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CURVATURE EFFECTS AND THE SHAPE OF BUNCHES
IN ELECTRON STORAGE RINGS

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A B S T R A C T

Longitudinal self-field effects of an azimuthally bunched beam moving on a circular trajectory, between two infinitely large perfectly conducting planes, are evaluated theoretically. For most accelerators the coherent radiation terms are negligible in so far as bunch shape is concerned, while the curvature terms add to the Coulomb term and, above transition, enhance the negative mass effect.

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I. INTRODUCTION

The possibility of coherent synchrotron radiation increasing the bunch length in electron storage rings has recently been discussed ⁽¹⁾. Further interest in the subject has arisen because of the observations made at the electron-positron storage ring ACO, which in fact show an increase of bunch length with the beam intensity ⁽²⁾.

The electromagnetic fields generated by an electron beam are strongly influenced by the presence of the metallic vacuum tank inside which the beam is moving. In the work of reference ⁽¹⁾ the electromagnetic field generated by the beam in vacuum is used to obtain the force acting on electrons, and it is then assumed that the shielding effect, while reducing the electromagnetic field intensity, does not change appreciably the spectrum of the field. This hypothesis, of Ref. (1), seems to be in contrast with the result obtained in the present work where we make a direct calculation of the electromagnetic field generated by an electron beam moving within two metallic plane surfaces.

We consider the case in which the shielding is provided by two infinitely large, perfectly conducting parallel planes at a distance H . The beam is assumed to move in a circular orbit, of radius R on a plane parallel to the shielding planes and equidistant from them. This situation is different from the real one in which the beam is moving inside a metallic vacuum chamber, but the study of this case should give us a qualitatively correct insight into the true situation - especially for those accelerators whose vacuum tank has a height to width ratio much less than one, and lateral holes from which synchrotron radiation is emitted. A rather general formalism may be found in Nielsen, Sessler and Symon ⁽³⁾; our analysis can be considered as an explicit evaluation of the expressions of Ref. (3).

We find, for typical accelerator parameters, that the self field has two terms : (1) the shielded Coulomb field proportional to $1/\gamma^2$ and, (2) a term in $(\beta H/R)^2$ which is γ -independent. (The particle velocity is βc , and $\gamma^{-2} = 1 - \beta^2$.) The curvature term adds to the Coulomb term - and can, for some accelerators, be larger than the Coulomb term. Consequently, above transition self field effects should produce a bunch shortening : the observations at ACO will require some other mechanism for their explanation.

II. Longitudinal Electric Field

We derive the field which has the beam as a source, and satisfies the appropriate boundary conditions on the two shielding planes and at infinity.

Using polar coordinates (r, θ) in the plane of the orbit and calling z the orthogonal direction, the beam charge density and current are assumed to be given by

$$j_r = j_z = 0, \quad (1)$$

$$j_\theta = e\omega_0 \delta(r - R) \delta(z) N(\theta - \omega_0 t), \quad (2)$$

$$\rho = j_\theta / \beta c \quad (3)$$

where R is the orbit radius, ω_0 the revolution frequency, and βc is the azimuthal velocity. The function $N(\theta - \omega_0 t)$, describing the azimuthal beam density, is periodic and can be expanded in a Fourier series :

$$N(\theta - \omega_0 t) = \sum_n N_n e^{in(\theta - \omega_0 t)} \quad (4)$$

We now assume the scalar and vector potential to be of the form

$$\underline{A} = \sum_n \underline{A}_n(r, \theta, z) e^{-in\omega_0 t}, \quad (5)$$

$$\underline{\phi} = \sum_n \underline{\phi}_n(r, \theta, z) e^{-in\omega_0 t}. \quad (6)$$

Then the equations

$$\square \underline{A} = - \frac{4\pi}{c} \underline{j}, \quad (7)$$

$$\square \phi = - 4\pi q, \quad (8)$$

can be solved with the help of a Green function for Eq. (8), satisfying the given boundary conditions (4); namely :

$$G(r, \theta, z; r', \theta', z') = \frac{i}{2H} \sum_{j=1}^{\infty} \sum_{m=-\infty}^{+\infty} \sin j\pi(\frac{1}{2} + \frac{z}{H}) \sin j\pi(\frac{1}{2} + \frac{z'}{H}) \exp \left[im(\theta - \theta') \right] J_m(\gamma_{nj} r_{<}) H_{\pi}^{(1)}(\gamma_{nj} r_{>}), \quad (9)$$

where $r_{<}$ and $r_{>}$ are respectively the smallest and the largest of the two radii, r, r' . The quantity γ_{nj} is given by

$$\gamma_{nj} R = \left\{ n^2 \beta^2 - j^2 \left(\frac{\pi R}{H} \right)^2 \right\}^{1/2} \quad (10)$$

