sigma \)-type transitions can be seen to result from pure \( L = 0 \) diquark transitions. This is a testable prediction in as much as the population of the helicity states in the decay baryon is fixed resulting in a characteristic angular decay pattern of the subsequent decays. More difficult is a test of the relation between the \( \Lambda \)-type and the \( \Sigma \)-type form factors. In the test one would have to compare \( \Lambda_b \to \Lambda_c \) and \( \Omega_b \to \{\Omega_c, \Omega^*_c\} \) transitions (where there are additional \( SU(3) \) breaking effects), since these are the transitions that are experimentally accessible. The \( \Sigma_b \to \{\Sigma_c, \Sigma^*_c\} \) branching fraction is expected to be too small to be measurable.

For the transitions to the \( p \)-wave charm baryon states one similarly reduces the number of reduced form factors when invoking \( SU(2N_f) \otimes O(3) \) symmetry in addition to HQS. For the transition into the \( K \)-multiplet one has a reduction from five HQS reduced form factors to the two form factors \( A_K(\omega) \) and \( B_K(\omega) \) in Eq. (11), whereas for transitions into the \( k \)-multiplet one can relate two HQS reduced form factors to the one spin-orbit form factor \( B_k(\omega) \) [5]. The one-pion and photon transitions can be treated in a similar manner. Again one finds a significant simplification of the HQS structure, i.e. the number of coupling factors is reduced from those listed in Eqs. (6) and (7) when \( SU(2N_f) \otimes O(3) \) is invoked in addition to HQS. Results for the one-pion transitions can be found in [8]. Corresponding results for the photon transitions are presently worked out.
5 Concluding Remarks

We have studied the consequences of Heavy Quark Symmetry for current, pion and photon transitions between heavy baryons. For the three types of transitions we discussed how the most general Heavy Quark Symmetry structure can be further simplified by invoking a constituent quark model $SU(2N_f) \otimes O(3)$ symmetry for the light-side transition. All of these predictions lead to testable results for rates and angular decay distributions. The future will show how well these predictions work.

References

[1] J.G. Körner, M. Krämer and D. Pirjol,


Table Captions

Tab. 1: Spin wave functions (s.w.f.) of heavy Λ-type and Σ-type s- and p-wave heavy baryons

Figure Captions

Fig. 1: Generic picture of bottom to charm current transitions, and pion and photon transitions in the charm sector in the HQS limit $m_Q \to \infty$

Fig. 2: One-pion transition strengths for the transitions $\left\{ \Lambda_{QK}^{**} \right\} \to \left\{ \Sigma_Q \right\} + \pi$. Degeneracy levels are split for illustrative purposes
\[
\begin{array}{|c|c|c|c|}
\hline
& \text{light side s.w.f.} & j^P & \text{heavy side s.w.f.} \\
\hline
& \hat{\phi}_{\mu_1 \ldots \mu_j} & j^P & \psi_{\mu_1 \ldots \mu_j} \\
\hline
\text{s-wave states } (l_k = 0, \ l_K = 0) \\
\Lambda_Q & \hat{\chi} & 0^+ & u & \frac{1}{2}^+ \\
\{\Sigma_Q\} & \chi^{1\mu_1} & 1^+ & \frac{1}{\sqrt{3}} \gamma^\perp_{\mu_1} \gamma_5 u & \frac{1}{2}^+ \\
& & & u_{\mu_1} & \frac{3}{2}^+ \\
\hline
\text{p-wave states } (l_k = 0, \ l_K = 1) \\
\{\Lambda^{**}_{QK1}\} & \hat{\chi}^0 K^{\mu_1}_\perp & 1^- & \frac{1}{\sqrt{3}} \gamma^\perp_{\mu_1} \gamma_5 u & \frac{1}{2}^- \\
& \frac{1}{\sqrt{3}} \chi^1 \cdot K_\perp & 0^- & u & \frac{1}{2}^- \\
\Sigma^{**}_{QK0} & \Sigma^{**}_{QK1} & \frac{i}{\sqrt{2}} \varepsilon(\mu_1 \chi^1 K_\perp v) & 1^- & \frac{1}{\sqrt{3}} \gamma^\perp_{\mu_1} \gamma_5 u & \frac{1}{2}^- \\
& & & u_{\mu_1} & \frac{3}{2}^- \\
\{\Sigma^{**}_{QK2}\} & \frac{1}{2} \{\chi^{1,\mu_1} K^{\mu_2}_\perp\}_0 & 2^- & \frac{1}{\sqrt{10}} \gamma_5 \gamma^\perp (\mu_1 u_{\mu_2})_0 & \frac{3}{2}^- \\
& & & u_{\mu_1 \mu_2} & \frac{5}{2}^- \\
\hline
\text{p-wave states } (l_k = 1, \ l_K = 0) \\
\{\Sigma^{**}_{Qk1}\} & \chi^0 K^{\mu_1}_\perp & 1^- & \frac{1}{\sqrt{3}} \gamma^\perp_{\mu_1} \gamma_5 u & \frac{1}{3}^- \\
& \frac{1}{\sqrt{3}} \chi^1 \cdot k_\perp & 0^- & u & \frac{1}{3}^- \\
\Lambda^{*}_{Qk0} & \frac{1}{\sqrt{2}} \varepsilon(\mu_1 \chi^1 k_\perp v) & 1^- & \frac{1}{\sqrt{3}} \gamma^\perp_{\mu_1} \gamma_5 u & \frac{1}{3}^- \\
& & & u_{\mu_1} & \frac{3}{2}^- \\
\{\Lambda^{*}_{Qk1}\} & \frac{1}{2} \{\chi^{1,\mu_1} k^{\mu_2}_\perp\}_0 & 2^- & \frac{1}{\sqrt{10}} \gamma_5 \gamma^\perp (\mu_1 u_{\mu_2})_0 & \frac{5}{2}^- \\
& & & u_{\mu_1 \mu_2} & \frac{5}{2}^- \\
\hline
\end{array}
\]

Table 1