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Method of background estimation in invariant mass distributions due to
higher resonances

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Abstract A method is given for estimating background in invariant mass distributions due to particles produced in the decay chain of heavy resonances. As illustration few examples are calculated, which were considered as possible background in an experiment [1].

1. - Introduction

In estimating the background of resonant states in the invariant mass distributions, as well as the corresponding phase space and possible final state interactions between the particles participating in the mass distribution, there is very often another source of background to be considered, namely the effect of higher resonances. Suppose a resonance of large mass is decaying step by step via short lived particle or resonant states and gives rise to several particles in the final state, some of which take part in the mass distribution. Although such events may be infrequent, they may considerably enhance the background level in only a small interval of the investigated mass distribution (see e.g. Fig. 2a).

In the present paper we give a method for estimating background of this type, finding an expression for the invariant mass in terms of independent variables, whose distributions may reasonably be estimated. One could then calculate the resultant mass distribution by Monte Carlo methods. We shall, however, discuss a different method, using the formula

$$f(\mu) \propto \prod_i \int f_i(z_i) dz_i \delta(\mu - S_n(\dots, z_i, \dots)), \quad (1)$$

where $f(\mu)d\mu$ is the probability of finding S_n , the square of the n particle invariant mass, between μ and $\mu + d\mu$ and z_i are the independent variables having the distributions $f_i(z_i)dz_i$. The main advantage of this method, compared with the Monte Carlo methods, is that the computation is much easier and can usually be done manually.

In Section 2 we elaborate the method assuming no more than three outgoing particles in each individual decay of the subsequent decay chain. This condition seems to be fulfilled in most practical cases. In Section 3 we illustrate the method by several examples, which were considered as possible background in experiment [1].

2. - Description of the method

2.1 - The case of 2-particle mass distribution ($n = 2$). Consider a particle, the square of whose mass has the distribution function $F(S)$, which decays into particles 1 and 2, the latter decaying further into particles 3, 3' and 3'' (Fig. 1a). Following Equ.(1), the distribution function $f(\mu)$ of the square of mass of particles 1 and 3 will have the following form:

$$f(\mu) \propto \left\{ F(S) dS \int_{s_1} f(s_1) ds_1 \dots \int_{s_{3'}} f(s_{3'}) ds_{3'}, \int_{u_3} f(u_3) du_3 \int_{p_3} f(p_3) dp_3 \right. \\ \left. \delta(\mu - S_2(S, \dots, p_3)) \right\}, \quad (2)$$

where $s_1, \dots, s_{3'}$ are the squares of masses of particles $1, \dots, 3'$, respectively and v_3 ($u_3 = \cos \theta_3$) and p_3 are the polar angle and the magnitude of the momentum of particle 3 evaluated in the rest system of particle 2. S_2 is given by the expression:

$$S_2(S, \dots, p_3) = s_1 + s_3 + 2s_1^{1/2} \gamma_{12} (p_3 u_3 v_{12} + e_3) \quad (3)$$

with $\gamma_{12} = (S - s_1 - s_2)/(4s_1 s_2)^{1/2}$, $v_{12} = (\gamma_{12}^2 - 1)^{1/2}/\gamma_{12}$ and $e_3 = (p_3^2 + s_3)^{1/2}$.

In practice, the s_i 's in Equ.(2) need not be taken as variables, their distributions, f_{s_i} , being approximately δ -functions, as is f_{p_3} in the case where particle 2 decays into two particles. In general, experimentally determined distributions are available for f_{p_3} and $f_{u_3}^{++}$.

⁺⁺ The upper and lower limits of the u_3 integral are +1 and -1, respectively.

p_3 may vary generally between some $p_3^{\min}$ and $p_3^{\max}$. If the s_i 's are fixed, $S_2^{\max}$ and $S_2^{\min}$ are given by the formulae:

$$S_2^{\max} = s_1 + s_3 + 2s_1^{1/2} \gamma_{12}^{\max} (e_3^{\max} + p_3^{\max} v_{12}^{\max}) \\ S_2^{\min} = s_1 + s_3 + 2s_1^{1/2} \tilde{\gamma}_{12} \tilde{e}_3 (1 - \tilde{v}_3 \tilde{v}_{12}),$$

where we put $v_3 = p_3/e_3$ and $\tilde{v}_3$ and $\tilde{v}_{12}$ belong to the minimum value of $d = |v_3 - v_{12}|$. If $d_{\min} = 0$ then $S_2^{\min} = (s_1^{1/2} + s_3^{1/2})^2$.

If the original particle decays first into 3 particles (Fig. 1b), a 2-body invariant mass distribution from 3-particle decay has to be used in formula (2) for $F(S)$.

The procedure can be easily generalized to the case where further decays of the daughter particles occur. For example, if in the above case the particle 3 undergoes decay to particles 4 and 4' and the invariant mass of particles 1 and 4 is desired, one takes for S in Eqs.(2) and (3) the invariant mass square of particles 1 and 3 already calculated, and proceeds in just the same way as above. This can be described mathematically by means of a recursive relation with an initial condition given by Eqs.(2) and (3). The recursion can be solved by integrating over the δ -functions, after which one arrives at a formula similar to (1), where $S_2(\dots, z_i, \dots)$ is obtained by successive insertion of the intermediate masses. Not solving the recursion, however, has the advantage that the integrals can often be handled more easily and one problem can be related to others already solved (see example 3'2).

