Possible Implications of the Atmospheric, the Bugey, and the Los Alamos Neutrino Experiments *

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Abstract

A combined analysis of the Los Alamos and the Bugey neutrino experiments and the Kamiokande observation of atmospheric neutrino anomaly is performed using the three-flavor mixing scheme of neutrinos. It is pointed out that the constraints from these three experiments are so strong that the patterns of mass hierarchy and flavor mixing of neutrinos are determined almost uniquely depending upon the interpretation of the atmospheric neutrino anomaly.

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There have been accumulating indirect evidences for nonvanishing masses and the flavor mixings of neutrinos. They include the solar neutrino deficit [1] which may be interpreted by either the Mikheyev-Smirnov-Wolfenstein mechanism [2] or the vacuum neutrino oscillation [3], both being based upon the notion of flavor mixing. The second in the list is the atmospheric neutrino anomaly first observed by the Kamiokande experiment [4] and subsequently confirmed by other detectors [5,6], which strongly indicates the large-angle flavor mixing of neutrinos.

The recent announcement of the discovery of nonzero neutrino mass by the Liquid Scintillator Neutrino Detector (LSND) experiment [7,8] in Los Alamos may have brought the first direct evidence for the neutrino masses and the flavor mixing. The experiment seems to have observed the neutrino oscillation $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ with oscillation parameters $\Delta m^2 \simeq 1 - 12$ eV$^2$ and $\sin^2 2\theta \simeq 10^{-2} - 10^{-2.5}$, if interpreted by the two-flavor mixing scheme. The result may be marginally compatible with the earlier results obtained by the Los Alamos [9] and the BNL experiments [10] and by the KARMEN Collaboration experiment [11]. Needless to say the result, if confirmed by the succeeding runs, has tremendous implications for particle physics and cosmology [12-14].

In this paper we try to extract the possible implications of the LSND result in the light of the experimental informations from the underground detectors and the reactor experiments. We first observe, as many authors do [12-14], that one cannot explain the three phenomena mentioned above simultaneously by the three-flavor mixing scheme without introducing sterile neutrinos. It is simply due to the fact that the three-flavor scheme cannot accommodate three hierarchically different mass scales, $\Delta m^2 \simeq (1 - 12)\text{eV}^2$ for LSND, $\Delta m^2 \simeq 10^{-2}\text{eV}^2$ for the atmospheric neutrino anomaly, and $\Delta m^2 \simeq 10^{-6} - 10^{-5}\text{eV}^2$ ($\simeq 10^{-10}\text{eV}^2$) for the MSW (vacuum mixing) solution of the solar neutrino problem.

In this paper we derive the constraints imposed on neutrino masses and mixing angles via a combined analysis of the LSND result and the atmospheric neutrino anomaly. We will recognize in this analysis that it is crucially important to include the constraint from the reactor experiments, whose most stringent one is reported by the Bugey group [15]. We
employ the mixing scheme based on three-generation neutrinos, as beautifully confirmed by the LEP experiments [16]. It will be demonstrated that it is essential to use the three-flavor mixing scheme, rather than optional use of various two-flavor mixings, for drawing correct interpretation of the data. We will also consider the restrictions imposed by the neutrinoless double $\beta$ decay [17].

Amazingly, the constraints imposed by the atmospheric, the Bugey, and the LSND experiments are so restrictive as to determine the masses and the mixing patterns of three flavor neutrinos. Only a few patterns are allowed: (A) light “$\nu_e$” and almost degenerate strongly mixed heavy “$\nu_\mu$” and “$\nu_\tau$”, and its mass-inverted version, or (B) light “$\nu_\tau$” and almost degenerate strongly mixed heavy “$\nu_e$” and “$\nu_\mu$”, and its mass-inverted one. The choice of the solutions (A) or (B) is dictated by the interpretation of the atmospheric neutrino anomaly. The pattern (A) follows if we interpret the atmospheric neutrino anomaly as due to the $\nu_\mu \rightarrow \nu_\tau$ oscillation, and (B) results if it is due to the $\nu_\mu \rightarrow \nu_e$ oscillation.

