INTEGRAL RESONANCE SLOW EXTRACTION FROM THE ISR

by

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1. **INTRODUCTION**

Each of the Storage Rings can be used as a beam stretcher for the CPS. Pulses of particles accelerated by the CPS are injected at regular time intervals into one of the storage rings and slowly extracted from it, in the time interval until another pulse is injected. One can obtain in this way an extracted beam having a duty cycle close to unity. In particular, it is planned to extract particles from s.s. 701 (see Fig. 1) of Storage Ring I towards the future 25-GeV Experimental Hall. This paper gives possible parameters and performances of a system for integral resonance slow extraction.

Calculations are worked out using a normalized system of units, as defined in Appendix I of Ref. 1). The $\beta$-value there denoted by $\beta_n$, is in this report equal to the maximum $\beta$-value, i.e. $\beta_n = \beta_{\text{max}} = 41.3 \text{ m}$ for radial motion, and $\beta_n = \beta_{\text{max}v} = 50.3 \text{ m}$ for vertical motion. Normalized variables will be denoted by the same symbols as the corresponding unnormalized variables, with a bar above. No subscript is used to denote radial motion, whereas the subscript $v$ denotes vertical motion.

Numerical computations have been made with the computer programme described in Appendix II of Ref. 1). Details are given in Ref. 3).

2. **FUNDAMENTAL PRINCIPLES**

This section gives the fundamental principles of integral resonance extraction. Reference is made to 4), 5) and 6) for the detailed theory.
Betatron motion in the machine without any equipment for resonant extraction is hereafter called "unperturbed". In order to obtain integral resonance extraction in the radial plane, the following perturbations are introduced:

i) a quadrupole lens shifts the radial Q-value to the nearest integer, in this case 9. This perturbation makes the radial betatron oscillations unstable;

ii) by adding a suitable non-linear perturbation, only particles having large amplitudes of oscillations are betatron unstable. It can be shown that non-linearities of even order (sextupoles, decapoles, etc.) are required. In the following the non-linear perturbation is assumed to be given by sextupole lenses.

At each azimuth, radial phase-plane diagrams are characterized by two fixed-points, i.e. perturbed closed orbits, and by a separatrix curve, which by definition separates regions of the phase-plane inaccessible one from the other. Small amplitude motion is stable around one of the two fixed-points (stable fixed-point) and unstable around the other (unstable fixed-point). The separatrix curve crosses itself at the unstable fixed-point. The region enclosed by the closed branch of the separatrix contains the unperturbed closed orbit and the stable fixed-point. Inside this region particle motion is stable whereas outside it is unstable.
Fig. 2 shows a typical phase-plane diagram. In order to shift the Q-value from 8.75 to 9, a quadrupole having normalized strength $\tilde{K}_L = -2$ is required. One sextupole having normalized strength $\tilde{K}_S = 0.006 \text{ mm}^{-1}$ is placed at $\frac{3}{8}$ betatron wavelengths downstream to the quadrupole ($\psi_{LS} = 0.75\pi$). The symbols $L'$, $L''$, $S'$ and $S''$ denote the entrance and the exit of the quadrupole (called $L$) and of the sextupole (called $S$) and indicate at which azimuth fixed-points and separatrix curves are given. Figure 2 shows a trajectory starting at point 1 on the outward going separatrix at $L''$. The coordinates of this trajectory at $S'$, $S''$, $L'$, $L''$, $S'$ etc. in the following revolutions are represented by points 2, 3, 4, 5, 6 etc. The sextupole gives a phase advance which at $L'$ produces a positive displacement from the centre of the quadrupole. Consequently, the quadrupole gives a larger kick than in the preceding revolution and produces an increase of amplitude of oscillation, while the phase remains approximately constant. Such resonant oscillations bring betatron unstable particles into the extractor.

Numerical computations show that the effect of the sextupole field components of the ISR bending magnets is fairly small. In general, these sextupole fields are not in phase with resonant oscillations. Therefore, their effects average out over one revolution.

\[ \text{The strength of a lens is defined as follows: if a lens gives to a particle displaced by $\Delta y$ from its centre the kick $\Delta y' = K \Delta y$}, \]

where $y$ stands for $x$ of $z$ according to the plane which is considered and $j$ is the integer, the lens is said to have unnormalized strength $K$. The normalized lens strength $\tilde{K}$ is defined in a similar way and is related to $K$ by $\tilde{K} = \sqrt{\frac{8}{\pi}} \left(\frac{\beta}{\beta_n}\right)^{\frac{1}{2}} K$.

\[ \text{The betatron phase function } \int (ds/\beta) \text{ is denoted by the capital letter } \psi, \text{ whereas the phase (measured in the clock-wise direction from the x-axis) of the outward-going separatrix curve in a normalized phase-plane is denoted by } \psi. \text{ The advance of } \psi \text{ from an azimuthal location } A \text{ to another location } B \text{ is denoted by } \psi_{BA}. \text{ The phase of the outward-going separatrix curve in the normalized phase-plane at azimuthal location } A \text{ is denoted by } \psi_A. \]
The size of the stable region is strongly dependent on the radial position of the unperturbed closed orbit, which in Fig. 2 is taken as origin of the phase plane. In particular one can show\(^6\) that it is specially influenced by the displacement \((x_L)\) of the unperturbed closed orbit from the centre of the quadrupole. This feature is utilized to extract at a constant rate the beam circulating in the synchrotron. The stable area is initially large enough such that all the circulating particles are betatron stable. By displacing radially the unperturbed closed orbit, the stable area is reduced to zero. All the particles, starting from those having the largest amplitudes of oscillations, are in this way split over the stability limits. A small reduction of the stable area causes a layer of betatron unstable particles to be produced along the closed separatrix. These spilt particles move along the closed separatrix and then along the outward-going separatrix, which, if the stable area is shrunk adiabatically, during this motion can be considered unchanged. A one-revolution step taken by a trajectory along the outward-going separatrix defines the width of a beam which contains all spilt particles. Such a beam, called spilt beam, moves out along the outward-going separatrix and from revolution to revolution becomes wider and wider. Therefore the linear density of spilt particles on the outward-going separatrix decreases with increasing amplitude of oscillation.

The spilt beam is extracted by means of a septum magnet, the septum of which cuts the separatrix curve. The particle loss is proportional to the septum thickness, to the slope of the outward-going separatrix in the phase-plane at the septum and the particle density along the outward-going separatrix.
Hence the septum must be placed where resonant oscillations have large displacements and as far as possible from the centre of the vacuum chamber. The maximum normalized amplitude of oscillation that particles having their equilibrium orbit at the centre of the aperture can have without exceeding the aperture limits of the ISR is:

\[ r_m = \frac{150 - 33}{2} = 58.5 \text{ mm} \] (2.1)

where 150 mm is the width of the vacuum chamber and 33 mm are the closed orbit distortions²).

