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THE EFFECT OF MOMENTUM SPREAD IN SLOW EXTRACTION AT AN INTEGER
RESONANCE - A THEORETICAL STUDY

P. Strolin and E.J.N. Wilson

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1. Introduction

Measurements of slow extraction efficiency at the CERN PS suggest that the loss during extraction is about three times that given by existing theory. In an attempt to explain this discrepancy, the effect of momentum spread in the circulating beam has been considered.

Computations of the effect of momentum spread in an integer resonance extraction system have already been presented in an earlier study of the design of the CERN Intersecting Storage Rings (ISR) extraction system⁽¹⁾. The theory is, in general, applicable to any integer extraction system but in this paper we restrict our discussion to its implications for the particular case of the CERN Proton Synchrotron (CPS) extraction system.

It seems that in an extraction system operating at an integer resonance, displacement of the unperturbed closed orbit from the axis of perturbing quadrupoles due to momentum spread, closed orbit distortion or deliberate misalignment, results in a considerable displacement of the unstable and stable fixed points. The perturbed closed orbits corresponding to these points oscillate about the unperturbed orbit with an amplitude which is several times greater than the unperturbed displacement.

In this paper we first discuss a simple analytic model to illustrate the nature of the effect and then present the results of a computation for a more realistic model of the CPS.

2. Brief Description of the CPS Resonant Slow Extraction

In the CPS slow extraction system the radial Q of the machine is shifted from the nominal working point ($Q = 6.25$) to $Q = 6.0$ by a single quadrupole in a mid F straight section of the FOFDOD

lattice. Sextupoles in six other mid F straight sections provide the radial variation of Q necessary to control the spill rate and at the same time drive the exponential resonant growth along the outward-going separatrix.

Throughout this report we use units of displacement and divergence normalised so that, in mid D, unit displacement is 1 mm and the locus of the trajectory of a particle in subsequent mid D straight sections is a circle in phase space. The unit of divergence therefore represents $1/\beta_D$ milliradian in mid D (for the CPS, $\beta_D = 12.0$ metres)

Ideally, the normalised strength of the quadrupole, $K_1 = 2$. The value of the strength of each sextupole is $k_2 = 0.002$ in the above units⁽²⁾. The increment in normalised divergence is $K_1 x$ at a quadrupole and $k_2 x^2$ at a sextupole.

We choose, arbitrarily, to refer to the phase plane at the quadrupole exit where the separatrix is inclined at about 45° to the displacement axis.

The spill of the CPS system is controlled by slowly expanding the radial position of unperturbed beam. This shrinks the stable area which eventually becomes zero (i.e. the stable and unstable fixed points converge on a double fixed point) when the unperturbed beam lies on the quadrupole axis. The controlled expansion of the radial beam position is achieved by a combination of a slight declination of guide field on flat top and a dipole magnet which is servo controlled from a spill monitor.

3. Simple Model of the Ejection System

In order to illustrate our arguments and provide a qualitative check of the computations which follow, we first consider a simple model of an integer resonance extraction system.

The extraction quadrupole has strength $K_1 = 2$, which is independent of momentum. The effect of the six sextupoles in the CPS can be represented by a single sextupole diametrically opposite the quadrupole and having a strength, K_2 , m times that of each of the six sextupoles. If the amplitude of the oscillations about the axis of the system is small, $m = 3.4$ (see below). This simplification can in general be made for any machine. p_0 is the momentum whose perturbed and unperturbed closed orbits are coincident with the axis of the quadrupole and of the vacuum chamber. There are no closed orbit bumps or distortions. The double fixed point is therefore at $x = x' = 0$ for all momenta and, at the instant considered, the stable area for the p_0 population is zero.

We now consider a sample of the beam having momentum $p_0 + \Delta p$. The unperturbed closed orbit has radial phase space coordinates at the quadrupole: $x = e = \alpha_p \Delta p / p$, $x' = 0$. ($\Delta p < 0$).

Since the unperturbed closed orbit is not on the axis of the quadrupole it is distorted by the dipole component of the quadrupole field (and by the sextupole) when the perturbation is switched on. The perturbed closed orbit, which is of course the stable fixed point, is therefore displaced from the unperturbed closed orbit and oscillates around the machine with an amplitude a_0 about the unperturbed closed orbit. In other words, the stable fixed point follows a circular trajectory in normalised phase space about the point $x = e$. The radius of the circle is a_0 (Fig. 1). The beam centre (zero betatron amplitude) also follows this trajectory.

Since the effective Q for the perturbed machine is very close to an integer, it is not surprising that the perturbing kicks produce a large amplitude of oscillation.

