Implications of supersymmetric models with natural R-parity conservation

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ABSTRACT: In the minimal supersymmetric standard model, the conservation of $R$-parity is phenomenologically desirable, but is \textit{ad hoc} in the sense that it is not required for the internal consistency of the theory. However, if $B - L$ is gauged at very high energies, $R$-parity will be conserved automatically and exactly, provided only that all order parameters carry even integer values of $3(B - L)$. We propose a minimal extension of the supersymmetric standard model in which $R$-parity conservation arises naturally in this way. This approach predicts the existence of a very weakly coupled, neutral chiral supermultiplet of particles with electroweak-scale masses and lifetimes which may be cosmologically interesting. Neutrino masses arise via an intermediate-scale seesaw mechanism, and a solution to the $\mu$ problem is naturally incorporated. The apparent unification of gauge couplings at high energies is shown to be preserved in this approach. We also discuss a next-to-minimal extension, which predicts a pair of electroweak-scale chiral supermultiplets with electric charge 2.
1. Introduction

One of the successes of the Standard Model of particle physics is the automatic conservation of baryon number ($B$) and total lepton number ($L$) at the renormalizable level. These conservation laws follow simply from the particle content and $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge invariance, and do not entail additional assumptions.

The simplest supersymmetric extension of the Standard Model does not share this appealing feature, because the existence of scalar partners of the quarks and leptons allows for renormalizable violation of $B$ and $L$. The most general renormalizable and $SU(3)_C \times SU(2)_L \times U(1)_Y$-invariant superpotential is given schematically by

$$W = W_0 + W_1 + W_2,$$
$$W_0 = \mu H_u H_d + H_u Q u + H_d Q d + H_d L e,$$
$$W_1 = u d d, \quad W_2 = \mu' L H_u + Q L d + L L e.$$

[Here $Q$ and $L$ are chiral superfields for the $SU(2)_L$-doublet quarks and leptons; $u$, $d$, $e$ are chiral superfields for the $SU(2)_L$-singlet quarks and leptons, and $H_u$, $H_d$ are the two $SU(2)_L$-doublet Higgs chiral superfields. It is possible to eliminate $\mu'$ by a suitable rotation among the superfields $H_d$ and $L$; but in general it is somewhat misleading to do so because in many extensions of minimal supersymmetry $H_d$ and $L$ will not have the same quantum numbers, and the appropriate rotation cannot be performed.] The terms in $W_0$ are just the supersymmetric versions of the usual standard model Yukawa couplings and Higgs mass, and they conserve $B$ and $L$. However, $W_1$ violates $B$ by one unit and $W_2$ violates $L$ by one unit. To prevent the proton from decaying within seconds or hours, either the couplings in $W_1$ or those in $W_2$ (or both) must be extremely small. In this sense, the supersymmetric standard model appears to be less successful or at least less elegant than the Standard Model, since the observed conservation of $B$ and $L$ is no longer automatic, but requires some additional assumptions about the structure of the theory.

The most common way to save the proton from the supersymmetric threat is to forbid all of the terms occurring in $W_1$ and $W_2$ by imposing the discrete $Z_2$ symmetry $[1,2]$ known as $R$-parity or matter parity. The matter parity of each superfield may be defined as

$$(\text{matter parity}) \equiv (-1)^{3(B-L)}. \quad (1.1)$$

Then multiplicative conservation of matter parity forbids all terms in $W_1$ and $W_2$, while allowing the phenomenologically necessary ones in $W_0$. Equivalently, the $R$-parity of any component field is defined by $(-1)^{3(B-L)+2s}$, where $s$ is the spin of the field. Since $(-1)^{2s}$
is of course conserved in any Lorentz-invariant interaction, matter parity conservation and
$R$-parity conservation are precisely equivalent. The description in terms of matter parity
makes clear that there is nothing intrinsically “$R$-symmetric” about this symmetry; in
other words, it admits a formulation at the superfield level. Conversely, the description in
terms of $R$-parity is convenient in phenomenological discussions, because it happens that
all Standard Model states have $R$-parity +1, while all superpartners have $R$-parity −1.
Conservation of $R$-parity then immediately implies that superpartners can be produced
only in pairs, and that the lightest supersymmetric particle (LSP) is absolutely stable.

The minimal supersymmetric standard model (MSSM) with $R$-parity conservation can
provide a description of nature which is consistent with all known observations. However,
the assumption of $R$-parity conservation might appear to be ad hoc, since it is not required
for the internal consistency of the theory. Alternative discrete symmetries have in fact been
proposed (see for example [3,4]). Perhaps the simplest of these is the $Z_3$ discrete “baryon
parity” of Ibáñez and Ross [4], which turns out to imply the falsifiable predictions that
the proton is absolutely stable and there can be no neutron–antineutron oscillations even if
there are isosinglet quark superfields near the TeV scale [5]. One might also entertain the
possibility of small $R$-parity violation, with intriguing phenomenological consequences (see
for example [6-9]). However, if $R$-parity is not exact, the LSP is unstable and so cannot be
a candidate for the cold dark matter, unless its lifetime is of order the age of the universe.