One can show that sextupoles placed where resonant oscillations have large displacements and excited with such a polarity that they advance phases of resonant oscillations, cause the largest amplitude increase per turn of resonant oscillations. Any suitable sextupole perturbation gives at \( L' \) and \( L'' \) a phase-plane diagram very similar to that represented in Fig. 2. This means that the phase of resonant oscillations is determined by the quadrupole.

The quadrupole lens and non-linear lenses perturb the optical beam properties in vertical plane, thus causing an increase of maximum beam height. In particular, the quadrupole lens causes a static perturbation, whereas non-linear lenses cause a perturbation, the magnitude of which increases with increasing amplitude of oscillation of the split beam. In the ISR the permissible increase of beam height is relatively small: the unperturbed beam is higher than in the CPS due to the larger betatron amplitude function and to beam transfer errors, and the vertical aperture is smaller. Fig. 3 gives
the permissible beam height increase as function of the height of the unperturbed beam at \( \beta_{\text{max}} \) for different vertical closed orbit distortions. The dotted curve refers to the case where vertical closed orbit distortions have the value given in Ref. 2). One realizes that particular attention must be paid to the vertical optics.

3. **Extraction Point**

The extraction point, called E, is at the upstream end of s.s. 701. One metre from the upstream end of s.s. 701 the radial and vertical \( \beta \) and \( \alpha \)-values are:

\[
\begin{align*}
\beta_E &= 29 \text{ m} & \beta_{Ev} &= 17 \text{ m} \\
\alpha_E &= 0.62 & \alpha_{Ev} &= 0.56
\end{align*}
\]

The extracted beam is deflected out of the machine aperture by means of one or more septum magnets, placed on s.s. 701. The most upstream of these magnets is placed at E. The thickness of its septum must be at least of the order of 1 mm (Section 5.6). It is assumed that during the operation of the storage ring as a beam stretcher, septums remain plunged inside the machine aperture.

4. **Quadrupole Location**

A fundamental parameter of an extraction system is the location of the quadrupole, which determines the phase of resonant oscillation with respect to the machine.

In order to have a good extraction efficiency resonant oscillations must have large displacements at E. This is
achieved if the advance \( \psi_{LE} \) of the radial betatron phase function \( \psi \) from L to E is approximately \( 0.75\pi + k\pi \), \( k \) being an integer. Since the amplitude increase of resonant oscillations is produced by the quadrupole (Fig. 2) the split beam reaches the largest amplitude of oscillation between the quadrupole and the extraction magnet. In this section of the ring there should be as few extremes of resonant oscillations as possible, i.e. \( k \) should be as small as possible.

The normalized quadrupole strength required to shift the radial Q-value from 8.75 to 9 is:

\[
\overline{K}_L = -2 \quad (4.1)
\]

A quadrupole having this strength is called "normal". The unnormalized quadrupole strength is \( K_L \), expressed by:

\[
K_L = \frac{\ell (dB/dx)}{(B_0)} = \frac{\overline{K}_L}{\beta_L} \quad (4.2)
\]

where \( \ell \) is the effective length of the quadrupole, \( (dB/dx) \) the field gradient, \( (B_0) \) the magnetic rigidity of the protons and \( \beta_L \) the radial \( \beta \)-value at the quadrupole. Eq. (4.2) shows that a large \( \beta_L \) is required.

The normalized quadrupole strength for vertical motion is \( \overline{K}_{LV} \), expressed by:

\[
\overline{K}_{LV} = - \overline{K}_L \frac{\beta_{LV}}{\beta_L} \quad (4.3)
\]
where $\beta_{LV}$ is the vertical $\beta$-value at the quadrupole. Since the static perturbation of the vertical optics increases with increasing $\bar{K}_{LV}$, a large $\beta_L/\beta_{LV}$ is required.

Suitable quadrupole locations are the downstream end of s.s. 733 and the downstream end of s.s. 801. The two corresponding extraction systems, called Extraction System L733 and Extraction System L801, are discussed in Sections 5 and 6 respectively.

5. **EXTRACTION SYSTEM L733**

5.1. Quadrupole

The downstream end of s.s. 733 is the nearest location upstream to L suitable to accommodate the quadrupole. 1.5 m from the downstream end of s.s. 733 one has:

$$\beta_L = 18.5 \text{ m} \quad \beta_{LV} = 13.3 \text{ m} \quad (5.1)$$

The advances of the radial and of the vertical betatron phase functions from this location to L are respectively:

$$\Psi_{EL} = 0.33 \times 2\pi \text{ rad} \quad \Psi_{ELV} = 0.34 \times 2\pi \text{ rad} \quad (5.2)$$

The required unnormalized strength of a normal quadrupole is (Eqs. (4.1), (4.2) and (5.1)):

$$K_L = -0.108 \text{ m}^{-1} \quad (5.3)$$

At 26 GeV this corresponds to:

$$\left(\frac{dB}{dx}\right) = 10.4 \text{ T} \quad (5.4)$$

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From Eqs. (4.1), (4.3) and (5.1) one obtains:

$$K_{LV} = 1.44 \quad .$$  \hfill (5.5)

5.2. Sextupoles

The phase of the outward-going separatrix in the phase-plane at $L'$ is denoted by $\psi_{L'}$. Let $\psi$ be the radial phase advance, measured from the location of the quadrupole. One period of the function $\cos (\psi_{L'} + \psi)$, which for $\psi < 0$ is proportional to the displacement of resonant oscillations, is represented in the upper part of Fig. 4. The lower part of Fig. 4 shows the layout of the long straight sections in the Storage Ring I. The intervals $0 \leq \psi/2\pi \leq -1, \ldots, -8 \leq \psi/2\pi \leq -8.75$ are drawn on consecutive lines. Sextupole lenses should be placed near to extreme values of $\cos (\psi_{L'} + \psi)$.

The relation between the normalized strength ($K_S$) and the unnormalized strength ($K_S$) of a sextupole placed where the radial $\beta$-value is $\beta_S$, is:

$$\bar{K}_S = K_S \sqrt{\frac{\beta_S^3}{\beta_{max}}} \quad \hfill (5.6)$$

The product of the second radial derivative of the vertical magnetic field times the effective sextupole length $\ell$ is:

$$\left( \frac{d^2 B}{dx^2} \right) \ell = 2(Bc) K_S \quad \hfill (5.7)$$

Combining Eqs. (5.6) and (5.7) one obtains:

$$\left( \frac{d^2 B}{dx^2} \right) \ell = 2(Bc) \sqrt{\frac{\beta_{max}}{\beta_S}} \bar{K}_S \quad . \hfill (5.8)$$

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A large $\beta_S$ is therefore required. Moreover, a large $\beta_S/c_{Sv}$ is required to make the perturbation of the vertical optics small.

The above criteria indicate that s.s. 117, 149, 517 and 549 are suitable to accommodate sextupoles. For ejection to the outside of the ring, the polarity of sextupoles placed in these straight sections must be positive, negative, negative and positive, respectively. Assuming $\beta_S = \beta_{\text{max}}$, for 28 GeV protons, Eq. (5.8) gives:

$$\left(\frac{4a_B^2}{dx}\right)_T = 4680 \overline{K}_S \quad \text{[m]}$$

where $\overline{K}_S$ is expressed in mm⁻¹.

