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Abstract

In this work, the behaviour of the system with N massive parallels rigid wires are analyzed. The aim is to explore its resemblance with a system of multiple cosmic strings. Assuming that it behaves like a “gas” of massive rigid wires, we use a thermodynamics approach to describe this system. We obtain a constraint relating the linear mass density of the massive wires, the number of the massive wires in the system and the dispersion velocity of the system.

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In this work we analyze the behaviour of the system with N massive rigid wires. The motivation of this analysis is that such system have some resemblance with a system of multiple cosmic strings [1]. These objects formed during a phase transition in an early universe are considered as one of the possible seeds for the large-scale structure formation. Nevertheless, the understanding of its role to the formation of these structures is far from completeness. The difficulties range from a better understanding of how to perform the embedding of the cosmic strings in the Friedman universes (see [2] and references therein) to the difficulties inherent to numerical simulations performed to study cosmic strings networks [3, 4]. Therefore the analysis of the system with massive wires may improve on an understanding of cosmic string dynamics. The N massive wires behaviour are analyzed using a thermodynamics approach, assuming that the system behaves like a “gas” of massive rigid wires.

The description of the gravitational interactions using thermodynamics is an unusual approach to the subject. The main difficulty is due to non-existence of equilibrium state in the gravitational systems. Nevertheless, if the gravitational system is evolving slowly, in such a way that we not consider the coalescence of the system, the thermodynamics approach can be useful [5, 6]. In this case, we can get some insights about gravitational system that would be difficult to be obtained with an usual approach. This is true specially in the N -body simulations, in which the results are difficult to be obtained without a powerful computational capabilities

To get the thermodynamics properties of the massive wires system, we first

determine the partition function

$$\mathcal{Z} = \int e^{-\beta E(\mathbf{x}, \mathbf{p})} d\mathbf{x}_1 d\mathbf{p}_1 \dots d\mathbf{x}_N d\mathbf{p}_N \quad (1)$$

where E is the energy density of the system, $\beta = (kT)^{-1}$, k is the Boltzmann constant, and

$$kT = \frac{\lambda \sigma^2}{2} \quad (2)$$

where σ is the dispersion velocity of the system. The kinetic term of E depend only on the linear momentum and the interaction term is

$$\frac{1}{2} \sum_{i \neq j}^N \Phi_{ij} = \frac{1}{2} \sum_{j=1}^N \sum_{i \neq j}^N 2G\lambda_i \lambda_j \ln r_{ij} \quad (3)$$

where λ_i is the linear density of i th wire. Therefore, the partition function can be factorized as $\mathcal{Z} \equiv \mathcal{Z}_p \mathcal{Z}_q$ where $\mathcal{Z}_p$ is due to kinetic term and $\mathcal{Z}_q$ is due to interaction term.

Thus the pressure P is obtained using the standard thermodynamics relation

$$P = \frac{1}{\beta} \left. \frac{\partial \ln \mathcal{Z}}{\partial V} \right|_{T,N} = \frac{1}{\beta} \left. \frac{\partial \ln \mathcal{Z}_q}{\partial V} \right|_{T,N} \quad (4)$$

Due to symmetries of the system, it is adequate to use a cylindrical coordinate ρ, θ, z with wires parallels to the z -axis, such that $r_{ij} = \rho_{ij}$. We enclose our system in a box in such a way that the wires becomes periodic in z -axis. With this choice, we avoid troubles with the infinity size of wires. Furthermore due its symmetries, all calculations can be reduced to 2+1 dimensional case. Then we effectuate a coordinate transformation $\rho_i = V^{1/2} \hat{\rho}_i$, $\theta = \hat{\theta}$ and where V is an area perpendicular to z -axis. Note that in this approximation, we are not taking in account any momentum component in the z -axis directions. This is done to be

consistent with true cosmic string, which is invariant by Lorentz boosts along its length [7]. Thus, the partition function result in

