Double beta decay in left-right symmetric models

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ABSTRACT

Left-right symmetric models provide a natural framework for neutrinoless double beta ($0
\nu \beta \beta$) decay. In the analysis of $0\nu \beta \beta$ decay in left-right symmetric models, however, it is usually assumed that all neutrinos are light. On the other hand, heavy right-handed neutrinos appear quite naturally in left-right symmetric models and should therefore not be neglected. Assuming the existence of at least one right-handed heavy neutrino, absence of $0\nu \beta \beta$ decay of $^{76}$Ge currently provides the following limits on the mass and mixing angle of right-handed $W$-bosons: $m_{W_R} \geq 1.1$ TeV and $\tan(\zeta) \leq 4.7 \times 10^{-3}$ for a particular value of the effective right-handed neutrino mass, $\langle m_N^{V} \rangle = 1$ TeV, and in the limit of infinitely massive doubly charged Higgs ($\Delta^{-+}$). The effects of the inclusion of the Higgs triplet on $0\nu \beta \beta$ decay are also discussed.

PACs: 11.30, 12.30, 13.10, 13.15, 14.60, 14.80, 23.40

keywords: Left-right symmetric models, double beta decay, neutrino mass, right-handed $W$-bosons, Higgs triplet models

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Left-right symmetric models (LR) [1] aim at explaining two of the most puzzling questions of the standard model (SM), both of which are intimately related to neutrinoless double beta decay ($0\nu\beta\beta$) [2, 3]; i.) The weak interaction violates parity, and ii.) in the standard model neutrino masses are zero. Especially if current hints on finite neutrino masses are correct, LR models provide a very attractive explanation for their small values - when compared to those of the charged leptons - via the well-known seesaw mechanism [4].

Of course, $0\nu\beta\beta$ decay has been studied in connection with LR models by many theoretical groups before, see [5] for reviews. However, the analysis of $0\nu\beta\beta$ decay is usually either restricted to the case where all neutrinos are light [3, 6, 7] or simplified by considering only left-handed neutrinos [5]. Although both approximations look reasonable from a standard model point of view, the situation is very different in LR models in general. Actually, taking the see-saw mechanism as a motivation for LR models one has to expect the existence of some heavy, right-handed neutrino.

The importance of heavy right-handed neutrinos for $0\nu\beta\beta$ decay has been pointed out by Mohapatra [8], while Doi and Kotani [9] derived a quite general decay rate, keeping terms for both left- and right-handed neutrinos. Both of these papers, however, are not complete. While Mohapatra [8] considered only the contribution proportional to $(m_W^2/m_W^2)$, Doi and Kotani [9] did not calculate the relevant nuclear matrix elements. In view of the experimental progress on double beta decay [10, 11] we therefore felt motivated to reconsider $0\nu\beta\beta$ decay in LR models and derive constraints on the various parameters of the decay rate in a more general way. For this purpose we have calculated matrix elements in the limit of heavy neutrino exchange in a pn-QRPA model [7, 12].

Furthermore, we discuss modifications of the formalism once the contribution of the Higgs triplet is taken into account. Assuming the validity of the SM gauge group and simply adding an Higgs triplet to the particle content opens up new decay channels for $0\nu\beta\beta$ decay [13], which however were shown to be negligible by Schechter and Valle [14], Wolfenstein [15] and Haxton et al. [16]. Again, the situation is different in LR models. While an Higgs triplet is fairly exotic an extension of the SM, in LR models it could provide an attractive explanation of the Majorana nature of the neutrino [17]. Moreover, Rizzo [17] has argued that the contribution of the doubly-charged Higgs to the inverse $0\nu\beta\beta$ decay is a necessary ingredient to preserve the unitarity of the cross section. Thus, although of quite modest numerical importance for limits on $W_R$ in usual $0\nu\beta\beta$ decay, as we will show at the end of this work, we felt

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the necessity to include the Higgs triplet in our analysis.