5.3. Thin septum magnet

The beam to be extracted can be separated from the circulating particles, before it enters the extraction magnet, by means of a septum magnet (T) which produces a displacement at E just sufficient to fit into it the septum of the extraction magnet. Particle loss occurs only on the septum of T. Since the required strength of T is smaller than that of the extraction magnet, its septum can be made thinner. It is usually called the "thin" septum magnet.

In order to obtain a small particle loss per unit septum thickness T should be placed where resonant oscillations have large normalized displacements. On the other hand, the required septum thickness decreases with increasing ratio between the displacement $\Delta x_B$ produced at E and the kick $\Delta x_T$ given by T. Therefore a normalized factor of merit f of T can be defined as follows:

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\[ f = \cos^2 \theta_T \frac{\Delta x_{E}}{q(x_T)} \frac{\Delta x_{E}^2}{\Delta x_T^3} \] (5.10)

where \( q(x_T) \) is the normalized particle density on the outward-going separatrix at the amplitude of oscillation \( x_T \), at which \( T \) cuts the outward-going separatrix. Since the unnormalized particle density is inversely proportional to \( \beta_T^2 \) and:

\[ \frac{\Delta x_{E}}{\Delta x_T^2} = \sqrt{\beta_E^2 \beta_T^2} \frac{\Delta x_{E}^2}{\Delta x_T^3} \] (5.11)

where \( \beta_T \) is the radial \( \beta \)-value at the location of \( T \), the unnormalized factor of merit is proportional to \( \beta_T \sqrt{\beta_E f} \).

The thin septum magnet can be placed upstream to \( L \) or between \( L \) and \( E \). In the case where the thin septum magnet is placed between \( L \) and \( E \),

\[ \frac{\Delta x_{E}}{\Delta x_T^2} = \sin \psi_{ET} \] (5.12)

The thin septum magnet, called in this case T717, could be placed near to the downstream end of s.s. 717. Three metres from the downstream end of s.s. 717, one has:

\[ \beta_{T717} = 39 \text{ m} \quad \beta_{T717v} = 15 \text{ m} \]
\[ \alpha_{T717} = -0.35 \quad \alpha_{T717v} = -0.42 \]
\[ \psi_{ET717} = 0.12 \times 2\pi \text{ rad} \quad \psi_{ET717v} = 0.12 \times 2\pi \text{ rad} \] (5.13)
From Eqs. (5.11), (5.12) and (5.13) one obtains:

\[ \frac{\Delta x_E'}{\Delta x'}_{T717} = 0.68 \text{ mm/mm} \]

\[ \Delta x_E'/\Delta x'_{T717} = 23 \text{ mm/mrad} . \quad (5.14) \]

In the case where the thin septum magnet is placed upstream to L:

\[ \frac{\Delta x_E}{\Delta x'}_T = \sin \psi_{ET} + \bar{k}_L \sin \psi_{LT} \sin \psi_{EL} \quad (5.15) \]

Introducing Eq. (5.15) into Eq. (5.10) one obtains:

\[ f q(x_T) = \frac{1}{2} \left[ A + B \cos (2 \psi_{LT} - \gamma) \right] \quad (5.16) \]

where:

\[ A = \sin (\psi_{EL} + \phi_L) + \bar{k}_L \sin \phi_L \sin \psi_{EL} \]

\[ B = \sqrt{1 + \bar{k}_L \sin 2\psi_{EL} + \bar{k}_L^2 \sin^2 \psi_{EL}} \]

\[ \sin \gamma = (\cos (\psi_{EL} - \phi_L) + \bar{k}_L \sin \psi_{EL} \cos \phi_L) / B \]

\[ \cos \gamma = (\sin (\psi_{EL} - \phi_L) - \bar{k}_L \sin \psi_{EL} \sin \phi_L) / B \]

The upper part of Fig. 4 shows the right hand sides of Eqs. (5.15) and (5.16) as functions of \( \psi_E - \psi_{LT} \). Fig. 4 shows that a suitable location for the thin septum magnet is the downstream end of s.s. 701. A thin septum placed
there is called T701. Three meters from the downstream end of s.s. 701 one has:

\[ \beta_{T701} = 21 \text{ m} \quad \beta_{T701v} = 15 \text{ m} \]
\[ \alpha_{T701} = 0.034 \quad \alpha_{T701v} = -0.39 \quad (5.17) \]
\[ \psi_L T701 = 8.33 \times 2\pi \text{ rad} \quad \psi_L T701v = 8.27 \times 2\pi \text{ rad} \]

From Eqs. (5.11), (5.15) and (5.17) one obtains:

\[ \frac{\Delta x_E}{\Delta x_{T701}} = -2.4 \text{ mm/mm} \]
\[ \frac{\Delta x_E}{\Delta x_{T701}'} = -59 \text{ mm/mrad} \quad (5.18) \]

It should be noted that Eq. (5.15) is valid in the case where between T and L there are no sextupole lenses, or if they are an integral number of half betatron wavelengths downstream to T. In general, sextupoles are near to such positions because sextupoles and thin septum magnets are placed where betatron oscillations have large displacements. One can see that sextupoles placed slightly upstream to those positions increase the displacement given at E by the thin septum magnet. This is the case for sextupoles placed as described in Section 5.2. The theoretical extraction efficiencies which can be achieved by using T717 or T701 are worked out in Sections 5.7 and 5.8 respectively.

5.4. Radial phase-plane

The sum of the absolute values of the normalized strengths of the sextupoles, placed according to Section 5.2, is called total normalized sextupole strength and is hereafter denoted
by $\overline{K}_S$. Figure 5 shows the normalized phase-plane diagram at $L', L''$, $E$, T701 and T717, in the case where $Q = 8.75$, $\overline{K}_L = -2$ and $\overline{K}_S = 0.005 \text{ mm}^{-1}((d^2 E/dx^2) \ell = 23.4 \text{ T/m at 28 GeV}.)$. The solid line refers to the case where the unnormalized stable area ($A$) corresponds to a practical ISR beam size, whereas the dotted line refers to the case $\ell = 0$. A particle situated on the outward-going separatrix and having normalized amplitude of oscillation $r$, after one revolution has a larger amplitude of oscillation $R$. Figure 6 gives $R$ vs. $r$ and the normalized linear particle density $q(r)$ on the outward-going separatrix (expressed in $\%$ of the intensity of the spilt beam per mm of normalized displacement along the outward-going separatrix) for different values of the normalized stable area $\overline{A}$. The dotted line, having equation $R = r$, is the locus of amplitude of oscillations of the fixed-points.