Fortunately, there is a particularly compelling scenario which does automatically pro-
vide for exact $R$-parity conservation due to a deeper principle. This is suggested imme-
diately by (1.1), which shows that matter parity is simply a $Z_2$ subgroup of $B − L$. If
$U(1)_{B−L}$ is gauged at high energies, it will forbid each of the terms in $W_1$ and $W_2$ [10-
13]. Of course, there is no massless gauge boson found in nature which couples to $B − L$,
so $U(1)_{B−L}$ must be spontaneously broken. The question then becomes how to break
$B − L$ without also breaking matter parity. To guarantee that matter parity should re-
main unbroken even after a gauged $U(1)_{B−L}$ is broken, it is necessary and sufficient to
require that all scalar vacuum expectation values (VEVs) or other order parameters carry
$3(B − L)$ charges which are even integers. Following the general arguments of Krauss
and Wilczek [14], the gauged $U(1)_{B−L}$ symmetry breaks down to a $Z_2$ subgroup which,
in view of (1.1), is nothing other than matter parity. Unlike a global discrete symmetry,
such a gauged discrete symmetry must be respected by Planck scale effects, and satisfies

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discrete anomaly cancellation conditions [12,15,16]. Note that it is a contradiction in terms to speak of explicit $R$-parity breaking in a supersymmetric model with gauged $U(1)_{B-L}$; $R$-parity will either be exactly conserved (if all order parameters carry only even integer values of $3(B - L)$) or spontaneously broken [17] (if some order parameter carries an odd integer $3(B - L)$).

Of course, this scenario for the origin of matter parity is hardly mandatory, since it is technically natural to forbid the terms in $W_1$ and $W_2$ “by hand” as an unexplained assumption. However, it is worthwhile to take seriously the idea that $R$-parity conservation is explainable, since we then obtain some quite non-trivial information about physics at very high energy scales. Not only do we gain an indication that the unbroken gauge group should contain $U(1)_{B-L}$, but we also obtain information about how it should (and should not!) be broken.

In ref. [13], the criteria for maintaining natural $R$-parity conservation in models with gauged $U(1)_{B-L}$ were considered for various extended gauge groups. Consider, for example, the possibility of a natural explanation for $R$-parity conservation in a supersymmetric $SO(10)$ grand unified theory (GUT). Now, $SO(10)$ contains $U(1)_{B-L}$ as a subgroup, so that $R$-parity conservation is automatic before spontaneous symmetry breaking. However, the smallest “safe” representations for a scalar which can break $U(1)_{B-L}$ without breaking the gauged matter parity subgroup in the process are $120, 126, 210, \ldots$ and their conjugates. This is unfortunate, since experience has shown that it is quite difficult to build a successful supersymmetric GUT with such large representations. The alternative is to break $SO(10)$ with an order parameter in a $16$ representation, and that is what is usually done. However, the neutral component of a $16$ carries $3(B - L) = 3$, so that the original automatic $R$-parity conservation is forfeit. Indeed, renormalizable matter parity violation appears in the low energy superpotential from non-renormalizable operators of the form $(1/M)\langle 16 \rangle \times 16 \times 16 \times 16$. (Here and in the following $M$ is some physical cutoff scale, perhaps $M = M_{\text{Planck}}/\sqrt{8\pi}$.) As another example, one might consider an extension of the MSSM with a Pati-Salam gauge group $SU(4)_{PS} \times SU(2)_L \times SU(1)_{R}$. Again, $R$-parity conservation is automatic before spontaneous symmetry breaking since $SU(4)_{PS} \supset SU(3)_C \times U(1)_{B-L}$. To avoid breaking matter parity in the process of breaking $SU(4)_{PS}$, it is necessary and sufficient that all order parameters have even $SU(4)_{PS}$ quadrality, since $SU(4)_{PS}$ quadrality $= 3(B - L) \mod 4$. The smallest such “safe” representation for an order parameter which breaks $U(1)_{B-L}$ is the $10$ of $SU(4)_{PS}$.

In ref. [18], the dynamical issues associated with automatic $R$-parity conservation have
been considered in the case of left-right symmetric models. It was found that in a wide class of such models, \( R \)-parity must be spontaneously broken because of the form of the scalar potential, although this can be evaded if non-renormalizable interactions are included.

In this paper, we will consider instead a minimal extension of the MSSM in which the gauge group is extended by only \( U(1)_{B-L} \). Anomaly cancellation for \( U(1)_{B-L} \) implies the existence of three neutrino chiral superfields \( \nu \) which carry \( B-L = 1 \) and are singlets of the standard model gauge group. A VEV for the scalar component of \( \nu \) would spontaneously break matter parity, so we will require it to be absent. While such a weak-scale VEV for \( \nu \) is not yet ruled out phenomenologically, we adopt for this paper the point of view that this is unacceptable, since we want to explore here only possibilities with exact and automatic \( R \)-parity conservation.

To obtain a realistic theory of neutrino masses, we may invoke the seesaw mechanism [19] by means of the superpotential

\[
W \supset y_\nu H_u L \nu + \frac{y_S}{2} S \nu \nu .
\]

Here \( S \) is a chiral superfield which must carry \( B-L = -2 \). Assuming that \( \langle S \rangle \) is much larger than the electroweak scale \( m_W \), one finds that the lighter neutrino mass eigenstates have tiny masses \( \sim (y_\nu \langle H_u \rangle)^2/(y_S \langle S \rangle) \). Now, the role of \( S \) within this framework might be played by a composite field \( S = \bar{\nu} \nu / M \). However, this again cannot be consistent with our criteria for automatic \( R \)-parity conservation, since then a VEV \( \langle S \rangle \neq 0 \) implies a VEV \( \langle \bar{\nu} \nu \rangle \neq 0 \). Therefore, we prefer the possibility that \( S \) is a fundamental chiral superfield, so that the VEV \( \langle S \rangle \) cannot break the matter parity subgroup of \( U(1)_{B-L} \). The field \( S \) must be accompanied by a field \( \bar{S} \) in the conjugate representation, in order to cancel the anomalies and to allow spontaneous symmetry breaking in a nearly \( D \)-flat direction. (Otherwise there would be catastrophically large supersymmetry-breaking \( D \)-terms, which would destabilize the electroweak scale.) For the models in this paper, the scale \( \langle S \rangle \) is an intermediate one, roughly the geometric mean between the electroweak scale and the Planck scale. Assuming that the Yukawa couplings \( y_\nu \) are of the same order as those of the charged leptons, one then expects light neutrino masses in the range relevant [20] to solar or atmospheric neutrino oscillations and hot dark matter.