The azimuthal field at the beam position ($r=R, z=0$) can be readily obtained :

$$E_{n\theta} = \frac{2\pi^2 c}{H R} n N_n e^{in\theta} \left\{ \frac{2f_n}{\gamma_{nj}^2} - \beta^2 \varepsilon_n \right\}. \quad (11)$$

In (11) $\gamma^2 = (1 - \beta^2)^{-1}$ and

$$f_n = \sum_j J_n(\gamma_{nj} R) H_n^{(1)}(\gamma_{nj} R), \quad (12)$$

$$\varepsilon_n = \sum_j \left\{ J_{n+1}(\gamma_{nj} R) H_{n+1}^{(1)}(\gamma_{nj} R) + J_{n-1}(\gamma_{nj} R) H_{n-1}^{(1)}(\gamma_{nj} R) - 2J_n(\gamma_{nj} R) H_n^{(1)}(\gamma_{nj} R) \right\}, \quad (13)$$

where $\sum_j'$ indicates a sum over all odd non-negative values of j and $\gamma_{nj}R$ is defined in Eq. (10).

For the case in which $\gamma_{nj}R$ is imaginary one can use the formula

$$I_\ell(x) K_\ell(x) = \int_0^\infty \frac{J_{2\ell}(2t)}{(t^2 + x^2)^{1/2}} dt,$$

and write f_n and g_n as

$$f_n = \frac{2}{\pi i} \sum_j' \int_0^\infty \frac{J_{2n}(2t)}{(t^2 + x^2)^{1/2}} dt \quad (14)$$

$$g_n = \frac{-2}{\pi i} \sum_j' \int_0^\infty J_{2n}(2t) \frac{x^2 - 2t^2}{(t^2 + x^2)^{5/2}} dt, \quad (15)$$

where

$$x^2 \equiv -\gamma_{nj}^2 R^2 > 0.$$

For $n \ll n_1$, where n_1 is the integer part of $\pi R/H$, we can make an approximate evaluation of f_n and g_n using the representations of (14) and (15). In fact, the function $J_{2n}(2t)$ peaks at $t \approx n$ and for larger values of t oscillates. Therefore if $n_1 \gg n$, and hence $x \gg n$, we can ignore t compared to x in (14), (15) and thus obtain

$$f_n \approx \frac{1}{\pi i n_1} \sum_j' \frac{1}{j}, \text{ for } n \ll n_1; \quad (16)$$

$$g_n \approx \frac{-1}{\pi i n_1^3} \sum_j' \frac{1}{j^3} \approx \frac{-1}{\pi i n_1^3}, \text{ for } n \ll n_1. \quad (17)$$

The logarithmic divergence in f_n has as its source the assumed infinitely narrow beam, and as is well known ⁽⁵⁾ a more realistic assumption produces a cut-off on the sum.

Introducing the cut-off via the beam radius a , we obtain from (16), (17) and (11)⁽⁶⁾:

$$E_{ne} \approx - \frac{2\alpha N_{inc}^{in\theta}}{R^2} \left\{ \frac{1}{\gamma^2} \left(1 + 2 \log \frac{2H}{\pi a} \right) + \left(\frac{\beta H}{\pi R} \right)^2 \right\}$$

for $n \ll n_1$. (18)

III. PARTICLE DYNAMICS

To evaluate the effect of E_{ne} we must compute N_n . In fact, we shall see, in this way, that only a restricted range of n values needs to be considered in most applications.

We estimate the magnitude of N_n by assuming N particles distributed equally amongst B bunches, each with a Gaussian density distribution having mean squared angular width of $2\langle\theta^2\rangle$; namely

$$N(\theta) = \frac{N}{B} \left(\frac{1}{2\pi\langle\theta^2\rangle} \right)^{1/2} \exp \left[\frac{-(\theta - \theta_j)^2}{2\langle\theta^2\rangle} \right], \quad (19)$$

where $\theta_j = \frac{2\pi}{B}j$ and $j = 0, 1, \dots, B-1$. Then

$$N_n = \delta_{n,kB} \left(\frac{N}{2\pi} \right) \exp (-n^2\langle\theta^2\rangle/2) \quad (20)$$

where $\delta_{n,kB}$ is the Kronecker symbol and k assumes all positive and negative integer values.