Now Bovet⁽²⁾ gives an expression for the coordinates of the stable fixed point in just such a model. Seen in phase space $1/8$ of a betatron wavelength after the quadrupole

$$x_{\text{SFP}} = - \sqrt{\frac{-e\sqrt{2}}{K_2}} \quad (1a)$$

$$x'_{\text{SFP}} = e/\sqrt{2} \quad (1b)$$

By subtracting the vector $x = e$, $x' = 0$ we have simply the value of a_0 :

$$a_0^2 = (x_{\text{SFP}} - e)^2 + x_{\text{SFP}}'^2, \quad (2)$$

i.e.

$$a_0^2 = \left[\left(\frac{-\sqrt{2}e}{K_2} \right)^{1/2} - e \right]^2 + \frac{e^2}{2} \quad (3)$$

We show in Fig. 2 the results of this calculation. a_0 is plotted in effective mm at the quadrupole as a function of e . The curves are for different K_2 values. It can be seen that if $e = -5$ mm and $K_2 = 0.6$, $a_0 = 30$ mm, i.e. six times the unperturbed displacement.

The e value, 5 mm, taken in this example corresponds to $\Delta p/p = 2.2 \times 10^{-3}$, almost an order of magnitude greater than the theoretical momentum spread in the CPS before debunching. It is our hypothesis that after debunching, which we know is used to deliberately inflate the momentum spread to improve the time structure of the spill, the momentum spread is as high as this.

At the start of extraction, when debunching is over, the lower momenta may therefore have a closed orbit whose oscillation amplitude, a_0 , is greater than the available radial aperture when other effects like beam width, closed orbit

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* A more general formula, covering the case $K_1 \neq 2.0$ has been derived by Baconnier⁽³⁾. This gives a variation of a_0 with e in agreement with the computed results ($K_1 = 1.7$) presented below.

distortion^{*} and deliberate closed orbit bumps are taken into account. In such an eventuality, the lower momenta would be lost immediately after debunching and appear as a pre-ejection loss.

In the simple model we have intentionally excluded the dispersive properties of the perturbing elements, the variation of Q with momentum, and the effect of having more than one sextupole. This is in order to demonstrate that the phenomenon which we consider is not caused by these effects but is principally due to the action of the quadrupole on the non-axial unperturbed orbit.

4. Computations with a more complete model

In order to verify the above predictions of the simple model and to extend the discussions to situations where $K_1 \neq 2$ we have used to program PRTAGS⁽⁴⁾. This includes the dispersive properties of perturbing magnets and the variation of unperturbed Q with momentum. The model now becomes more like the CPS insofar as the position and strength of the actual sextupoles is reproduced. There are six sextupoles of equal normalised strength ($k_2 = 0.002$) and of alternate polarity. Their betatron phase (in units of 2π) with respect to the quadrupole exit are 1.5, 2.125, 2.75, 4.0, 4.625 and 5.25 respectively. For small amplitude motion about

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* A displacement of the unperturbed closed orbit at the quadrupole due to closed orbit distortion would produce an oscillation of the unstable fixed point of the same amplitude as an equal momentum displacement. However, in setting up an ejection system the timing is normally adjusted so that the highest momentum p_0 lies just inside the quadrupole axis. This removes any contribution to a_0 due to closed orbit distortion. The radial aperture consumed by closed orbit distortion remains equal to that for the unperturbed machine, i.e. that at the working Q -value.

the machine axis they are equivalent to the single sextupole of the simple model having strength $K_2 = m \times 0.002 = 0.0068$.

The program computes the stable and unstable fixed points and the separatrices for different momenta (Figs. 3 and 4).

(a) $K_1 = 2$ - The Ideal Situation

The theoretical strengths of perturbation assumed in the design of the CPS extraction system are $K_1 = 2.0$, $k_2 = 0.002$. The position of the stable fixed point was computed as a function of $\Delta p/p$ in order to investigate the momentum effects in this ideal situation. From the coordinates of the stable fixed point (x_{SFP} , x'_{SFP}) the amplitude of its oscillation a_0 was derived using equation (2).

Figure 2 shows a_0 plotted as a function of e (curve A). We also repeated the computation with the momentum dependence of Q , K_1 and k_2 switched off (non-dispersive case). The a_0 values are less than 10% different from those shown. Thus we conclude that the influence of these dispersive effects is small.

It can be seen that the curve A, although in qualitative agreement with the family of curves given by the simple model, is not simply a member of that family with $K_2 = mk_2 = 0.0068$. This can be attributed to the fact that in the complex model the six sextupoles are distributed and the simple vector addition which yields the value $m = 3.4$ is therefore only valid for small amplitudes. In fact Fig. 2 shows that curve A is consistent with $K_2 = 0.0068$ at small amplitude. At larger e and a_0 one must choose a lower value of m to reconcile curve A with the simple model.

When the effective m at large amplitude is calculated by more rigorous vector addition, taking into account the effect of one sextupole on the displacement at the next, one does indeed find a lower value of m . When the simple model is used to evaluate a_0 for this value of m the results are in exact agreement with those computed for the non-dispersive case.

The measure of agreement between the two models gives one some confidence in the physical significance of the results of the computation.