In section 2 of this paper we will propose a minimal extension of the supersymmetric standard model which successfully implements automatic and unbroken \( R \)-parity conservation from gauged \( B-L \), and discuss some of its implications. Section 3 contains some discussion of the subtleties associated with \( U(1) \) mixing in this model, and the effect of
intermediate scale thresholds on the sparticle spectrum. In section 4 we will discuss a next-to-minimal model [with gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$]. Section 5 contains some concluding remarks.

2. A minimal model of automatic $R$-parity conservation

We consider a supersymmetric model with gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$. The MSSM chiral superfields (plus $\nu$) transform under this gauge group as three copies of

- $Q \sim (3, 2, \frac{1}{6}, \frac{1}{3})$
- $L \sim (1, 2, -\frac{1}{2}, -1)$
- $d \sim (\bar{3}, 1, \frac{1}{3}, -\frac{1}{3})$
- $e \sim (1, 1, 1, 1)$
- $u \sim (\bar{3}, 1, -\frac{2}{3}, -\frac{1}{3})$
- $u \sim (\bar{3}, 1, -\frac{2}{3}, -\frac{1}{3})$
- $\nu \sim (1, 1, 0, 1)$

and two Higgs doublets

$H_u \sim (1, 2, \frac{1}{2}, 0)$
$H_d \sim (1, 2, -\frac{1}{2}, 0)$.

In order to break $U(1)_{B-L}$, we introduce two chiral superfields

$S \sim (1, 1, 0, -2)$
$S \sim (1, 1, 0, 2)$.

Because this gauge group contains two abelian factors, in general one must consider the possibility of arbitrary mixing between $U(1)_Y$ and $U(1)_{B-L}$. (Indeed, it would be quite unexpected if the gauge interactions were diagonal in the $Y, B-L$ basis.) We will be able to postpone a discussion of this until the next section, however.

Besides the interaction between $S$ and $\nu$ given in (1.2), $S$ and $\overline{S}$ participate in a non-renormalizable superpotential interaction\footnote{We do not allow tree-level superpotential mass terms $SS$ and $H_uH_d$ or their soft supersymmetry breaking counterparts, in sympathy with general results in superstring models.}

$$W = \frac{\lambda}{2M} S^2 \overline{S}^2$$

and soft supersymmetry-breaking terms

$$V_{\text{soft}} = m^2 |S|^2 + m^2 |\overline{S}|^2 - \left( \frac{A}{2M} S^2 \overline{S}^2 + c.c. \right).$$

(We use the same symbol for each chiral superfield and its scalar component.) The parameters $m^2$, $\overline{m}^2$ and $A$ should each be of order the electroweak scale $m_W$ in order not
to upset the hierarchy. Note that the $A/M$ term, while dimensionless, should nevertheless be treated as “soft” because of its tiny magnitude. Such terms should naturally arise in supergravity models, and this one will play a crucial role on several accounts, as we shall soon see. By a suitable phase rotation, we take $A$ to be real and positive, while the phase of $\lambda$ can be arbitrary. The full scalar potential for the $S$ and $\bar{S}$ degrees of freedom is given by the sum of $V_{\text{soft}}$ and

$$V_{\text{SUSY}} = \frac{|\lambda|^2}{M^2} |S\bar{S}|^2(|S|^2 + |\bar{S}|^2) + \frac{g_X^2}{2} \left(|S|^2 - |\bar{S}|^2\right)^2$$

(2.3)

where the latter term is the $D$-term contribution. The parameter $g_X$ is related to the gauge couplings of the $U(1)$s, and will be explicitly identified in the next section when we discuss the effects of $U(1)$ mixing.

A familiar method of inducing spontaneous gauge symmetry breaking in supersymmetric models is to arrange for the running soft (mass)$^2$ of the appropriate scalar to become negative at some scale. In the usual MSSM, this radiative symmetry breaking is achieved by means of a large top-quark Yukawa coupling which drives the Higgs (mass)$^2$ negative. In the model discussed here, the parameter $m^2$ obtains a negative radiative correction due to the Yukawa coupling $y_S$ in (1.2), and this has been exploited to obtain radiative symmetry breaking in similar models [21-26]. However, in the present case one also obtains a large positive radiative correction to both $m^2$ and $\bar{m}^2$ from $U(1)$-gaugino loops. Indeed, the large ($\pm 2$) $B - L$ charges of $S$ and $\bar{S}$ make it seem somewhat problematic to achieve radiative symmetry breaking in the traditional way; an examination of the renormalization group (RG) equations shows that very large (or numerous) Yukawa couplings $y_S$ seem to be required. Fortunately, it is not really necessary for $m^2$ or $\bar{m}^2$ to be driven negative in this model, since the $A$ term in (2.2) always favors spontaneous symmetry breaking. A non-trivial local minimum of the scalar potential will be obtained provided that

$$A^2 - 6|\lambda|^2(m^2 + \bar{m}^2) > 0.$$  

(2.4)

This minimum will be global if

$$A^2 - 8|\lambda|^2(m^2 + \bar{m}^2) > 0.$$  

(2.5)

These conditions can be satisfied either by driving $m^2$ negative, or simply by taking the free parameter $A$ to be sufficiently large (while still roughly of order the electroweak scale), or perhaps by a combination of these effects. In any case, the minimum of the potential occurs along a nearly $D$-flat direction:

$$\langle S \rangle^2 \approx \langle \bar{S} \rangle^2 \approx \frac{M}{6|\lambda|^2} \left(A + \sqrt{A^2 - 6|\lambda|^2(m^2 + \bar{m}^2)}\right)$$