As a starting point for the calculation the following effective Hamiltonian (in the notation of [3]) is used:

\[ H_{CC}^W = \frac{G}{\sqrt{2}} \left\{ J_{\mu L}^\dagger j_{\mu L}^- + \kappa J_{\mu R}^\dagger j_{\mu R}^- + \eta J_{\mu L}^\dagger j_{\mu R}^- + \lambda J_{\mu R}^\dagger j_{\mu R}^- \right\}. \] (1)

Here, \( J_{\mu L/R} \) and \( j_{\mu L/R}^- \) are left- and right-handed hadronic and leptonic currents, respectively. \( \kappa, \eta \) and \( \lambda \) are the right-handed parameters, defined such that the SM charged weak current Hamiltonian is obtained in the limit when \( \kappa, \eta \) and \( \lambda \) approach zero [3].

Since \( \lambda, \eta \ll 1 \) one could think of deriving the decay rate considering only contributions of \( \lambda \) and \( \eta \) in lowest order. However, such a procedure leads to the neglect of important terms. In general, keeping also higher order terms, the decay rate can be written as a fourth-order polynomial in \( \lambda \) and \( \eta \) as derived in [9]. However, to separate the particle from the nuclear physics part of the calculation, it is convenient to assume that there are no neutrinos with mass eigenstates in the range of \( O(10-1000) \) [MeV]. Using this well-motivated assumption, after some lengthy but straightforward calculation, we write the inverse half-life for \( 0\nu\beta\beta \) decay in a factorized form as [18],

\[
\left[ T_{1/2}^{0\nu\beta\beta}(0^+ \rightarrow 0^+) \right]^{-1} = \left( \frac{\langle m_\nu \rangle}{m_e} \right)^2 C_{LL}^{mm} + \left( \frac{m_p}{\langle m_N \rangle} \right)^2 C_{NN}^{mm} + \langle \lambda \rangle^2 C_{LL}^{m\lambda} + \langle \eta \rangle^2 C_{LL}^{m\eta} + \langle \xi \rangle^2 C_{NN}^{m\xi} + \left( \frac{\langle m_\nu \rangle}{m_e} \right) \langle \lambda \rangle C_{mLL}^{N} + \left( \frac{\langle m_\nu \rangle}{m_e} \right) \langle \eta \rangle C_{mLL}^{N} \\
+ \left( \frac{m_p}{\langle m_N \rangle} \right) \langle \xi \rangle C_{mLL}^{N} + \left( \frac{m_p}{\langle m_N \rangle} \right) \langle \lambda \rangle C_{mNN}^{L} + \left( \frac{m_p}{\langle m_N \rangle} \right) \langle \eta \rangle C_{mNN}^{L} + \left( \frac{m_p}{\langle m_N \rangle} \right) \langle \xi \rangle C_{mNN}^{L} \]

\[ + \left( \frac{m_p}{\langle m_N \rangle} \right) \langle \lambda \rangle C_{mNN}^{L} + \langle \lambda \rangle \langle \eta \rangle C_{mLL}^{N} + \langle \lambda \rangle \langle \xi \rangle C_{mLL}^{N} + \langle \eta \rangle \langle \xi \rangle C_{mNN}^{L} \] (2)

\footnote{Terms proportional to \( \kappa \) can be safely neglected [3].}
where \( C^{\alpha \beta}_{\nu_{\gamma}} \) are products of nuclear matrix elements and phase space integrals. In the limit when all neutrinos are light, eq. (2) reduces to the expression previously used [3, 7]. Correspondingly, all coefficients with “LL” superscripts coincide with those of the light neutrino case, see [3, 7]. Complete definitions for the coefficients for the heavy neutrino case are given in [18].