5.5. Spill-out

The stable region is shrunk by varying the main guiding field or by means of a closed orbit bump at the quadrupole. Figure 7 gives the stable area vs. $|\overline{X}_L|$, in the case where the unperturbed closed orbit is not displaced at the sextupoles. Denoting by $N(A)$ the number of particles contained inside the stable region when its area is $A$, the intensity $I$ of the extracted beam is given by:

$$ I = -\frac{dN(A)}{dt} = -\frac{dN(A)}{dA} \frac{dA}{dx_L} \frac{dx_L}{dt} \quad (5.19) $$

The time dependence of $x_L$ must be such that $I = \text{const}$. A deviation $A|\frac{dx_L}{dt}|$ from the correct value of $|\frac{dx_L}{dt}|$ causes a variation of intensity.

* Since this bump has very small amplitude, it could be given by a single dipole magnet, placed at about $3/8$ wavelengths from $L$.

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\[ \frac{\Delta I}{I} = \frac{\Delta \left| \frac{dx_L}{dt} \right|}{\left| \frac{dx_L}{dt} \right|} \]  \hspace{1cm} (5.20)

It follows that a ripple on \( x_L \) of amplitude \( \varepsilon \) and frequency \( \omega \) causes a peak variation of \( I \):

\[ \left( \frac{\Delta I}{I} \right)_{\text{max}} = \frac{\omega \varepsilon}{\left| \frac{dx_L}{dt} \right|} \]  \hspace{1cm} (5.21)

In particular, a ripple on \( x_L \) is caused by the ripple on the dc voltage of the main magnet system. Denoting by \( v \) the amplitude of this ripple, by \( \omega \) its frequency and by \( V \) the d.c. magnet voltage, the peak deviation of the guiding magnetic field is:

\[ \left( \frac{\Delta B}{B} \right)_{\text{max}} = \frac{1}{\omega \tau} \frac{v}{V} \]  \hspace{1cm} (5.22)

where \( \tau \) is the time constant of the magnet system. The closed orbit displacement caused by \( (\Delta B/B)_{\text{max}} \) at \( L \) is:

\[ \varepsilon = \alpha_{PL} (\Delta B/B)_{\text{max}} \]  \hspace{1cm} (5.23)

where \( \alpha_{PL} \) is the value of momentum compaction function at \( L \).

Combining Eqs. (5.21), (5.22) and 5.23) one obtains:

\[ \left( \frac{\Delta I}{I} \right)_{\text{max}} = \alpha_{PL} \frac{v}{\tau \sqrt{V}} \frac{1}{\left| \frac{dx_L}{dt} \right|} \]  \hspace{1cm} (5.24)

Since \( \alpha_{PL} = 2.3 \) m and \( \tau = 5.5s \), an intensity fluctuation of less than \( \pm 10\% \) requires:

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\[ \left| \frac{1}{|\Delta x_{L}/\Delta t|} \right| v < 2.4 \times 10^{-4} \, \text{sec/mm} \]  (5.25)

The closed orbit displacement which is necessary for extracting a beam of nominal size is of the order of 1 mm. Therefore \(1/|\Delta x_{L}/\Delta t|\) is of the order of magnitude of the extraction time. This shows that long extraction times impose very strict requirements on the magnet ripple. The effect of this ripple could be compensated by means of feedback acting on the closed orbit bump at the quadrupole.

5.6. Extraction (no thin septum)

In the section of the ring from the quadrupole to the extraction point, where a good closed orbit control is foreseen, the amplitude of oscillation of the extracted beam can be larger than \(r_m\). On this assumption, the amplitude of oscillation \(r_E\) where the septum of the extraction magnet is placed can be \(r_E = r_m = 58.5 \, \text{mm}\). This corresponds (Fig. 5) to a position and a slope of the septum of the extraction magnet with respect to the equilibrium orbit:

\[ \overline{x}_E = 58.2 \, \text{mm} \quad \overline{x}'_E = 6 \, \text{mm} \]
\[ x_E = 49 \, \text{mm} \quad x'_E = -0.9 \, \text{mrad} \]

Figure 8 shows the maximum amplitude of oscillation \(R_E\) of the extracted beam and \(q(r_E)\) as functions of \(K_S\) and \(A\). These curves are obtained from Fig. 6 using the scaling laws introduced by H.G. Horwood. Since \(r_E\) is large compared to the size of the stable region, the motion of particles having this amplitude of oscillation is roughly independent from \(A\).
It follows that \( q(x) \) and \( R_E \) are approximately constant during the spill-out time. The normalized width of the extracted beam at \( E \) is \( \bar{w}_E \), expressed by:

\[
\bar{w}_E = (R_E - x_E) \cos \phi_E
\]  

(5.26)

Figures 9 and 10 show the extracted beam in the plane \( \bar{x}, \bar{x}' \) and in the plane \( \bar{y}, \bar{x} \), respectively.

The deflection of the beam away from the machine can be given by three septum magnets \( E_1, E_2 \) and \( E_3 \), the parameters of which are given in Table 4.1. Their lengths are 2 m, 1 m and 1 m respectively. The width of the useful field is 50 mm in all those septum magnets. The magnet \( E_1 \) is placed near to the upstream end of s.s. 701. 2.5 m downstream to its centre it produces between the extracted beam and the circulating particles a gap which is sufficient to accommodate the 4 mm water-cooled septum of \( E_2 \). The magnet \( E_3 \), having an 8 mm water-cooled septum is placed immediately downstream to \( E_2 \). At the downstream end of s.s. 701 these magnets give a displacement of approximately 175 mm. The displacement of the inner trajectory of the extracted beam from the centre of the vacuum chamber is approximately 210 mm. The yoke of \( \text{m.u.} \) 667 should be reversed. From Fig. 8 taking into account that \( \cos \psi_E \approx 1, \beta_E = 29 \) m., one obtains a theoretical extraction efficiency of the order of 96%.
### TABLE 4.1
Parameters of Septum Magnets

<table>
<thead>
<tr>
<th>Parameter</th>
<th>T</th>
<th>E₁</th>
<th>E₂</th>
<th>E₃</th>
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<tr>
<td>Deflection per unit length [mrad/m]</td>
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<td>0.7</td>
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<td>Magnetic field [Tesla]</td>
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<td>0.67</td>
<td>0.67</td>
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<td>Septum thickness [mm]</td>
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<td>Gap height [mm]</td>
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<td>20</td>
<td>20</td>
<td>20</td>
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<tr>
<td>Number of coil turns</td>
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<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Current [KA]</td>
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<td>Power per unit length [kW/m]</td>
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<tr>
<td>Coil cross-section [mm²]</td>
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<td>20</td>
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<tr>
<td>Resistance per unit length [mΩ/m]</td>
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<td>0.36</td>
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<td>Current density [A/mm²]</td>
<td>58</td>
<td>55</td>
<td>196</td>
<td>80</td>
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</tbody>
</table>