(2.6)
with the deviation from $D$-flatness given by

$$\langle S \rangle^2 - \langle \overline{S} \rangle^2 \approx (\overline{m}^2 - m^2)/(2g^2_X).$$

(2.7)

We see from (2.6) that the characteristic scale of $B - L$ breaking is roughly a geometric mean between the electroweak scale and the Planck scale: $(m_WM)^{1/2} \sim 10^{10}$ GeV. Since only $S$ and $\overline{S}$ obtain VEVs, it is clear that $R$-parity conservation is automatic (and in fact unavoidable) in this model. Note that the minimum of the scalar potential is stable against $\nu$ obtaining a VEV (and more generally against arbitrary perturbations of $S$, $\overline{S}$ and $\nu$); this can be understood from the fact that the scalar potential contains large positive semi-definite contributions $|F_\nu|^2 = |y_S \langle S \rangle \nu|^2$.

After spontaneous symmetry breaking, a gauge boson and gaugino obtain masses

$$M_I = 2g_X \langle S \rangle$$

(2.8)

by eating the would-be Nambu-Goldstone boson degree of freedom $\text{Im}[S - \overline{S}]/\sqrt{2}$. (Without loss of generality, we take $\langle S \rangle$, $\langle \overline{S} \rangle$ to be real and positive.) Here $g_X$ is the same coupling appearing in (2.3). In addition, one real scalar degree of freedom (given approximately by $\text{Re}[(S - \langle S \rangle) - (\overline{S} - \langle \overline{S} \rangle)]/\sqrt{2}$) and one Weyl fermion from $S, \overline{S}$ get masses $M_I$, forming a complete massive vector supermultiplet. There remains one light neutral chiral supermultiplet, given approximately by

$$\Phi \approx [S - \langle S \rangle + \overline{S} - \langle \overline{S} \rangle]/\sqrt{2},$$

(2.9)

whose components all obtain electroweak-scale masses. These degrees of freedom consist of a Weyl fermion $\psi$ (with $R$-parity $-1$) of mass

$$m_\psi = 3|\lambda|\frac{\langle S \rangle^2}{M}$$

(2.10)

and two real scalar degrees of freedom $a$ and $b$ (of $R$-parity $+1$) with squared masses

$$m_a^2 = 4A\frac{\langle S \rangle^2}{M}, \quad m_b^2 = 2A\frac{\langle S \rangle^2}{M} - 2(m^2 + \overline{m}^2).$$

(2.11)

Let us pause to remark on several interesting features of this spectrum of electroweak-scale, neutral particles. First, note that in the limit $A \to 0$, $m_a^2$ vanishes and the scalar $a$ becomes a Nambu-Goldstone boson. This corresponds to the spontaneous breaking of a continuous $R$-symmetry of the superpotential, which is explicitly broken only by the $A$ term. Fortunately, there is no reason for the parameter $A$ to be small compared to the
electroweak scale; on the contrary, it is likely that $A$ should be large in order to achieve
the necessary condition (2.4) for spontaneous symmetry breaking, as we have already
discussed. Perhaps a more plausible limit physically is $A \gg |\lambda|^2 (m^2 + \overline{m}^2)$ (but still very
roughly of order the electroweak scale), which leads to

$$\langle S \rangle^2 \approx \frac{AM}{3|\lambda|^2}; \quad m_\psi \approx \frac{A}{|\lambda|}; \quad m_a \approx \sqrt{\frac{4}{3}} \frac{A}{|\lambda|}; \quad m_b \approx \sqrt{\frac{2}{3}} \frac{A}{|\lambda|}.$$ (2.12)

In general, the masses of the component fields of the supermultiplet $\Phi$ satisfy the sum rule

$$m_a^2 + m_b^2 - 2m_\psi^2 = m^2 + \overline{m}^2.$$ (2.13)

It is not difficult to show that the lightest member of the supermultiplet $\Phi$ is always one
of the scalars ($a$ or $b$).

An important byproduct of this symmetry breaking scenario follows from the existence
of an allowed term in the non-renormalizable superpotential which is of the same order as
(2.1):

$$W \supset \frac{\lambda'}{M} H_u H_d S \overline{S}.$$ (2.14)

After symmetry breaking, one obtains the usual $\mu H_u H_d$ term of the MSSM, with

$$\mu = \frac{\lambda' \langle S \rangle^2}{M}$$ (2.15)

which is naturally of order the electroweak scale. The corresponding soft MSSM Higgs mass
term (often denoted $B\mu$) is generated in the same way from the soft term corresponding to
(2.14). This is a solution to the problem of generating an electroweak-scale $\mu$ term along
the lines of [27].

The interaction (2.14) also plays another crucial role in this model; it allows $a$, $b$, and
$\psi$ to decay in a cosmologically timely fashion. These fields clearly have only tiny couplings
to the particles of the MSSM. The most important such interactions actually follow from
(2.14); one finds the coupling of $\Phi \supset (a, b, \psi)$ to MSSM states

$$W \supset \frac{\sqrt{2} \lambda' \langle S \rangle}{M} \Phi H_u H_d.$$ (2.16)