$$\mathcal{Z}_q = V^N \int \exp \left[-\frac{\beta}{2} \sum_{i \neq j}^N \Phi_{ij} \right] d\mathbf{q}_1 d\mathbf{q}_2 \dots d\mathbf{q}_N \quad (5)$$

where $\mathbf{q}_i \equiv (\tilde{\rho}_i, \tilde{\theta}_i)$. Using equation (5) we obtain

$$\left. \frac{\partial \mathcal{Z}_q}{\partial V} \right|_{T,N} = \frac{N \mathcal{Z}_q}{V} - \beta V^N \int \exp \left[-\frac{\beta}{2} \sum_{i \neq j}^N \Phi_{ij} \right] \frac{1}{2} \sum_{i \neq j} \frac{\partial \Phi_{ij}}{\partial V} d\mathbf{q}_1 d\mathbf{q}_2 \dots d\mathbf{q}_N. \quad (6)$$

Nevertheless, $\Phi_{ij} = 2G\lambda_i\lambda_j \ln \rho_{ij} = 2G\lambda_i\lambda_j \ln V^{1/2} \tilde{\rho}_{ij}$, then

$$\frac{\partial \Phi_{ij}}{\partial V} = \frac{G\lambda_i\lambda_j}{V}. \quad (7)$$

Thus, assuming that $\lambda_i \equiv \lambda$ for all i , we get the equation of the state

$$P = \frac{NkT}{V} \left\{ 1 - G(N-1) \frac{\lambda^2}{kT} \right\} \quad (8)$$

We can obtain some insights on the behaviour of the system comparing the above equation with van der Waals equation

$$\left(P + \frac{a}{V^2} \right) (V - b) = NkT \quad (9)$$

where a is related to the reduced pressure due to long-range interaction, and b is related to the short-range interaction. Then from equations (8) and (9), we obtain that

$$a = \frac{GN(N-1)\lambda^2 V}{2} \quad (10)$$

$$b = 0. \quad (11)$$

Therefore at first sight, we may consider the massive wire system like a non-ideal gas described by van der Waals equation without a hard core. Moreover,

although these resemblance, these two system have a different behaviour. In the van der Waal case, the pressure rise monotonously with N for fixed V, T . On the other hand, in the massive wire case, the pressure reach a maximum at

$$N_1 = \frac{kT}{G\lambda^2} + \frac{1}{2} = \frac{\sigma^2}{2G\lambda} + \frac{1}{2} \quad (12)$$

and decrease after this point.

For fixed N, T there are no differences between the perfect gas and massive wire. This is because the equation (8) is of first degree on the area V , like the perfect gas equation. Then unlike the van der Waals case, the massive wire system do not undergo a phase transition. Therefore the similarity between these two system are only cursory.

Besides the constraint N_1 , we can obtain other one from the equation (8). Note that the temperature of the massive wires system, cannot assume arbitrary values. From equation (8) we get that for fixed N , at temperature

$$T_c \equiv G(N-1) \frac{\lambda^2}{2k} \quad (13)$$

the behaviour of the system change drastically: the pressure become zero, and the coefficient of thermal expansion

$$\alpha = \frac{1}{V} \left. \frac{\partial V}{\partial T} \right|_{N,P} \quad (14)$$

$$= \frac{1}{T} \left[1 - G(N-1) \frac{\lambda^2}{2kT} \right]^{-1} \quad (15)$$

tend to infinity. Therefore the system at this point is highly unstable. Any small perturbation would trigger the collapse of the system, and the thermodynamics approach breakdown.

On the other hand, for temperatures $T \gg T_c$, the collisions are infrequent, and system behaves like a perfect gas. However, with decreasing of the temperature, the system behaviour change. Near to critical temperature T_c , the influence of the collision between massive wires cannot be neglected.

Due to resemblance of the massive wires description with cosmic strings [1], we can expect that our results may be translated qualitatively to the cosmic strings case.