The emittance of the extracted beam (Fig. 9) is roughly equal to 1/3 of the initial emittance of the circulating beam. Its shape can be described by a complex number \( Z \), defined as in Ref. 8), or by its reciprocal \( Y \). In this paper these complex numbers are denoted by \( \overline{Z} \) and \( \overline{Y} \) if evaluated on the normalized phase-plane and by \( Z \) and \( Y \) if evaluated on the unnormalized phase-plane. One can show that:

\[
Y = (\overline{Y} + i\alpha)/\beta \quad (5.27)
\]
From Fig. 9 one obtains that at $E$ the normalized $Y$-value is:

$$\overline{Y}_E = 0.08$$

Therefore:

$$Y_E = \left(\overline{Y}_E + i\alpha_E\right)/\beta_E = 0.0028 + 0.0214i$$

$$Z_E = \frac{1}{Y_E} = 6 - 46i$$

5.7. Extraction (thin septum at s.s. 717)

The septum magnets $E_2$ and $E_3$ can in this case be placed one after the other near to the upstream end of s.s. 701. Because of the increased distance to the downstream ends of s.s. 701, the required strengths are smaller than in the preceding section, namely 6 mrad each. The septum magnets of Table 4.1, could be run with a current of 9.2 KA instead of 10.6 KA. The power dissipation is 30 KW/m and 13 KW/m instead of 42 KW/m and 17 KW/m respectively. T717 must give a reflection of 0.30 mrad. This includes the deflection (0.17 mrad) necessary for obtaining a useful displacement of 4 mm at $E$ and the spread (0.12 mrad) of the slope of the extracted beam at T717. If T717 has the parameters given in column T of Table 4.1, its effective length must be 1 m. The septum of T717 should be placed at

$$\overline{x}_{T717} = 37 \text{ mm} ; \quad \overline{x}_{T717} = 42 \text{ mm}$$

$$x_{717} = 36 \text{ mm} ; \quad x'_{717} = 1.4 \text{ mrad}$$

The theoretical extraction efficiency is approximately 98%. The emittance of the extracted beam is practically the same as in section 5.6.
5.8. Extraction (thin septum at s.s. 701)

Allowing between L and E an amplitude of oscillation larger than \( r_m \), the amplitude of oscillation \( r_T \) corresponding to the thin septum is such that at the end of the extraction process, (when the spilt beam is wider) its one-revolution shadow has amplitude of oscillation \( r_m \). Figure 11 gives \( r_T', r_E', R_E \) and \( q(r_T) \) as functions of \( K_S \). These curves can be deduced from Fig. 6 using scaling laws. The arrangement of the septum magnets in s.s. 701 can be the same as in Section 5.7. The magnet T701 must provide a deflection which compensates for the variation of the width of the spilt beam in the last revolution, from the beginning to the end of the extraction process. In the case where the initial beam emittance is \( A = 4\pi \) mrad mm, this variation in normalized units is \( 8 \) mm (Fig. 6). A useful displacement of \( 4 \) mm at E is obtained if T701 provides a displacement:

\[
\Delta X_E = 13 \text{ mm} \\
\Delta X_E = 11 \text{ mm}
\]

Inserting this displacement into Eqs. (5.18) one obtains:

\[
\Delta x'_T701 = -5.4 \text{ mm} \\
\Delta x'_T701 = -0.19 \text{ mrad}
\]

If T701 has the parameters given in column T of Table 4.1, its effective length must be 0.65m. The width of the useful field must be at least 15 mm. In the case where \( K_S = 0.005 \text{ mm}^{-1} \), which as Fig. 11 shows is a convenient sextupole strength, one has:
from Figs. 5 and 11:

\[ x_T = 35 \text{ mm} ; \quad \bar{x}_{T701} = 31 \text{ mm} ; \quad x_{T701} = -12 \text{ mm} \]

\[ x_{T701} = 22 \text{ mm} ; \quad x_{T701}^1 = 0.5 \text{ mrad} \]

T701 bends the beam inwards. Hence an additional particle loss occurs along its length. This is expressed by saying that the effective septum thickness is larger than the physical one by \((\ell \Delta x_{T701}^1)/2\), where \(\ell\) is the length of T701. There is a minimum of particle loss in the case where the particle losses on the septum thickness and on the septum length are approximately equal. In practice it is convenient to have thicker and shorter septums. The theoretical extraction efficiency, which varies during the spill-out process (Fig. 11), is 92% when \(A/\pi = 4\) mrad mm and 95% when \(A = 0\). Those efficiencies have been obtained by multiplying \(\varepsilon(r_T)\sqrt{\beta_{T701}/\beta_{\text{max}} \cos \psi_{T701}}\) by the effective septum thickness (0.5 mm). Figure 12 shows the extracted beam in the normalized phase-plane. The emittance of the extracted beam is practically the same as in Section 5.6.

5.9. Thin septum lens in s.s. 701

The width of the extracted beams can be reduced introducing a quadrupole component in the field of T701. Denoting by \(\Delta x_{T701}^1\) and \(\Delta x_{T701}^0\) the deflections given by T701 to the inner and to the outer trajectory of the extracted beam, the width of the extracted beam at \(E\) is changed by:

\[ \Delta x_E = (\Delta x_{T701}^1 - \Delta x_{T701}^0) \frac{\Delta x_E}{\Delta x_{T701}^1} \quad (5.2) \]

where \(\Delta x_E/\Delta x_{T701}^1\) is given by Eq. (5.18). Denoting by
\( \overline{w}_{T701} \) the normalized width of the extracted beam at T701, the normalized quadrupole strength of T701 is \( \overline{K}_{T701} \), expressed by:

\[
\overline{K}_{T701} = \frac{\Delta x'_{T70l} - \Delta x'_{T70li}}{\overline{w}_{T701}} = \frac{\Delta w_E}{\overline{v}_{T701} \left( \frac{\Delta x_E}{\Delta x'_{T70l}} \right)} \quad (5.29)
\]

In order to obtain \( \Delta w_E = -30 \text{ mm} \), one requires:

\[
\Delta x'_{T70l} - \Delta x'_{T70li} = 12.5 \text{ mm} \quad \Delta x'_{T70l} - \Delta x'_{T70li} = 0.42 \text{ mrad} \\
\overline{w}_{T701} = 0.6 \quad \overline{K}_{T701} = 0.028 \text{ m}^{-1} \quad (5.30)
\]

where it has been taken \( \overline{w}_{T701} = 21 \text{ mm} \). Figure 13 shows the phase-plane diagram which is obtained.

The septum of a quadrupole septum magnet must be curved in the vertical plane, in order to follow the equipotential lines of the magnetic field. Hence T must provide at E an additional displacement, equal to the shadow at E of the sagitta of the part of the thin septum which intercepts the beam. Because of the small amplitude at which the thin septum must be placed, the additional strength required and the fact that the effective septum thickness is larger than the physical septum thickness, a thin septum placed between L and E does not in the present case give any gain in extraction efficiency. However, a thin septum lens gives an important reduction of the emittance of the extracted beam and facilitates keeping the extracted beam inside the machine aperture from L to E. If the thin septum does not have a quadrupole field component, it increases the amplitude of oscillation of the extracted beam from L to E.
5.10. Momentum dependence

The displacement of the unperturbed closed orbit and the lens strengths depend on the particle momentum. Therefore, particles having different momenta have a different phase-plane diagrams as shown in Fig. 14, where particles in resonant condition \((A = 0)\) have \(\Delta p/p = 0\). Betatron stable particles move about the stable fixed point, the amplitude of oscillation of which, called \(r_s\), is an increasing function of \(\Delta p/p\) (Fig. 15).