The dimensionless coupling $\sqrt{2} \lambda' \langle S \rangle/M$ is very roughly of order $(m_W/M)^{1/2}$ if $\lambda'$ is of
order unity, leading to decay widths of order $\Gamma \sim m_W^2/(8\pi M) \sim 10^{-15}$ GeV for the
component fields of $\Phi$ into MSSM states. Depending on kinematic and mixing angle
factors, one has\(^\dagger\) two-body decays \(a, b \to \text{Higgs} + \text{Higgs}\) or \(a, b \to \text{Higgsino} + \text{Higgsino}\) and \(\psi \to \text{Higgs} + \text{Higgsino}\) decays with lifetimes of order \(10^{-9}\) seconds, give or take several orders of magnitude. (This should be compared to the notorious problem of the lifetime of an electroweak-scale gravitino, which can be estimated to be of order \(M/m_W\) times as long.) Thus it is unlikely that late decays of these particles could jeopardize the successful predictions of big-bang nucleosynthesis, although they might certainly have other interesting cosmological effects which should be carefully investigated. Note in particular that each decay of \(\psi\) results in the production of one stable LSP. (It does not appear viable to allow \(\psi\) itself to be the LSP, because its annihilation cross-section is so tiny that it would cause the universe to become matter dominated too early.)

3. U(1) mixing and the sparticle spectrum

The model described in the previous section contains two \(U(1)\) factors which can mix in an \textit{a priori} arbitrary way. We chose to specify the charges of the chiral superfields in the \(Y, B – L\) basis, but this does not completely specify the gauge interactions of these fields. In fact, it would be rather surprising if the gauge interactions at high energies were diagonal in this basis. We will choose instead to use the “\(SO(10)\)-inspired” basis given by \(U(1)_R, U(1)_{B-L}\), with the \(B – L\) charges as before, and \(R = Y – (B – L)/2\). [There would be no mixing in this basis if the gauge group were imbedded in e.g. unbroken \(SO(10)\).] One can always perform a rotation on the \(U(1)\) vector supermultiplets so that the kinetic terms are diagonal and canonically normalized for the \(U(1)_R, U(1)_{B-L}\) gauge bosons and gauginos. Then the interactions with matter fields \(\phi_i\) are specified by the covariant derivative

\[
D_\mu \phi_i = (\partial_\mu + i\bar{g}_i^R A^R_\mu + i\bar{g}_i^{B-L} A^{B-L}_\mu) \phi_i ,
\]

\[
\bar{g}_i^R = g_R R_i + g_{B-L,R} \sqrt{3/8} (B-L)_i ,
\]

\[
\bar{g}_i^{B-L} = g_{B-L} \sqrt{3/8} (B-L)_i + g_{R,B-L} R_i .
\]

The charges \(R_i\) and \((B-L)_i\) are constants and are not renormalized. However, in general the couplings \(g_{B-L}, g_R, g_{B-L,R}\), and \(g_{R,B-L}\) all require counterterms and are renormalized [28]. The mixing couplings \(g_{B-L,R}\) and \(g_{R,B-L}\) cannot avoid counterterms unless the matter content is special, e.g. in complete multiplets of a non-abelian group containing at least one of the \(U(1)\)’s. It is therefore not consistent in general, and in particular in the model of the previous section, to set \(g_{B-L,R}\) and \(g_{R,B-L}\) equal to 0. At any particular renormalization

\(^\dagger\) The components of \(\Phi\) can also decay into light (s)neutrino pairs, but these decays turn out not to be competitive because they are suppressed by the seesaw mixing angle squared.
scale one can perform a rotation on the vector superfield basis to set either $g_{B-L,R}$ or $g_{R,B-L}$ equal to 0 [28]. This condition is not renormalization scale-invariant, however, so it is sometimes convenient to keep all four couplings as free parameters.

In terms of these parameters, the coupling $g_X$ appearing in the previous section is

$$g_X^2 = (g_R - \sqrt{3/2} \ g_{B-L,R})^2 + (\sqrt{3/2} \ g_{B-L} - g_{R,B-L})^2.$$  \hspace{1cm} (3.1)

At the scale of symmetry breaking, the surviving $U(1)_Y$ gauge coupling is given by (in a GUT-like normalization)

$$g_Y = \sqrt{5/2} \ (g_R g_{B-L} - g_{B-L,R} g_{R,B-L})/g_X.$$  \hspace{1cm} (3.2)

The one-loop RG equations for the gauge couplings are [$t = \ln(Q/Q_0)$]:

$$\frac{d}{dt} \begin{pmatrix} g_{B-L} \\ g_{R,B-L} \\ g_B \\ g_{R,B} \end{pmatrix} = \frac{1}{16\pi^2} \begin{pmatrix} g_{B-L} & g_{B-L,R} \\ g_{R,B-L} & g_B \\ g_B & g_{R,B} \\ g_{R,B} & g_{R,B,L} \end{pmatrix} \begin{pmatrix} b_{B-L} \\ b_{R,B-L} \end{pmatrix} ,$$  \hspace{1cm} (3.3)

with

$$b_{B-L} = 9(g_{B-L}^2 + g_{R,L}^2) - 2\sqrt{6} \ g_{B-L} g_{R,B-L} , \hspace{1cm} b_R = 9(g_{R}^2 + g_{R,L}^2) - 2\sqrt{6} \ g_R g_{B-L,R} ,$$

$$b_{R,B-L} = 9(g_R g_{R,B-L} + g_{B-L} g_{B,R,L}) - \sqrt{6} (g_{B-L} g_R + g_{B-L,R} g_{R,B-L}) .$$

Using these equations, one finds that $g_Y$ defined by (3.2) satisfies the one-loop RG equation

$$\frac{d}{dt} g_Y = \frac{1}{16\pi^2} \frac{33}{5} g_Y^3 .$$  \hspace{1cm} (3.4)

(just as in the MSSM) both above and below $M_I$, so that the condition for unification of $g_Y$ with the $SU(3)_C$ and $SU(2)_L$ gauge couplings $g_3$ and $g_2$ is unaffected by mixing, up to two-loop and threshold effects.