We see, from (20), that $N_n \approx \frac{N}{2\pi} \delta_{n,kB}$ for $n \leq \left[\frac{2}{\theta^2} \right]^{1/2} \equiv n_0$

and that N_n decreases quickly for $n > n_0$. In most machines ⁽⁷⁾ $n_0 < n_1$

(bunch length $>$ vacuum chamber vertical aperture), and since N_n is small for $n > n_0$ we need only consider the range $n \leq n_0$ in the evaluation of the force, unless E_n is very large for some $n > n_0$.

In fact, from (14) and (15) it is easy to see that E_n is not larger for $n_1 > n > n_0$ than it is for $n < n_0$. The range $n > n_1$ corresponds to real radiation, but with wavelengths much shorter than the bunch length ($n \gg n_0$), and consequently can have little effect on the overall bunch shape.

Thus, in the important range of n , for bunch shape considerations, we may employ (18). The first, and most important observation, is that the curvature corrections to the usual Coulomb (shielded) term is of the same sign as the Coulomb term. Since, above transition, the repulsive Coulomb force leads to a bunch shortening (negative mass effect) we see that the curvature term only enhances this effect (8).

The second observation is that the curvature term can, in some cases, be even larger than the Coulomb term. For example, in ACO at $\gamma = 500$, the curvature term is four times the Coulomb term and should cause a bunch shortening at high current - rather than the observed bunch lengthening.

IV. ACKNOWLEDGEMENTS

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REFERENCES

- 1) E. Ferlenghi in, "Proceedings of the International Symposium on Electron and Positron Storage Rings," Saclay (1966), contribution IV.8.
- 2) P. Marin - private communication. These preliminary observations seem to exclude a Touschek effect explanation since bunch length was independent (within experimental accuracy of about 10%) of transverse beam size. Also apparently excluded, is a beam-cavity interaction since with one intense bunch and one weak bunch the intense one is long and the weak short - while the Q of the cavity would seem to preclude such an effect. Wall resistivity effects also seem to be excluded because of their smallness.
- 3) C.E. Nielsen, A.M. Sessler, and K.R. Symon, in Proceedings of the International Conference on High Energy Accelerators and Instrumentation, Geneva, 1959, CERN, Geneva, 1959, p.239.
- 4) J.S. Nodvick and D.S. Saxon, Phys. Rev. 96, 180 (1954).
- 5) C.E. Nielsen and A.M. Sessler, Rev. Sci. Instr. 30, 80 (1959).
- 6) H. Hereward has pointed out - private communication - that (18) can be interpreted as a reduction in the inductive contribution to E_{ne} , of order $(\frac{H}{\pi R})^2$; which is quite a reasonable result, from a simple circuit theory point of view.
- 7) The DESY proposed e^-e^+ storage ring is a counter example. K. Steffen - private communication.

- 8) This result - bunch shortening due to curvature effects - is in marked contrast with the result of Ref. (1) where coherent radiation yielded a bunch lengthening. Ref. (1), however, has an error in sign in Eq. (1), as has been pointed out by D. Petaux - private communication. That is :Ferlenghi's work is valid below transition, but must be changed in sign when applied in the region of negative mass and hence (when corrected) predicts a bunch lengthening.

However, from (15), for $n \approx n_1$, $\epsilon_n \approx \frac{2}{\pi \sin}$ which is of the same magnitude as (17), but of the opposite sign. Thus if $n_0 \approx n_1$ (situation in which shielding is relatively less important) we find a bunch lengthening.

In going from the regime $n \ll n_1$ to the regime $n \approx n_1$, the (purely reactive) field E_{n0} changes from capacitive in sign to inductive in sign.

If we consider the regime $n_0 \gg n_1$ (situation in which shielding is unimportant) then we must recover the field employed by Ferlenghi; namely that of Ref. (9). The unshielded field has both resistive and reactive terms, as may be seen from (11), (12) and (13) evaluated in the regime where $\gamma_{nj}R$ is real [$H \rightarrow \infty$, in (10)]. In fact (9), the resistive term is larger than the reactive term; the reactive term is inductive in sign, but particle motion is essentially determined by the resistive (real radiated power) term which causes a bunch lengthening (above transition). In going from the regime $n \approx n_1$ to the regime $n \gg n_1$, the purely reactive field E_{n0} has become augmented with a (dominant) resistive part.

- 9) L.V. Iogansen and M.S. Rabinovich, Soviet Phys. JETP 37, (10), 83, (1960).