The intersections of the curves in Fig. 15 with the straight line \(r_s = r_m\) give the largest \(\Delta p/p\) that particles can have without being lost, in the extreme case of infinitely small betatron emittance. This shows the difficulty of extracting a beam having a large momentum spread.

5.11. Higher order non-linear fields

The particle loss is determined by the increase rate of resonant oscillations at amplitudes smaller than the amplitude at which the septum is placed, while the increase rate of resonant oscillations at amplitudes larger than this determines the width of the extracted beam. This shows that using sextupole lenses, the field of which at larger amplitudes is weaker than the nominal sextupole field, one can achieve a reduction of the width of the extracted beam without any relevant loss of extraction efficiency. A reduction of the field at large amplitudes corresponds to adding symmetrical non-linear fields of higher order and opposite polarity (decapole fields) to the sextupole fields. Denoting by \(\text{\textbf{K}}_S\) and \(\text{\textbf{K}}_D\) the normalized sextupole and decapole strengths, the normalized strength \(\text{\textbf{K}}_N\) of the non-linear lens

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consisting of the sextupole and the decapole is defined as follows:

\[ \bar{K}_N = \bar{K}_S + \bar{K}_D \bar{x}_0^2 \]  

(5.31)

At \( \bar{x} = \bar{x}_0 \) the kick given by the non-linear lens is equal to the kick given by a sextupole having normalized strength \( \bar{K}_N \). (Fig. 16a.) Figure 16b gives the maximum amplitude of oscillation of the extracted beam as function of \( \bar{K}_N \), for \( \bar{x}_0 = 50 \text{ m} \) and different values of \( \bar{K}_D \bar{x}_0^2/\bar{K}_S \). A similar effect on the width of the extracted beam has been computed in the case where the quadrupole contains an octupole field component of positive polarity. These higher-order non-linear fields at large amplitudes also cause a bending of the outward-going separatrix in the anti-clockwise direction.

5.12. Static perturbation of vertical optics

The ratio between the major and the minor axes of perturbed, normalized emittance ellipses (unperturbed, normalized emittance ellipses are circular) is called "beaf factor" and denoted by G. The expression \( \sqrt{G-1} \) gives the increase of the maximum beam height, taking the unperturbed beam height as unit. The beaf factor caused by the quadrupole is 6):

\[ G = \frac{2 + \bar{K}_{LV} \tan(\pi Q_v)}{2 - \bar{K}_{LV} \cot(\pi Q_v)} \]  

(5.32)

where \( Q_v \) is the Q-value for vertical motion. If the right hand side of Eq. (5.32) is smaller than 1, its reciprocal should be taken, such that \( G > 1 \). The minimum value of G as function of \( Q_v \):
\[ G_{\text{min}} = \left| \frac{X_{LV}}{2} \right| + \sqrt{1 + \left( \frac{X_{LV}}{2} \right)^2} \]  

is reached in the case where

\[ Q_v = 8.75 + \frac{\arctan \left( \frac{X_{LV}}{2} \right)}{2\pi} \]

Fig. 17 shows \( \sqrt{C-1} \) vs. \( Q_v \) in the case where \( X_{LV} = 1.44 \) (Extraction System L733) and in the case where \( X_{LV} = 1.18 \) (Extraction System L801). The dotted line is the locus of the minima of \( \sqrt{C-1} \) taking \( X_{LV} \) as variable parameter. The dependence of the unperturbed beam height on \( Q_v \) can be neglected compared to \( \sqrt{C-1} \).

Using \( \Upsilon \)-values to describe emittance ellipses, one can represent perturbations to betatron optics on admittance diagrams as used in electrical transmission-line calculations. When the beam traverses a quadrupole field, the imaginary part of \( \Upsilon \) decreases by the normalized quadrupole strength. Along a section of unperturbed machine the \( \Upsilon \)-value moves in the clock-wise direction on one of the circles having their centre on the horizontal axis, the phase advance being equal to the betatron phase advance; each of these circles is characterized by a constant \( C \)-value. On the adiabatic approximation, the trajectory of \( \Upsilon \) on the admittance diagram is determined by the condition that it must close itself after one revolution. The rectangular admittance diagram of Fig. 18 shows the static perturbation caused by \( L \). The \( \Upsilon \)-values at the upstream and downstream at s.s. 733 are denoted by 733 u and 733 d, respectively. The same notation applies to other straight sections.

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The beam height increase caused by L could be reduced by means of a correcting quadrupole \( 6^\prime \) (c) placed at s.s. 725. Although this is impractical because s.s. 725 already accommodates a sextupole lens, Figure 19 shows the static perturbation of vertical optics in the case where such a correcting quadrupole has strength \( \bar{K}_{QV} = -0.4 \) \( (dE/dx) \ell = 0.74 \) T at 26 GeV. The G-value is reduced over all the ring except from L to C. However, in this section the beating phase is such that the beam height is not increased. Figure 20a and 20b show \( \sqrt{G-1} \) vs. \( \bar{K}_{QV} \), for different values of \( Q_v \), in the case where \( \bar{K}_{LV} = 1.44 \) (Extraction System L733) and the case where \( \bar{K}_{LV} = 1.18 \) (Extraction System L801) respectively.

5.13. Dynamic perturbation of vertical optics

Sextupole lenses perturb the split beam with quadrupole fields which for vertical motion have positive polarity, if sextupoles are placed as described in Section 5.2. The strength of these fields increases with increasing amplitude of oscillation. On the assumption that the unperturbed closed orbit is undistorted at the sextupoles, the maximum normalized strength reached is:

\[
\bar{K}_{QV} = \frac{2|F_S| r_m \beta_{Sv}}{\beta_S} \tag{5.35}
\]

In Fig. 21 the solid line shows the perturbation caused by L and by a sextupole lens placed in s.s. 517, under the adiabatic approximation and for a practical value of \( \bar{K}_{QV} \) (in this case denoted by \( \bar{K}_{Q517v} \)). The shape of the normalized vertical emittance of the extracted beam at E is given by:
\[ \bar{Y}_{Ev} = 2.0 + 0.8i \]

Therefore:

\[ Y_{Ev} = \frac{\bar{Y}_{Ev} + i\alpha_{Ev}}{\beta_{Ev}} = 0.118 + 0.080i ; \quad Z_{Ev} = \frac{1}{\bar{Y}_{Ev}} = 5.8 - 4.0i. \]

The dotted line shows the non-adiabatic perturbation caused by a thin septum lens of normalized quadrupole strength \( \bar{K}_{T701v} = -0.4 \). Figure 22 concerns the case where a correcting quadrupole, \( \bar{K}_{CV} = -0.4 \) is placed at s.s. 725. In this case:

\[ \bar{Y}_{Ev} = 1.62 - 0.05i \quad Y_{Ev} = 0.095 + 0.030i \]

\[ Z_{Ev} = 9.6 - 3.0i. \]

6. **EXTRACTION SYSTEM L801**

6.1. Quadrupole

The quadrupole is placed near to the downstream end of s.s. 801. 1.5 m from the downstream end of s.s. 801 one has:

\[ \beta_L = 28.5 \quad \beta_{LV} = 16.7 \text{ m} \quad (6.1) \]

\[ \psi_{EL} = 0.73 \times 2\pi \text{ rad} \quad \psi_{ELV} = 0.79 \times 2\pi \text{ rad} \]

A normal quadrupole has the strength given below:

\[ \bar{K}_L = -2 \]

\[ K_L = -0.070 \text{ m}^{-1} \quad (6.2) \]

\[ \ell(dE/dx) = 6.8 \text{ T} \]

\[ \bar{K}_{LV} = 1.18 \quad (6.3) \]
Extraction Systems L733 and L801 mainly differ in the phase of resonant oscillation with respect to the magnetic structure of the ring. Computations not involving this phase are therefore valid for both extraction systems.