It therefore is sensible to impose a gauge coupling unification condition on all the couplings, as could follow from a superstring or a GUT model. At the unification scale $t_U$, one might therefore take

$$g_3 = g_2 = g_R = g_{B-L} \equiv g_U , \hspace{1cm} g_{B-L,R} = g_{R,B-L} = 0 .$$  \hspace{1cm} (3.5)

We will assume these boundary conditions for the remainder of this section, although it cannot be overemphasized that alternative boundary conditions are certainly possible. At lower scales, one can then solve the one-loop RG equations analytically (for example by rotating to the multiplicatively renormalized basis), with the result

$$g_R = g_{B-L} = g_U (\kappa_+ + \kappa_-)/2 ,$$  \hspace{1cm} (3.6)

$$g_{R,B-L} = g_{B-L,R} = g_U (\kappa_- - \kappa_+)/2 ,$$  \hspace{1cm} (3.7)

$$\kappa_\pm = \left[ 1 + \frac{g_U^2}{8\pi^2} (9 \pm \sqrt{6}) (t_U - t) \right]^{-1/2} .$$  \hspace{1cm} (3.8)
(The first equality in each of (3.6) and (3.7) is due to a coincidental symmetry of the RG equations in this model.) On this “unification trajectory”, the mixed couplings $g_{B-L,R}$ and $g_{R,B-L}$ remain fairly small ($< .04$).

Since the apparent unification [29] of gauge couplings observed at LEP can be maintained in this model, it is sensible to explore features of the low-energy theory which follow from unified supergravity-inspired [30] boundary conditions. These boundary conditions include the supposition that at some scale $M_U \geq 2 \times 10^{16}$ GeV the scalars in the theory have a common soft supersymmetry breaking (mass)$^2$ (denoted $m_0^2$) and there is a common mass $m_{1/2}$ for each gaugino. One can then integrate the RG equations from $M_U$ down to the electroweak scale, and study the resulting low-energy theory. Here we will restrict ourselves to some brief comments regarding the impact of the extension of the MSSM of section 2, using the MSSM (with no new fields below $M_U$) as a template.

It is possible to show that the well-known gaugino mass unification prediction

$$(M_3/g_3^2) = (M_2/g_2^2) = (M_1/g_Y^2) = (m_{1/2}/g_U^2) \quad (3.9)$$

at low energies is precisely maintained by the one-loop RG equations of this model along the unification trajectory, provided that the gaugino masses are unmixed at the scale $t_U$ in the $R, B - L$ basis. [There is mixing induced among the gaugino mass parameters in the $R, B - L$ basis by RG running, yet the surviving $U(1)_Y$ gaugino mass parameter does satisfy (3.9).] Therefore the predictions for chargino, neutralino, and gluino masses are essentially unaffected in the model of section 2, compared to the MSSM as a template. The condition (3.9) is modified by small two-loop corrections [31,32], of course.

The predictions for masses of squarks and sleptons are affected in two ways. First, one has $D$-term contributions to scalar masses [22-26] due to the spontaneous breaking of the $U(1)$ symmetry. In the model of section 2, one finds that each scalar $\phi_i$ obtains a contribution to its (mass)$^2$ of

$$\Delta m_i^2 = (m^2 - m)^2 \left[ \frac{3}{20} X_i + \frac{3}{10} Y_i(g_R^2 - g_{B-L}^2 + g_{R,B-L}^2 - g_{B-L,R}^2) - \sqrt{\frac{1}{6}} [g_R g_{B-L,R} + g_{B-L} g_{R,B-L}] \right] / g_X^2 \quad (3.10)$$

where $X_i = \frac{4}{3} Y_i - \frac{5}{3} (B - L)_i$ and $m^2 - m^2$ is evaluated at the scale $M_I$. For the unification trajectory, the term proportional to $Y_i$ is non-vanishing but small, however it is important to keep in mind that it need not be so with more general boundary conditions on the gauge couplings. The corrections (3.10) should be added to the scalar masses at the intermediate
scale $M_I$ and must be renormalized down to the electroweak scale; this turns out [26] to not affect the contributions proportional to $X_i$, while inducing a quite small change in the term proportional to $Y_i$.

The presence of the additional $U(1)$ gauge interactions above $M_I$ also makes a contribution to scalar masses, because of terms in the RG equations due to gaugino loops. Evaluating these contributions for the unification trajectory of the RG equations (and taking into account all mixing effects) one finds the following approximate results for the slepton masses at the electroweak scale:

$$m^2_{\tilde{e}_R} = [m^2_0 + .15 m^2_{1/2} - \sin^2 \theta_W m^2_Z \cos 2\beta] - \frac{1}{20} (m^2 - m^2) + .015 m^2_{1/2}$$ (3.11)

$$m^2_{\tilde{e}_L} = [m^2_0 + .52 m^2_{1/2} + (\sin^2 \theta_W - \frac{1}{2}) m^2_Z \cos 2\beta] + \frac{3}{20} (m^2 - m^2) + .045 m^2_{1/2}$$ (3.12)

$$m^2_{\tilde{\nu}} = [m^2_0 + .52 m^2_{1/2} + \frac{1}{2} m^2_Z \cos 2\beta] + \frac{3}{20} (m^2 - m^2) + .045 m^2_{1/2}$$ (3.12)