2'2 - The case of $n > 2$. The n -particle invariant mass square can be expressed in terms of 2-particle mass squares by the formula:

$$S_n = \sum_{j=1}^n \sum_{k>j}^n S_2^{(j,k)} - (n-2) \sum_{i=1}^n s_i. \quad (4)$$

By means of the method developed in 2'1 $S_2^{(j,k)}$'s can be expressed in terms of the independent variables of particles j and k , $z_i^{(j,k)}$. Using then formula (4) there is given all to write down for $F(S_n)$ a corresponding equation similar to (1). $z_i^{(j,k)}$'s are essentially the same variables appearing in Equ.(2) with the exception that if particle 2 is a common origin of other

particles participating in the mass distribution, another variable should be considered, namely the polar angle of the relative motion of particles 3' and 3'' in the rest system of particle 2.

Unfortunately in this case one cannot always group the variables so to achieve a useful recursive solution of the final integral as was possible in the 2-particle case.

3. - Examples

3.1 Consider the $S_{\Lambda\gamma}$ distribution from the $Y_0^*(1670) \rightarrow \Lambda + \eta$; $\eta \rightarrow \gamma + \gamma$ decay. Neglecting the final half width of the Y_0^* and η and using the isotropic emission of the γ 's in the η rest system, Equ.(2) gives

$$F(S_{\Lambda\gamma}) \propto \Theta(1-u^2) \quad \text{with} \quad u = \frac{2S_{\Lambda\gamma} - (S_{Y_0^*} + s_{\Lambda} + s_{\eta})}{((S_{Y_0^*} - s_{\Lambda} - s_{\eta})^2 - 4s_{\Lambda}s_{\eta})^{1/2}} \quad (5)$$

where the Θ -function is defined by the equations:

$$\Theta(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x > 0. \end{cases}$$

If we take into account the final half-width of Y_0^* in a Breit-Wigner form the above distribution will have the following form:

$$F(S_{\Lambda\gamma}) \propto \int \frac{dS}{S^{1/2}((S^{1/2} - \frac{1}{2}M_{Y_0^*})^2 + \Gamma_{Y_0^*}^2/4)((S - s_{\Lambda} - s_{\eta})^2/4s_{\Lambda}s_{\eta} - 1)^{1/2}} \quad (6)$$

where u is the same as in Equ.(5), but now S stays instead of $S_{Y_0^*}$. Both results are shown in Fig. 2a.

3.2 We can use the result of (5) or (6) in computing the $S_{\Lambda\pi}$ distribution from the $Y_0^*(1405) \rightarrow \Sigma^0 + \pi^0$; $\Sigma^0 \rightarrow \Lambda + \gamma$; $\pi^0 \rightarrow \gamma + \gamma$ decay chain. Namely, neglecting the half-widths of all masses, the $S_{\Lambda\pi}$ distribution

is described by (5) with the corresponding masses. Using this distribution in the second step we can determine the desired $S_{\Lambda\gamma}$ distribution by manual integration, which yields:

$$F(S_{\Lambda\gamma}) \propto \begin{cases} C_1 - \text{arch}((A^2 + 1)/2A) & \text{if } S_{\Lambda\gamma}^{\min} \leq S_{\Lambda\gamma} \leq \mu_1 \text{ or } \mu_2 \leq S_{\Lambda\gamma} \leq S_{\Lambda\gamma}^{\max} \\ C_2 & \text{if } \mu_1 \leq S_{\Lambda\gamma} \leq \mu_2 \\ 0 & \text{if } S_{\Lambda\gamma} \geq S_{\Lambda\gamma}^{\max} \text{ or } S_{\Lambda\gamma} \leq S_{\Lambda\gamma}^{\min} \end{cases} \quad (7)$$

where $A = (S_{\Lambda\gamma} - s_{\Lambda})/(s_{\Lambda} s_{\pi})^{1/2}$; $\mu_{1,2} = s_{\Lambda} + (s_{\Lambda} s_{\pi})^{1/2} \gamma(S_{\Lambda\pi}^{\min})(1 \mp v(S_{\Lambda\pi}^{\min}))$,

C_1 and C_2 are constants chosen so that the distribution is continuous. Equ. (7) is plotted in Fig. 2b.

3'3 Finally, let us only mention even more complicated cases. Three steps are required for the calculation of the $S_{\Lambda\gamma}$ distribution from a $\sum^0 \pi^0 \pi^0$ resonance. For a 4-step process we can mention the $F(S_{\Lambda\gamma})$ determination from an eventual $\sum^0 A_1$ resonance.

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L i t e r a t u r e

- 1 V.P. Dželepov et al., E. Fenyves et al., Investigation on electromagnetic decays of strange baryon and mesonic resonances. Presented in the Annual Sessions of Nuclear Physics Department of Academy of Science, USSR, 1966.

Figure capture

Fig. 1 A particle with square of mass S decays into a) 2 particles; b) 3 particles. Particle 2 decays further into three particles

Fig. 2 The $S_{\Lambda\gamma}$ invariant mass square distributions from a) $Y_0^*(1670)$, b) $Y_0^*(1405)$ resonances. The solid curve on a) corresponds to $\Gamma_{Y_0^*} = 0$ while the dotted line is obtained with $\Gamma_{Y_0^*} = 60$ MeV

a)

b)

a)