Therefore, it is instructive to compare our results with standard cosmic string N-body simulations. Nevertheless, it is more interesting to use the number of the massive wires instead of the critical temperature T_c . Thus, from the equations (2) and (13) we obtain that

$$N_c = \frac{2\sigma^2}{G\lambda} + 1 > N. \quad (16)$$

Furthermore, because we have $\sigma^2 < c^2$, where c is the light velocity, we obtain a restriction

$$\frac{2c^2}{G\lambda} + 1 > N_c > N. \quad (17)$$

Note that $N_1 < N_c$, and because $N_1 \rightarrow 0$ when $\lambda \rightarrow \infty$, the system becomes unstable well before the critical value N_c is reached. Therefore, we cannot have an arbitrary high value for λ . Using the constraint $\sigma < c$, and equations (2) and (13) we obtain the maximal linear density

$$\lambda_m = \frac{c^2}{G(N-1)} \simeq \frac{1.35 \times 10^{28}}{N-1} \text{ g cm}^{-1}. \quad (18)$$

Now, from the theory of cosmic string formation, we know that $G\lambda$ introduce an azimuthal deficit angle $\delta = 8\pi G\lambda/c^2$, and its magnitude is related to symmetry

breaking scale η as [7]

$$G\lambda \sim \left(\frac{\eta}{m_p}\right)^2 \ll 1 \quad (19)$$

where we used $c = 1$ in the above equation, and m_p is the Planck mass. In the grand scale unification scale we have $\eta \sim 10^{16} \text{ GeV}$, which result in $\lambda \simeq 10^{22} \text{ g} \cdot \text{cm}^{-1}$, a value compatible with galactic formation scenario [3]. From equation (18) we obtain that $\lambda_m \simeq 10^{22} \text{ g} \cdot \text{cm}^{-1}$ only if $N \simeq 1.35 \cdot 10^6$. To perform a numerical simulation with such high value for N is a difficult task with nowadays computational capabilities. However in numerical simulations with cosmic string, for example, in [4] we have $N \simeq 1000 - 16500$, in such a way that the simulations are far from the onset of coalescence of the system. Thus, we can use $\lambda \simeq 10^{22} \text{ g} \cdot \text{cm}^{-1}$ without a trouble, and from equation (16) we obtain $N_c \simeq 0.75 \times 10^{-15} \sigma^2 + 1$. Now using equation (16) we obtain that $\sigma > 1.15 \cdot 10^9 \text{ cm} \cdot \text{s}^{-1} \simeq 0.04c$ for $N_c = 1000$, and $\sigma > 4.69 \cdot 10^9 \text{ cm} \cdot \text{s}^{-1} \simeq 0.16c$ for $N_c = 16500$. Therefore, if the mean velocity of the system reach a lower value than above obtained σ , the system would collapse.

Although our results are suited to describe a system with parallels massive wires, the existence of a threshold for a coalescence should be considered carefully in any gravitational N-body simulations. This remark is specially true in the simulations of cosmic strings networks. Therefore, while our computational facilities are far from capabilities to perform simulations with $N \simeq 10^6$ strings, it should be premature to conclude that cosmic string networks result in a good candidate as a seeds for large-scale structure formations. It is necessary to analyse more carefully the threshold of coalescence in the case of cosmic string networks. In this sense, it is desirable a more detailed analysis of these networks, perhaps using

an approach like a used in this paper. One advantage of this approach is that we can avoid a highly complex and time consuming numerical simulations, or at least reduce this computational time. Nevertheless, it is important to remember that in the analysis of the dynamics of large scale structure formations it is important to determine a characteristic time of these process. Moreover, in a thermodynamics approach we assume a system in equilibrium, therefore we cannot obtain any information concerning a time evolution of the system. Then it would be interesting to change from a thermodynamics approach to kinetic theory approach. With these improvement, a more accurate description of characteristics times involved may be obtained. This would be done elsewhere.

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