The particle physics parameters of the decay rate are defined as:

\[
\langle m_{\nu} \rangle = \sum_{j}^\prime U_{e j}^2 m_j \tag{3}
\]

\[
\langle m_N^{(U)} \rangle^{-1} = \sum_{j}^\prime U_{e j}^2 m_j^{-1} \tag{4}
\]

\[
\langle \lambda \rangle = \sum_{j}^\prime U_{e j} V_{e j} \lambda, \tag{5}
\]

\[
\langle \eta \rangle = \sum_{j}^\prime U_{e j} V_{e j} \eta, \tag{6}
\]

\[
\langle \xi \rangle = [\lambda^2 + \eta^2 - 2\lambda \eta \left( \frac{M_{GT}^N + M_{FT}^N}{M_{GT}^N - M_{FT}^N} \right)] \sum_{j}^{''} V_{e j}^2 \left( \frac{m_p}{m_j} \right). \tag{7}
\]

\( U_{e j} \) and \( V_{e j} \) are the elements of the neutrino mixing matrix for the left- and right-handed sectors, which satisfy the completeness \( (\sum_j |U_{e j}|^2 = \sum_j |V_{e j}|^2 = 1) \) and the orthogonality relation \( (\sum_j U_{e j} V_{e j} = 0) \) [3]. As usual the primed sum indicates [3] that the sums extend over light mass eigenstates only, whereas the double primed sums extend over the heavy mass eigenstates. \( \langle \xi \rangle \) describes right-handed neutrino exchange and the first term in \( \langle \xi \rangle \) corresponds to the one considered by Mohapatra [8]. Neglecting all other terms and assuming no mixing between the W-bosons, our decay rate reproduces the one considered by Mohapatra [8]. Note, that in deriving eq. (2) we have neglected light right-handed neutrinos, as well as terms proportional to \( \sum_j^{''} U_{e j} V_{e j} \lambda m_j^{-2} \) and \( \sum_j^{''} U_{e j} V_{e j} \eta m_j^{-2} \), since the latter are suppressed by additional powers of large neutrino masses.

We have calculated the matrix elements for heavy neutrino exchange within the pn-QRPA model of Muto et al. [7, 12] Numerical results for the experimentally most interesting isotopes are given in table 1. Corresponding matrix elements for light neutrino exchange can be found in ref. [7]. Table 1 shows that, with the possible exception of the two heaviest isotopes, all matrix elements have rather similar numerical values, in agreement with the expectation.
Table 1: Nuclear matrix elements for heavy neutrino exchange in 0νββ decay for the experimentally most interesting isotopes calculated within pn-QRPA.

<table>
<thead>
<tr>
<th>AY</th>
<th>76Ge</th>
<th>82Se</th>
<th>100Mo</th>
<th>116Cd</th>
<th>128Te</th>
<th>130Te</th>
<th>136Xe</th>
<th>150Nd</th>
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<tbody>
<tr>
<td>$M^N_{GT}$</td>
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<tr>
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<td>156</td>
<td>248</td>
<td>219</td>
<td>121</td>
<td>344</td>
<td></td>
</tr>
<tr>
<td>$M^N_F$</td>
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<td>-64</td>
<td>-36</td>
<td>-55</td>
<td>-48</td>
<td>-27</td>
<td>-78</td>
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</table>

With the calculated matrix elements at hand, using the half life limit on $^{76}\text{Ge}$ $0\nu\beta\beta$ decay as recently measured by the Heidelberg-Moscow collaboration, $T_{1/2}^{0\nu\beta\beta}(^{76}\text{Ge}) \geq 7.4 \times 10^{24}$ years (90 \% c.l.) [10] we are ready to derive quantitative constraints on the various LR model parameters. In principle, the experimental half life limit and eq. (2) define an excluded area in a 5-dimensional parameter space. However, since in LR models heavy neutrinos are expected to be right-handed, we will restrict the discussion to $\langle m_\nu \rangle$, $\langle \lambda \rangle$, $\langle \eta \rangle$ and $\langle \xi \rangle$.  