In each case the first set of terms in square brackets is the result for the template (MSSM) model. Next is the $D$-term associated with breaking of the $U(1)$ gauge group (neglecting the small contribution proportional to $Y$, and with $m^2 - m^2$ evaluated at $M_I$), and the last term is the additional contribution from $U(1)$ gaugino loops above $M_I$. We have used here representative values $M_U = 2 \times 10^{16} \text{ GeV}$ and $M_I = 10^{10} \text{ GeV}$. Similar equations can be written down for the squarks, although the relative effect is much larger for the sleptons. As long as we are assuming scalar mass unification, there is good motivation for the expectation that $\overline{m}^2 > m^2$, since $m^2$ receives a negative RG contribution proportional to $|y_S|^2$ [c.f. eq.(1.2)]. Therefore, the change (compared to the MSSM) in the difference between charged slepton masses,

$$\Delta(m^2_{\tilde{e}_L} - m^2_{\tilde{e}_R}) \approx (\overline{m}^2 - m^2)/5 + .03 m^2_{1/2}$$ (3.13)

is expected to be positive. This reinforces the expectation in the MSSM that $\tilde{e}_L$ should be heavier than $\tilde{e}_R$; depending on the relative magnitudes of $m^2_0$, $m^2_{1/2}$, and $\overline{m}^2 - m^2$, the difference (3.13) could even be dramatic.

4. An extension of the minimal model

The model described in the section 2 is the simplest extension of the MSSM with gauged $B - L$ breaking to matter parity at an intermediate scale. Its other successful features include a natural solution to the $\mu$ problem and a potentially successful theory
of neutrino masses. It is interesting to consider extensions of the minimal model with
an enlarged gauge symmetry at the symmetry breaking scale. Note, however, that larger
gauge groups seem to be somewhat disfavored for the following reasons. In order to have a
viable seesaw mechanism, the order parameter field \( S \) ought to be in a symmetric product
of the conjugate of the representation which contains \( \nu \). If the gauge group is extended
to contain additional non-abelian factors, such a symmetric product representation will
generally be large, containing fields which are not neutral under the Standard Model
gauge group. This jeopardizes asymptotic freedom of the gauge couplings, and in any case
forces us to view the apparent unification of gauge couplings observed at LEP as merely
a perverse accident. Furthermore, symmetric product representations are quite difficult
to obtain in string models. Even worse, the positive gaugino-loop radiative corrections to
soft scalar masses are proportional to the quadratic Casimir invariant, and thus tend to
be large for symmetric product representations containing \( S \) and \( \bar{S} \). This effect seems to
strongly disfavor VEVs for such scalars, although sufficiently large term(s) analogous to
the \( A \) term in (2.2) might overcome it.

Therefore, we will consider here only the next-to-smallest gauge group containing
gauged \( U(1)_{B-L} \), namely \( SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \). Under this gauge

group, the chiral superfields transform as†

\[
Q \sim (\mathbf{3}, \mathbf{2}, 1, \frac{1}{3}) \quad L \sim (\mathbf{1}, \mathbf{2}, 1, -1) \quad H_u, H_d \sim (\mathbf{1}, \mathbf{2}, 2, 0)
\]

\[
u, \nu \sim (\mathbf{3}, \mathbf{1}, 2, -\frac{1}{3})
\]

\[
S \sim (\mathbf{1}, \mathbf{1}, 3, -2) \quad \bar{S} \sim (\mathbf{1}, \mathbf{1}, 3, 2).
\]

The lowest-order non-renormalizable superpotential for the \( S, \bar{S} \) degrees of freedom con-
tains two independent terms in general:

\[
W = \frac{\lambda_1}{2M} \text{Tr}[S\bar{S}]^2 + \frac{\lambda_2}{4M} \text{Tr}[S^2] \text{Tr}[\bar{S}^2].
\]

Here we use a notation in which \( SU(2)_R \) triplets are given by traceless \( 2 \times 2 \) matrices,
explicitly

\[
S = \begin{pmatrix}
S^-/\sqrt{2} & S^0 \\
S^- & -S^-/\sqrt{2}
\end{pmatrix}; \quad \bar{S} = \begin{pmatrix}
S^+/\sqrt{2} & S^{++} \\
S^0 & -S^+ /\sqrt{2}
\end{pmatrix}
\]

with the superscripts indicating the electric charge. The soft breaking terms are given by

\[
V_{\text{soft}} = m^2 \text{Tr}[S^\dagger S] + m^2 \text{Tr}[\bar{S}^\dagger \bar{S}] - \left( \frac{A_1}{2M} \text{Tr}[S\bar{S}]^2 + \frac{A_2}{4M} \text{Tr}[S^2] \text{Tr}[\bar{S}^2] + \text{c.c.} \right). \quad (4.2)
\]

† This is not a left-right symmetric model, given our (minimal) choice of particle content.
Similar models are considered in [18], but our treatment will be somewhat different.
By a suitable phase rotation, we take the parameter $A_1$ to be real and positive, while the phases of $\lambda_1, \lambda_2, A_2$ are arbitrary. There is a possible minimum of the full scalar potential for the neutral scalar components of $S$ and $\bar{S}$ with VEVs in a nearly $D$-flat direction:

$$\langle S^0 \rangle^2 \approx \langle \bar{S}^0 \rangle^2 \approx \frac{M}{6|\lambda_1|^2} \left( A_1 + \sqrt{A_1^2 - 6|\lambda_1|^2(m^2 + \bar{m}^2)} \right),$$  \hspace{1cm} (4.3)

with the deviation from $D$-flatness given by

$$\langle S^0 \rangle^2 - \langle \bar{S}^0 \rangle^2 \approx (m^2 - \bar{m}^2)/(3g_{B-L}^2 + 2g_R^2).$$  \hspace{1cm} (4.4)