We stress that neither the LSND-Atrmospheric nor the LSND-Solar neutrino combinations are compelling. The interpretation is somewhat more complicated in the LSND-Solar neutrino case because there still exist three types of solutions to the solar neutrino problem based on the neutrino flavor mixing; the small and the large-angle MSW solutions in addition to the vacuum mixing one. The analysis of this combination will be presented elsewhere [18]. Nonetheless, we should mention that our theoretical prejudice prefers the LSND-Atrmospheric neutrino combination over the other one. It is natural to introduce a sterile neutrinos to accommodate the third experimental data left over in both combinations. It is, however, difficult to explain the atmospheric neutrino anomaly by introducing the mixing with sterile neutrinos. In doing so one encounters the trouble with the light element nucleosynthesis [19].

We make use of one crucial aspect of the atmospheric neutrino data in our analysis. Namely, the Kamiokande group recently provided a new data set called the multi-GeV sample [20]. They consist of the events with higher energy, $\gtrsim 1.33$ GeV, than the previously reported data. The important feature of the new data is that, because of the higher en-
ergy, the path-length dependence of the oscillation probability can be probed by measuring the zenith angle dependence. It is striking that it can be perfectly fitted by the neutrino oscillation with mixing parameters $\Delta m^2 \approx 10^{-2} \text{ eV}^2$ and $\sin^2 2\theta \approx 1$ [20]. Such quantitative agreement with the zenith-angle dependence is the strongest support for the neutrino oscillation interpretation of the atmospheric neutrino anomaly.

We now make three basic observations in view of the data of the LSND and the Kamiokande atmospheric neutrino experiments. (1) To have gross deficit in the ratio $(\nu_\mu + \bar{\nu}_\mu)/(\nu_\tau + \bar{\nu}_\tau)$ we need at least one large mixing angle. (2) To explain the rate of the oscillation events of the order of $\sim 5 \times 10^{-3}$ at least one mixing angle has to be small. (3) The feature of the atmospheric neutrino data indicates, apart from a slim chance of the miraculous cancellation, that the one of the three $\Delta m_{ij}^2$ is of the order of $\sim 10^{-2} \text{ eV}^2$. Notice that we cannot have two $\Delta m_{ij}^2$ of the order of $10^{-2} \text{ eV}^2$ because it contradicts the LSND result, $\Delta m^2 \gtrsim 1 \text{ eV}^2$.

Based on these observations we classify the hierarchy of the neutrino masses into the following two types:

$$ a : m_3^2 \approx m_2^2 \gg m_1^2 \quad \quad \quad b : m_1^2 \gg m_2^2 \approx m_3^2 \quad (1) $$

Here the symbols $\approx$ and $\gg$ imply the difference by $\sim 10^{-2} \text{ eV}^2$ and $\sim 1-10 \text{ eV}^2$, respectively. Throughout the analysis in this paper the relative magnitude of the masses connected by $\approx$ does not matter. The other types of mass hierarchies which are obtained by permuting 1, 2, and 3 will be automatically taken care of because they merely represent relabelling of the mass eigenstates.

We recollect the basic formula of the oscillation probabilities with three flavors of neutrinos. We introduce the neutrino mixing matrix $U$ which relates the flavor- and the mass-eigenstates as $\nu_\alpha = U_{\alpha i} \nu_i$, where the flavor index $\alpha$ runs over $\epsilon, \mu$ and $\tau$ and the mass-eigenstate index $i$ runs over 1 to 3. We assume the CP invariance in the present analysis. In this case the mixing matrix $U$ is real and contains only three angles $\theta_{ij}$.

With the mixing matrix the oscillation probability of neutrinos of energy $E$ after travers-
ing the distance $L$ can be written as

$$P(\nu_\beta \to \nu_\alpha) = P(\bar{\nu}_\beta \to \bar{\nu}_\alpha) = \delta_{\alpha \beta} - 4 \sum_{j>1} U_{\alpha i} U_{\beta j} U_{\alpha j} U_{\beta j} \sin^2 \left( \frac{\Delta m_{ij}^2 L}{4E} \right),$$

(2)

where $\Delta m_{ij}^2 = |m_i^2 - m_j^2|$. As a convenient parametrization of the matrix $U$ we use the so-called standard form of the Kobayashi-Maskawa matrix, which is now adopted for the neutrino mixing matrix:

$$U = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{bmatrix},$$

(3)

where $c_{ij}$ and $s_{ij}$ are the short-hand notations for $\cos \theta_{ij}$ and $\sin \theta_{ij}$, respectively. We note that the three real angles can all be made to lie in the first quadrant by an appropriate redefinition of neutrino phases.