Constraints can be derived under the assumption that only one parameter contributes to the decay rate at a time ("on axis"), or for an arbitrary variation of all four parameters. Numerically we find: $\langle m_\nu \rangle = 0.66(0.56)$ [eV], $\langle \lambda \rangle = 1.1(1.0) \times 10^{-6}$, $\langle \eta \rangle = 6.4(5.5) \times 10^{-9}$ and $\langle \xi \rangle = 1.7(1.7) \times 10^{-8}$ for the "arbitrary" ("on axis") cases, respectively.

As is clear from eqs.(5)-(6), limits on $\langle \lambda \rangle$ and $\langle \eta \rangle$ can not be converted into limits on the mass or mixing angle of right-handed W-bosons, without making assumptions about the size of the neutrino mixing matrix coefficients and their respective CP eigenvalues. Instead, for example, $\langle \lambda \rangle$ defines an

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5Assuming only left-handed heavy neutrinos to contribute to $0\nu\beta\beta$ decay one could also derive the constraint $\langle m^{(W)}_N \rangle \geq 6.0 \times 10^7$ GeV. However, $\langle m^{(U)}_N \rangle$ incorporates $\left( \sum_j U_{ej}^2 \right)^{(-1)}$ which has to be expected to be small. Moreover, since the effective masses include the unknown mixing coefficients, it has to be noted that $\langle m^{(U)}_N \rangle$ is not necessarily positive definite. Thus, by an extreme fine-tuning it is possible to cancel the contributions from light and heavy left-handed neutrinos. We disregard such an unlikely situation in the following.
excluded area in the plane $\{\Sigma_j' U_{ej} V_{ej}, m_{W_R}\}$, as is shown in fig. 1. Although for large mixing very stringent limits would be obtained, for typical values of $\Sigma_j' U_{ej} V_{ej} \sim O(10^{-6})$ or so, only $m_{W_R} \leq m_{W_L}$ is excluded, certainly not a very stringent constraint.

Much more interesting in this sense is the limit on $\langle \xi \rangle$. From the completeness relation we know that there is at least one right-handed neutrino with $V_{ej}^2 \sim O(1)$, which in LR models should be quite heavy. Defining

$$\langle m_N^{(V)} \rangle^{-1} = \sum_j^N V_{ej}^2 m_j^{-1}$$

from the limit on $\langle \xi \rangle$ one can derive

$$m_{W_R} \geq 1.1 \left( \frac{\langle m_N^{(V)} \rangle}{1 \text{ TeV}} \right)^{-1/4} \text{[TeV]}, \quad (9)$$

$$\tan(\xi) \leq 4.7 \times 10^{-3} \left( \frac{\langle m_N^{(V)} \rangle}{1 \text{ TeV}} \right)^{1/2}. \quad (10)$$

To compare the limit on the mass of the $W_R$ to the one derived by Mohapatra [8], we mention that $m_{W_R} \geq 1.23 \left( \frac{\langle m_N^{(V)} \rangle}{1 \text{ TeV}} \right)^{-1/4} \text{[TeV]}$ can be derived, if we take the limit $\tan(\xi) \to 0$. Finally, following the argument [8] that vacuum stability requires $\langle m_N^{(V)} \rangle \leq gm_{W_R}$, where $g$ is of order $O(1)$, an absolute lower bound on $m_{W_R}$ can be derived. The quantitative difference between our result and that of ref. [8] is mainly due to the improved half life limit used in our calculation.

We stress that it is not due to errors or uncertainties in the matrix element calculation - uncertainties of limits on $m_{W_R}$ scale only as the fourth square root of the uncertainties in the nuclear matrix elements.