This minimum is stable against local perturbations provided that

$$A_1^2 - 6|\lambda_1|^2(m^2 + \bar{m}^2) > 0,$$  \hspace{1cm} (4.5)

$$|\lambda_1 + \lambda_2|^2 s^2 + \frac{m^2 + \bar{m}^2}{2} > \left[ s^2|A_1 + A_2 - 2\lambda_1^* (\lambda_1 + \lambda_2)s|^2 + 4g_R^4 \Delta^4 \right]^{1/2}.$$  \hspace{1cm} (4.6)

where $s = \langle S^0 \rangle^2/M$ and $\Delta^2 = \langle S^0 \rangle^2 - \langle \bar{S}^0 \rangle^2$ define two convenient parameters of order $m_W$. These stability conditions are satisfied in a non-vanishing region of the parameter space. In a smaller, but still non-vanishing, region of parameter space this is also a global minimum of the potential. (However, it is not clearly relevant to require that the desired minimum be global, since the lifetime of the false vacuum might be many orders of magnitude longer than the age of the universe.) The VEVs break the gauge symmetry according to

$$SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y \times \text{(matter parity)}.$$

A viable spectrum of neutrino masses can arise just as before via the seesaw mechanism, by taking the obvious extension of (1.2). The $\mu$ term of the MSSM can also arise in a way exactly analogous to that discussed in the model of the previous section, from a non-renormalizable superpotential term proportional to $H_u H_d \text{Tr}[S \bar{S}]$.

The resulting spectrum contains intermediate-scale states consisting of a pair of charge $\pm 1$ massive vector supermultiplets with mass $\sqrt{2} g_R \langle S \rangle$ and a neutral massive vector supermultiplet of mass $\sqrt{6g_{B-L}^2 + 4g_R^2 \langle S \rangle}$. The remaining uneaten components of $S$ and $\bar{S}$ obtain electroweak-scale masses, and consist of the pair of charge $\pm 2$ chiral supermultiplets $S^{--}, \bar{S}^{++}$ and one neutral chiral supermultiplet $\Phi$. The components of $\Phi$ get masses just given by eqs. (2.10)-(2.11) with $\lambda \rightarrow \lambda_1$ and $A \rightarrow A_1$; and all of the same comments apply as before. The fermionic components of $S^{--}$ and $\bar{S}^{++}$ obtain masses $|\lambda_1 + \lambda_2| s$, while their light scalar partners have squared masses

$$|\lambda_1 + \lambda_2|^2 s^2 + \frac{m^2 + \bar{m}^2}{2} \pm \left[ s^2|A_1 + A_2 - 2\lambda_1^*(\lambda_1 + \lambda_2)s|^2 + 4g_R^4 \Delta^4 \right]^{1/2}.$$  \hspace{1cm} (4.7)
These masses suffer renormalization between the scale of spontaneous symmetry breaking and the electroweak scale, since $S^{--}, \bar{S}^{++}$ are charged under the MSSM gauge group.

The most striking prediction of this model is therefore the presence of an exotic vector-like pair of chiral supermultiplets of electric charge $\pm 2$ which may well be accessible to future collider experiments. These particles will have unsuppressed two-body decays into pairs of like-sign leptons, because of the superpotential interaction $W \supset y_{See}S^{--}$ which derives from the analog of (1.2). This should yield a striking experimental signature. This feature is shared by left-right symmetric models with symmetry breaking near the electroweak scale [33]. In the present case, the lightness of these exotic states is due to the lack of renormalizable mass couplings in the underlying superpotential. One can check that the presence of these exotic states does not cause the gauge couplings to blow up below the Planck scale; however, they do completely modify the running. In this model, gauge coupling unification in the usual sense would require additional fields not considered here, and unlike in the model of section 2, the LEP observation of apparent unification would have to be viewed as entirely accidental.

5. Conclusion

In this paper we have analyzed what might be called the minimal supersymmetric standard model with automatic $R$-parity conservation. We do not include renormalizable tree-level mass terms in the superpotential. Instead, gauge symmetry breaking arises because of the interplay between non-renormalizable interactions and soft supersymmetry-breaking interactions. We then found that it is important to take into account dimensionless but “soft” supersymmetry-breaking couplings in this analysis, which can play a crucial role in the spontaneous symmetry breaking; indeed, it may not be able to understand the symmetry breaking mechanism without them. These models have several attractive features. First, the $\mu$ term of the MSSM is naturally generated by a mechanism familiar from [27]. Second, the masses of neutrinos are determined by an intermediate-scale seesaw mechanism and so may be phenomenologically interesting. The minimal version of the model in section 2 also has the nice property that the apparent unification of gauge couplings observed at LEP can still be considered non-accidental. In this model we found that, assuming supergravity-inspired boundary conditions on the soft terms, a discernible imprint may be left on the spectrum of MSSM sparticles. In particular, the masses of the left-handed sleptons are further increased over those of the right-handed sleptons.
One of the interesting consequences of this class of models is the existence of a supermultiplet (Φ) of neutral particles with electroweak scale masses and only very weak couplings to MSSM particles. The largest couplings of these particles to MSSM states are suppressed by at least $(m_W/M)^{1/2}$, and arise from the same non-renormalizable interaction which induces the $\mu$ term of the MSSM. These particles should therefore have relatively long (perhaps microsecond or nanosecond) lifetimes. While this may provide for interesting cosmological consequences, the weak couplings of these particles means that they cannot play a role in collider experiments. In the next-to-minimal model, we found that the symmetry breaking mechanism also predicts a pair of exotic chiral supermultiplets of particles with electric charge $\pm 2$ and electroweak-scale masses.

It is possible that $R$-parity conservation cannot be “explained”, but should simply be taken as a law of nature. It is also possible that an explanation exists, but lies only on the far side of the Planck or string scale. However, it is gratifying that one can construct field theory models which are consistent with all known observations, and in which $R$-parity conservation has its origin in terms of a deeper gauge principle. This may be taken as one of many clues to the nature of physics at very high energies.

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References