The expressions of the oscillation probability are rather cumbersome involving many angle factors. Therefore, we shall derive the approximate formulas by taking into account the mass hierarchies and the experimental parameters of the three experiments. The oscillation probability which corresponds to the LSND experiment is approximately given by

$$P(\bar{\nu}_\mu \to \bar{\nu}_e) = 4c_{12}^2 c_{13}^2 (s_{12} c_{23} + c_{12} s_{23} s_{13})^2 \sin^2 \left( \frac{\Delta m_{12}^2 L}{4E} \right),$$

(4)

where two terms with $\Delta m_{12}^2 \approx \Delta m_{13}^2$ (which differ only by $10^{-2}$eV$^2$) are combined and the term with $\Delta m_{23}^2$ is ignored. The former procedure can be neatly done by utilizing the orthogonality relation of the mixing matrix. The latter approximation is completely legitimate because the term is smaller than others by factor $10^{-4} - 10^{-6}$ owing to the mass hierarchy $\Delta m_{23}^2/\Delta m_{12}^2 \approx 10^{-2} - 10^{-3}$.

If the atmospheric neutrino anomaly is attributed to the $\nu_\mu \to \nu_\tau$ oscillation the relevant formula is

$$P(\nu_\mu \to \nu_\tau) = 2(s_{12} c_{23} + c_{12} s_{23} s_{13})^2 (s_{12} s_{23} - c_{12} c_{23} s_{13})^2 
+ 4c_{23} s_{23} c_{13} (c_{12} c_{23} - s_{12} s_{23} s_{13}) (c_{12} s_{23} + s_{12} c_{23} s_{13}) \sin^2 \left( \frac{\Delta m_{23}^2 L}{4E} \right).$$

(5)
In (5) the sine-squared factors with large \( \Delta m^2 \) of \( \gtrsim 1 \text{eV}^2 \) are replaced by the average value \( \frac{1}{2} \), which can be justified because of the rapid oscillations; with \( \Delta m^2 = 1 - 10 \text{eV}^2 \) and \( E= 1 \text{GeV} \) the argument of the sine is \( \sim 10 - 100 (10^4 - 10^5) \) for \( L= 10 (10^4) \text{ Km} \).

We note that, given the LSND result, there exist the possibility that the atmospheric neutrino anomaly is due to the \( \nu_\mu \rightarrow \nu_e \) oscillation, the possibility one might not naively expect. It is perfectly consistent with LSND result because the relevant scales of the path length and the energy involved in these two experiments are much different. In this case the formula for the oscillation probability to be used is

\[
P(\nu_\mu \rightarrow \nu_e) = 2c_{12}^2 c_{13}^2 (s_{12} c_{23} + c_{12} s_{23} s_{13})^2
- 4s_{12} s_{23} c_{13}^2 s_{13} (c_{12} c_{23} - s_{12} s_{23} s_{13}) \sin^2 (\frac{\Delta m^2_{23} L}{4E}),
\]  

(6)

Finally the formula for the Bugey experiment takes the form

\[
1 - P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 2c_{12}^2 c_{13}^2 (s_{12}^2 c_{13}^2 + s_{13}^2)
+ 4s_{12} c_{13}^2 s_{23}^2 s_{13} \sin^2 (\frac{\Delta m^2_{23} L}{4E}),
\]

(7)

where the terms with \( \Delta m_{12}^2 \) are averaged as before. It can be justified because the argument of the sine term is of the order of 10-100 with \( \Delta m^2 = 1 - 10 \text{eV}^2 \), \( E= 4 \text{MeV} \), and \( L= 40 \text{m} \), the typical parameters of the Bugey experiment. The second term of (7) may be neglected (as we will do) because sine-squared factor is \( \sim 10^{-2} \) for \( \Delta m^2 = 10^{-2} \text{eV}^2 \).

We first examine the case that the atmospheric neutrino anomaly is attributed to the \( \nu_\mu \rightarrow \nu_e \) oscillations. Our discussion does not distinguish the types a and b until we address the constraint due to the double \( \beta \) decay.