Let us now turn to a brief discussion of the Higgs triplet contribution to $0\nu\beta\beta$-decay. The generation of Majorana masses in left-right symmetric models is achieved quite naturally if the Higgs sector of the theory contains two additional triplets, $\Delta_{LR} = (\Delta^-, \Delta^-, \Delta^0)_{LR}$[20]. This implies that $0\nu\beta\beta$ decay can not only occur through the usual neutrino exchange diagram (fig. 2.a), but in addition also through the graph involving the exchange of a doubly-charged Higgs (fig. 2.b). It is straightforward to show that the contribution

\[ \text{In addition, there is the possibility that the two W-bosons of fig. 2.a are replaced by the singly-charged component of the triplet. This contribution, however, is negligible due to the small coupling of the Higgs to quarks [14], and, in addition, due to the small relevant nuclear matrix elements [16].} \]
of this graph to $0\nu\beta\beta$ decay is proportional to $\frac{1}{m_{W_R}} \frac{m_N}{m_{\Delta_{R}}}$. [20]. (In general, also $\Delta_R$ and $\Delta_L$ can mix with each other as is the case for the W-bosons. The left-handed doubly-charged Higgs, however, has a negligible coupling strength proportional to the light neutrino mass. We neglect this possibility for simplicity.)

The inclusion of the graph in fig 3.b therefore modifies the contribution of the $\lambda^4$-terms, which are then proportional to, (neglecting mixing among neutrinos for simplicity)

$$\left(\frac{m_{W_L}}{m_{W_R}}\right)^4 \left(\frac{1}{m_N} + \frac{m_N}{m_{\Delta_{R}}^2}\right).$$

(11)

Eq. (11) leads to a modified constraint on the mass of the right-handed W-bosons as shown in fig. 3, where limits are shown as a function of the heavy, right-handed neutrino mass and various values of $m_{\Delta_{R}}$. Given that there is no upper bound on the mass of the right-handed Higgs triplet, however, no more stringent constraints on $m_{W_R}$ can be inferred from $0\nu\beta\beta$ decay, than the one quoted in eq. (9).

To summarize, it is concluded that right-handed neutrino exchange in $0\nu\beta\beta$ decay leads to much more stringent limits on the mass and mixing angle of right-handed W-bosons, than the left-right mixing mechanism, usually expressed in terms of the effective parameters $\langle \lambda \rangle$ and $\langle \eta \rangle$. This is mainly due to the small mixing, which has to be expected between the left- and the right-handed neutrino sectors. Terms proportional to right-handed neutrinos can not be neglected in the decay rate of $0\nu\beta\beta$ decay in left-right symmetric models. Although limits on the mass of the right-handed W-boson do depend only weakly on nuclear matrix elements, it therefore seems to be desirable also to reconsider the calculation of nuclear matrix elements for heavy particle exchange more carefully than has been done up to now.

We have also discussed the modifications of the formalism, due to the contribution of the right-handed Higgs triplet. Although such contribution turns out to be numerically small (unless $\Delta^{-}$ is very light) for $0\nu\beta\beta$ decay, as is shown in fig. 3, it is necessary to include these terms if one wants to make a consistent comparison of the constraints on LR models as derived from $0\nu\beta\beta$ decay with those inferred from inverse neutrinoless double beta decay searched for at particle accelerators [21, 22].

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References


Fig. 1: Excluded area in the plane \( \{ \sum_j U_{ej} V_{ej}, m_{w_R} \} \), for \(^{76}\text{Ge}\). Combinations to the upper left of the thick line are not allowed.

Fig. 2: a) To the left: Heavy neutrino exchange contribution to neutrinoless double beta decay in left right symmetric models, and b) to the right: Feynman graph for the virtual exchange of a doubly-charged Higgs boson, see text.
Fig. 3: Limits on the mass of the right-handed W-boson from neutrinoless double beta decay (full lines) and vacuum stability (dashed line). Combinations below the lines are forbidden. The five full lines correspond to the following masses of the doubly charged Higgs, $m_{\Delta^--}$: a) 0.3, b) 1.0, c) 2.0, d) 5.0 and e) $\infty$ (TeV).