6.2. Sextupoles

Suitable locations for sextupoles are (Fig. 23) s.s. 117, 149, 517, and 549. The polarities must be positive, negative, negative, and positive, respectively.

6.3. Thin septum magnet

Downstream to the quadrupole no location is possible for a thin septum magnet, because resonant oscillations have small displacements at s.s. 717 and s.s. 749 (Fig. 25), and no space is available in s.s. 733. The function $f_0(x_n)$ is shown in the upper part of Fig. 23. One can see that upstream to the quadrupole a thin septum magnet, $(T701)$ could be placed at s.s. 701. One has:

\[
\begin{align*}
\beta_{T701} &= 21 \text{ m} \\
\alpha_{T701} &= 0.034 \\
\psi_L T701 &= 7.93 \times 2\pi \text{ rad} \\
\psi_L T701 &= 7.82 \times 2\pi \text{ rad} \\
\frac{\Delta x_E}{\Delta x_{T701}} &= -1.5 \text{ mm/mm} \\
\Delta x_E/\Delta x_{T701} &= -37 \text{ mm/mrad}.
\end{align*}
\]

None of the sextupoles given in Section 6.2 decreases the displacement produced at $E$ by $T701$. 

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6.4. Extraction (no thin septum)

Figure 24 shows the normalized emittance of the extracted beam. The septum of E is placed at amplitude \( r_E = r_m \). It follows:

\[
\bar{x}_E = 44 \text{ mm} \quad \bar{x}_E' = 37 \text{ mm} \quad (6.5)
\]

\[
x_E = 37 \text{ mm} \quad x_E' = 0.3 \text{ mrad}
\]

The emittance of the extracted beam is:

\[
\bar{y}_E = 0.11 - 0.65i
\]

\[
y_E = 0.0038 - 0.0010i \quad (6.6)
\]

\[
z_E = 250 + 70i
\]

Using the septum magnets described in Section 5.6, the theoretical extraction efficiency is 94\%. This is smaller than in the case of Extraction System L733, because here resonant oscillations do not have extreme displacements at E.

Figure 25 shows the extracted beam on the plane \( \varphi, \bar{x} \). The extracted beam at s.s. 733 reaches the machine aperture limits. However, at s.s. 733

i) the radial and vertical \( \beta \)-values are small;

ii) the beam height is not increased with respect to its unperturbed value (see Figs. 28 and 29, and Section 6.6);

iii) a good closed orbit control is foreseen;

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iv) A d.c. bump brings the injection closed-orbit at s.s. 733 close to the septum of the injection septum magnet in order to minimize the strength of the injection fast kicker. This bump given by backleg windings produces at s.s. 733 a displacement of approximately -11 mm \(^9\). At ejection a bump in the opposite direction has to be given by means of pulsed steering magnets, in order to miss the septum.

v) The septum magnets could be placed further inside than as described above, although this results in a somewhat lower extraction efficiency.

These arguments show that the inconvenience of having large displacements of resonant oscillations at s.s. 733 can be overcome.

6.5. Extraction (thin septum in s.s. 701)

A useful displacement at E of 4 mm, requires \(\Delta x_E = 11\) mm. Such a displacement is produced by:

\[
\Delta x_{T701} = -7.3 \text{ mm} \quad \Delta x'_{T701} = -0.25 \text{ mrad}
\]

In the case where \(k_S = 0.005 \text{ mm}^{-1}\), one has:

\[
x_T = 35 \text{ mm} \\
x_T = 33 \text{ mm} \quad x'_T = 10 \text{ mm} \\
x_T = 24 \text{ mm} \quad x'_T = 0.3 \text{ mrad}
\]

Figure 26 shows the extracted beam in the phase-plane \(\bar{x}, \bar{x}'\). The emittance of the extracted beam is approximately the same as in Section 6.4. If T701 has the parameters given in column T
of Table 4.1, an effective length of 0.35 m is required. The theoretical extraction efficiency varies from 92% to 95% when A varies from 4π mrad mm to 2π mrad. A thin septum lens is particularly useful because it reduces the amplitude of oscillation of the extracted beam at s.s. 733. Figure 27 shows the phase plane diagram in the case where Δx_E = -30 mm. Taking w_T701 = 24 mm, this requires:

\[
\Delta x'_T701 - \Delta x_T701 = 20 \text{ mm} ; \quad \Delta x'_T701 - \Delta x'_T701i = 0.68 \text{ mrad}
\]

\[
K_T701 = 0.8, \quad K_T701 = 0.038 \text{ m}^{-1}
\]

6.6. Vertical optics

The static perturbation caused by L has been dealt with in Section 5.12. In the present case a correcting quadrupole could be placed in s.s. 757. The combined effect of static and dynamic perturbations is shown in Figs. 28 and 29 in the case where K_{CV} = 0 and K_{CV} = 0.4 respectively. Sextupole lenses are assumed to be placed at s.s. 517 and s.s. 549. In the case where K_{CV} = 0:

\[
\bar{Y}_{EV} = 0.99 + 0.72i ; \quad \bar{Y}_{EV} = 0.058 + 0.075i
\]

\[
Z_{EV} = 6.4 - 8.3i
\]

In the case where K_{CV} = -0.4

\[
\bar{Y}_{EV} = 1.22 + 0.41i ; \quad \bar{Y}_{EV} = 0.072 + 0.056i
\]

\[
Z_{EV} = 8.6 - 6.8i
\]

The dotted lines in Figs. 28 and 29 represent the effect of a quadrupole field normalized strength K_{T701v} = -0.55, in T701.
7. **DEAD TIME BETWEEN INJECTION AND EJECTION**

The duty cycle is unity if injection and ejection take place one immediately after the other. Actually this is not possible because of the following reasons:

i) The nominal injection equilibrium orbit is displaced from the centre of the vacuum chamber. Before ejection starts, the beam must be accelerated to the centre of the vacuum chamber. The time required is of the order of two tenths of a second, for a 28 GeV beam.