We demand, for consistency with the gross features of the LSND, the Bugey, and the atmospheric neutrino experiments, the following constraints:

\[
c_{12}^2 c_{13}^2 (s_{12} c_{23} + c_{12} s_{23} s_{13})^2 \simeq 10^{-3} \equiv \epsilon
\]

(8)

\[
c_{12}^2 c_{13}^2 (s_{12}^2 c_{13}^2 + s_{13}^2) \leq 2.5 \times 10^{-2} \equiv \delta,
\]

(9)
\[(s_{12}c_{23} + c_{12}s_{23}s_{13})^2(s_{12}s_{23} - c_{12}c_{23}s_{13})^2 \leq 0.1, \quad (10)\]
\[4c_{23}s_{23}c_{13}^2(c_{12}c_{23} - s_{12}s_{23}s_{13})(c_{12}s_{23} + s_{12}c_{23}s_{13}) \simeq 1. \quad (11)\]

The constraints (8) comes from the rate \(\sim 5 \times 10^{-3}\) of the appearance events in the LSND experiment [8], while (9) is due to the bound \(1 - P(\bar{\nu}_e \to \bar{\nu}_e) \lesssim 5\%\) obtained in the Bugey experiment [15]. The latter number includes statistical and systematic uncertainties. The remaining two restrictions are from the features of the atmospheric neutrino data that the zenith angle dependence is well described by an effective two-flavor mixing with \(\Delta m^2 \simeq 10^{-2}eV^2\) and \(\sin^2 2\theta \simeq 1\). The constraint (10) arises from a mild requirement that the first term of (5) should be less than 0.2 so as not to disturb the effective two-flavor description.

We emphasize that the constraints from the atmospheric neutrino data takes the simple forms (10) and (11) because of the mass hierarchy \(\Delta m^2_{12} \approx \Delta m^2_{13} \gg \Delta m^2_{23}\).

We first notice that from the Bugey constraint(9) that \(X \equiv c_{12}^2 c_{13}^2\) must satisfy the inequality that \(X^2 - X + \delta \geq 0\). This inequality is so powerful that restricts the value of \(X\) into the two tiny regions \(0 \leq X \leq \delta\) and \(1 - \delta \leq X \leq 1\). On the other hand, we must have \(c_{13}^2 \simeq O(1)\) in order to satisfy the requirement (11). Thus, we have either \(c_{12}^2 \simeq \delta\) or \(c_{12}^2 \simeq 1\) corresponding to the small-\(X\) and the large-\(X\) solutions, respectively. It is also required that \(c_{23}s_{23} \simeq \frac{1}{2}\) in order to maximize (11). The small-\(X\) solution is then inconsistent with (11). We end up with the unique solution

\[(A) \quad s_{12}^2 \approx s_{13}^2 \simeq \epsilon, \quad c_{23}^2 \approx s_{23}^2 \simeq \frac{1}{2} \quad (12)\]

where we have also utilized the LSND constraint (8) to push \(s_{12}^2 \simeq \delta\) down to \(s_{12}^2 \simeq \epsilon = 10^{-3}\).

To confirm that the obtained solution (12) is physically unique we have repeated the similar analysis to obtain the solution for other types of mass hierarchies corresponding to the cyclic permutations of 1-3 of (1). We have explicitly verified that the the allowed mixing pattern implied by (12) is physically unique throughout the varying mass hierarchies, as they should be. Namely, the light \(\nu_e\) and the almost degenerate strongly mixed heavy
“νμ” and “ντ” for the type-a, and the heavy “νe” and the almost degenerate strongly mixed light “νμ” and “ντ” for the type-b cases.

We now turn to the case that the atmospheric neutrino anomaly is caused by the νμ → νe oscillation. In this case we replace the requirements (10) and (11) by

\[ c_{12}^2 c_{13}^2 (s_{12} c_{23} + c_{12} s_{23} s_{13})^2 \leq 0.1, \]  

and

\[ -4 s_{12} s_{23} c_{13} s_{13} (c_{12} c_{23} - s_{12} s_{23} s_{13}) \simeq 1, \]

respectively. By the similar procedure one can show that the consistent solution of the requirements (8), (9), (13), and (14) is uniquely given by

\[ c_{12}^2 \approx c_{23} \approx c, \quad c_{13}^2 \approx s_{13}^2 \approx \frac{1}{2}. \]  

The solutions of the other type of mass hierarchies can be obtained by the similar manner and correspond to the redefinition of the mass eigenstates. The allowed mixing pattern is again physically unique: The light “νe” and the almost degenerate strongly mixed heavy “νe” and “νμ” for the type-a, and the heavy “νe” and the almost degenerate strongly mixed light “νe” and “νμ” for the type-b mass hierarchies.