(b) The Practical Situation

We have repeated the computation for a non ideal situation, $K_1 = 1.7$ (the empirically optimised value for the CPS). Then the unperturbed closed orbit corresponding to the double fixed point lies no longer at $e = 0$ but at $e = e_{00}$, a slightly larger radius. The results are plotted also in Fig. 2 (curve B).

It can be seen that, for a given value of e , the amplitude of oscillation of the stable fixed point a_0 is significantly reduced (Fig. 2). Thus, reducing K_1 may be an effective way of minimising the undesirable consequences of a_0 . Moreover, the range of e values occupied by the momenta p_0 to $p_0 - \Delta p$ can be shifted to the right (radially outwards) by adjusting the unperturbed closed orbit at the start of extraction. This further reduces a_0 for the lower momenta.

So far we have not mentioned the unstable fixed point: the origin of the spill along the separatrix. This too is a perturbed closed orbit which, when e is finite, suffers oscillations of amplitude a_1 . For our simple model and if $K_1 = 2$, a_0 and a_1 are identical though the phases of the oscillations are opposed. However, if we look at the history of a particular momentum component it is only when the stable area has shrunk to the point

where it is comparable with the emittance of the beam that spill commences and the position of the unstable fixed point is of importance. In the CPS this occurs only as the double fixed point is approached (Fig. 4). Nevertheless, a_1 does change during the spill.

The phase of oscillation of the unstable fixed point is such that it is in phase with the resonant oscillations themselves. This means that the higher betatron amplitudes will be extracted from an unstable fixed point closer to the septum and therefore will be extracted less efficiently. This effect is emittance dependent and unavoidable. In the ideal situation it is insufficient to account for the discrepancy between measured and predicted efficiencies at the CPS.

But in the practical situation, when $K_1 = 1.7$, the double fixed point is displaced to e_{00} , which corresponds to a finite and significant amplitude, a_{00} . Hence, during spill of a particular momentum population, the small and unavoidable variation in a_1 due to finite emittance is augmented by a_{00} . This can be seen from Fig. 2. Thus, during the spill the unstable fixed point is a distance a_{00} closer to the septum than in the ideal situation.

5. Relevance to the Behaviour of the CERN PS

The CERN PS extraction efficiency, estimated from measurements, is 80% compared with a theoretical 95%. The optimum quadrupole strength, determined empirically, is close to $K_1 = 1.7$ rather than the theoretical value of 2.0. Even at this setting there can be an additional pre-extraction loss of as much as 6% if debunching is not carefully set. A large inflation of the natural momentum spread ($\Delta p/p = 4 \times 10^{-4}$) in the bunched beam is deliberately introduced to improve the time-structure of the spill.

It is possible to explain these effects qualitatively in terms of the theory presented above. Quantitative verification must await more detailed experimental work. The hypothesis is as follows:

(i) The debunching process inflates the momentum spread considerably so that when one also adds the pulse to pulse jitter in the peak momentum the spread in momenta is, say, $\Delta p/p = 2 \times 10^{-3}$. When set to the theoretical values ($K_1 = 2$) the e of the lower momenta is less than - 4.5 mm (α_p for the PS is 2.23 metres in mid D). This results in an amplitude of oscillation, $a_0 = 30$ mm. This is more than the available radial aperture when closed orbit distortion, beam width and deliberate closed orbit bumps are allowed for.

(ii) In an attempt to minimise a beam loss which we would attribute to the above effect, a lower $K_1 = 1.7$ is empirically selected together with an initial radial position which is displaced somewhat outwards. Thus a_0 is reduced to about half its previous value and, provided care is taken in adjusting the debunching, pre-injection loss is avoided.

(iii) The indirect consequence of this non ideal situation is that the unstable fixed point during spill has an additional amplitude of oscillation. The double fixed point is a distance $a_{00} = 12$ mm closer to the septum. The growth rate across the septum, which we have computed, is thus reduced by a factor 2 to 3 and the loss increased by the same factor. Alternatively the oscillation of the unstable fixed point may cause part of the resonant beam to be lost to the vacuum chamber.

6. Conclusions

It seems that the momentum spread in a beam undergoing extraction at an integer resonance could lead to a greater loss of particles than has previously been calculated. This would

seem to be a disadvantage of the integer resonance, where closed orbit distortion and momentum displacement are enhanced by the integer stop band.

The momentum effects described above do seem to be consistent with some of the experimentally observed features of the CPS system. One can envisage more specific experiments which test our hypothesis but these have yet to be performed. Should they confirm the hypothesis, any proposal to adopt integer resonance extraction for a future machine should be critically examined.

Acknowledgement

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References

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Amplitude of oscillation of Fixed Points as a function of e

- a_0 for the simple model, $K_1 = 2$
- - - a_0 computed, $K_1 = 2$
- a_0 and a_1 computed, $K_1 = 1,7$

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Fig. 2

Fig. 3

Fig. 6