ii) A d.c. closed orbit bump brings the injection equilibrium orbit at s.s. 733 close to the septum. In Extraction System 733 this d.c. bump displaces the closed orbit at the quadrupole. The centre of the quadrupole must therefore be placed out of the centre of the vacuum chamber by an equal amount. In Extraction System 1801, before the ejection starts, a bump of opposite polarity must be given by means of pulsed steering magnets (Section 6.4).

iii) Time is required to excite the quadrupole and sextupole lenses for slow ejection.

iv) One CPS pulse fills only 2/3 of the storage ring circumference. Before ejection the injected pulse should spread over the whole circumference. The low momentum head of the injected beam (having momentum deviation - Δp/p) catches up its high momentum...
tail (+Δp/p) after a time \( t_c \), expressed by:

\[
t_c = \frac{t_r}{6ΔC/C}
\]  

(7.1)

where \( t_r \) is the revolution time, \( C \) is the storage ring circumference and \( C ± ΔC \) is the length of the equilibrium orbit of particles having momentum deviation \( ± \frac{Δp}{p} \). Since:

\[
\frac{ΔC}{C} = \frac{1}{v_{tr}^2} \frac{Δp}{p}
\]  

(7.2)

where \( v_{tr} \) is the transition energy over the rest energy, one has:

\[
t_c = \frac{t_r v_{tr}^2}{6 Δp/p}
\]  

(7.3)

In the case where \( Δp/p = 0.1 \) 6/5, from Eq. (7.3) one obtains:

\[
t_c = 0.04 \text{ sec}
\]  

(7.4)

This shows that in a time of the order of a tenth of a second the injected beam is spread over the whole circumference.

As concerns possible overlappings of these dead times:

- dead time ii) can overlap with dead time i)
- dead time iii) can overlap with dead time ii) provided that no lens is situated on the closed orbit bump centered about s.s. 733. This is not the case if Extraction System L733 is used.
- dead time iv) overlaps with dead times ii) and iii).
After ejection is finished, one cannot inject another pulse before the reverse processes of ii) and iii) have been completed. These processes can occur simultaneously.

8. CONCLUSIONS

Two practical extraction systems have been discussed. In one of them (Extraction System L733) the quadrupole lens is placed near to the downstream end of s.s.733, whereas in the other (Extraction System L801) it is placed near to the downstream end of s.s.801. The theoretical extraction efficiency is approximately the same for both extraction systems and is 92% to 96%. Advantages of Extraction System L 801 are the weaker quadrupole lens required (the radial β-value at the quadrupole lens is larger) and the smaller perturbation to the vertical optics (the ratio vertical to radial β-value at the quadrupole lens is smaller). This advantage is particularly important. On the other hand due to the presence of the injection septum magnet, it would be difficult to accommodate in s.s.733 the slow ejection quadrupole of Extraction System L733 or to make the existing Torwilliger quadrupole strong enough for this purpose. A disadvantage of Extraction System L801 is the large displacement of the extracted beam at s.s.733. However, it has been shown that this inconvenience can be overcome. In conclusion, Extraction System L801 appears to be more convenient.
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FIG. 1

GENERAL LAYOUT OF MAGNETS AND STRAIGHT SECTIONS IN THE ISR

SCALE

55m
Fig. 2 Normalized phase-plane diagram

\[ q = 8.75; \ k_2 = -2; \ k_3 = 0.006 \text{mm}^{-1}; \ \psi_{LS} = 0.75 \pi; \ x_L = 1 \text{mm} \]
Fig. 3  Allowed increase of beam height for different vertical closed orbit distortions (c.o.)
Fig. 4 Displacement of resonant oscillations and merit factor of a thin septum (extraction system L733)
Fig. 5 Normalized phase-plane diagram (extraction system L733)

$Q = 8.75$; $E_2 = -2$; $E_3 = 0.003 \text{ mm}^{-1}$
Fig 6  Particle motion on the outward-going separatrix
\[ Q = 8.75; \frac{A}{\pi} = 2; \frac{Z}{Z} = 0.005 \text{ mm}^{-1} \]
Fig. 7 Dependence of the stable area on the displacement of the unperturbed closed orbit from the centre of the quadrupole.
Fig. 8 Dependence of $S_6(R_E)$ and $R_E$ on the total sextupole strength.
Fig. 9  Extracted beam (extraction system T733, no thin septum)

\[ Q = 0.75, \quad \kappa_L = -2, \quad \kappa_S = 0.005 \text{ mm}^{-1} \]
outer trajectory of the extracted beam ($A/N=0$)

outer trajectory of the extracted beam ($A/N=4$ mrad/mm)

vacuum chamber walls

extracted beam

split beam in the last revolution

stable beam

injection fast liner

$X_f = 0.2$ mm, $A/N=100$ mm², $A/N=4$ mrad/mm

$X_f = 0$, $A/N=A/N+0$

vacuum chamber walls

**Fig. 10 Extracted beam (extraction system L733)**

$q=0.15$, $K_2 = -2$, $K_6 = 0.005$ mm⁻¹
Fig. 11  Amplitude of oscillation and particle loss at $T_{101}$. 
Fig 12: Extracted beam (extraction system L333; T101 used).

\[ Q = 8.95; \ \bar{k}_L = -2; \ \bar{k}_s = 0.005 \text{ mm}^{-1} \]
Fig. 13: Extracted beam (extraction system L733; thin septum (ens.T701).

\[ Q = 8.15; \bar{K}_L = -2; \bar{K}_S = 0.005 \text{ mm}^{-1}; \bar{K}_{T701} = 0.6 \]
Fig. 14. Momentum dependence of the phase-plane diagram at $E$

$Q = 0.15$, $k_L = 2$, $k_S = 0.005$ mm$^{-1}$; $V_x = 0$ for $\Delta p/p = 0$

$\Delta p/p = 0.0010$

$\Delta p/p = 0.0005$

$\Delta p/p = 0.0002$

$\Delta p/p = 0.0001$
Fig. 15 Amplitude of oscillation of the stable fixed-point
$q = 0.7/3, k_L = -2, \Delta \tau = 0$ for $\Delta p/p = 0$
$\alpha_{PL} = 2.3 m$
Fig. 16 Decapole field in the nonlinear lens
Fig. 17 Beam height increase caused by the quadrupole
Fig. 18 Static perturbation of vertical optics (extraction system L32; O, 0, 0).
Fig. 19 Static perturbation of vertical optics (extraction system \( L=333; q_V=0.75; \bar{k}_{LV}=1.44; \bar{k}_{CV}=-0.4 \)).
Fig. 20 Effect of a correcting quadrupole C
Fig. 23 Displacement of resonant oscillations and merit factor of a thin septum (extraction system LE01).
Fig. 24 Extracted beam (extraction system L801; no thin septum).

$q = 0.75; \alpha_2 = -2; k_3 = 0.005 \text{ mm}^{-1}$
Fig. 25 Extracted beam (extraction system 1B01)

\( Q = 0.75 \), \( K_2 = -2 \), \( K_5 = 0.005 \text{ mm}^{-2} \)