In the case of Majorana neutrinos the additional constraint emerges from the non-observation of the neutrinoless double β decay. The quantity

\[ < m_{\nu_e} > = \sum_{j=1}^{3} \eta_j |U_{ej}|^2 m_j \]

is constrained to be less than \( \sim 1 \text{eV} \) by the experiments \[17\] where \( \eta_j = \pm 1 \) is the CP phase. Notice that we are working with the representation in which the mixing matrix is real under the assumption of CP invariance.

Generally speaking, the constraint from the double β decay distinguishes between the type-a and the type-b mass hierarchies. In the type-a case there is a chance for cancellation between nearly degenerate two masses, but no chance in the type-b case because the heavy mass is carried by a unique mass eigenstate.
New features, however, arise in our consistent solutions obtained above on the double \( \beta \)-decay constraint. We first discuss the case of atmospheric neutrino anomaly due to the \( \nu_\mu \to \nu_\tau \) oscillation. It can be shown that in the type-a mass pattern the double \( \beta \) constraint is automatically satisfied because the heavy masses are always multiplied by small angle factors. On the contrary, the angle factors in front of the unique heavy mass are always of the order of unity in the type-b mass hierarchy. Therefore, there is no consistent solution of the double \( \beta \)-decay constraint for Majorana neutrinos in the type-b hierarchy.

In the case of atmospheric neutrino anomaly due to the \( \nu_\mu \to \nu_\tau \) oscillation, the conclusion is reversed. In the type-a mass pattern there is always a trouble because almost degenerate heavy masses are multiplied by \( O(1) \) coefficients and a fine tuning, i.e., \( c_{13}^2 = s_{13}^2 \) to order \( \epsilon \), is required for cancellation in addition to the requirement of opposite CP parities. On the contrary, in the type-b hierarchy, there is no trouble with the double \( \beta \)-decay constraint because the heavy mass is always multiplied by small coefficients of the order of \( \epsilon \) in each case of mass patterns.

Thus, we have shown in this paper that the neutrino masses and the mixings are strongly constrained by the atmospheric, the Bugey, and the LSND experiments. The constraint is so severe that the mass and the mixing patterns are determined almost uniquely within the uncertainties of the neutrino types and the interpretations of the atmospheric neutrino anomaly. In the case of the Majorana neutrinos the additional constraint from the double \( \beta \) decay selects out the unique solution in each interpretation.

Finally we give a few remarks.

(1) Our analysis in this paper is less powerful in constraining the absolute values of the neutrino masses than in restricting the relative masses and the mixing angles. All the constraints would be cleared by, for example, the type-a solution with \( m_1 = 6 \) eV, \( m_2 = 6.5 \) eV, and \( m_2 = (6.5 + \epsilon) \) eV, which is also consistent with the direct measurements [21] and the cosmological considerations [22].

(2) We have not performed a full three-flavor analysis of the atmospheric neutrino anomaly but relied on the effective two-flavor interpretation of either \( \nu_\mu \to \nu_\tau \) or \( \nu_\mu \to \nu_\tau \) channels.
While the point deserves further study we are under a strong feeling that coexistence of the two channels does not spoil our solutions obtained in this paper. They certainly survive in the case of equal contributions of these two channels.

(3) We have not addressed the constraint which might be placed by the consideration of the r-process nucleosynthesis in supernova [23]. If we do so it would favor the inverted mass hierarchy, the type-b (A) and the type-a (B) solutions, at the price of possible troubles with neutrino events from SN1987a [13,14]. However, we do not regard the constraint compelling because it depends on the specific model of the nucleosynthesis. Also the region $\Delta m^2 = 1 - 10\text{eV}^2$ seems to be a delicate one even if the hot-bubble scenario is taken for granted, and we should wait for the final conclusion of the ongoing investigations [24].

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