1. General

A high field D.C. magnet is essential for studies of properties of superconductors. Considering the existing power limitations we have designed a magnet which will produce a field of \( \approx \) 100 kG over a useful diameter of 6 cm.

For coils with rectangular section the field at the coil centre is given by Fabry's formula:

\[
B_0 = G \left( \frac{p\lambda}{\rho a_1} \right)^{1/2} \text{ (kGauss)}
\]

where

\[ \begin{align*}
  p &= \text{power available (MW)} \\
  \lambda &= \text{filling factor} \\
  \rho &= \text{copper resistivity (} \Omega \text{ cm)} \\
  a_1 &= \text{internal coil radius (cm)}
\end{align*} \]

\( G \) is a factor which depends on \( \alpha \), which is the ratio between O.D. and I.D. of the coil, and on \( \beta \), ratio between coil height and I.D. \( G \) is maximum, and equal to 0.179, if \( \alpha = 3.09 \) and \( \beta = 1.88 \). Fortunately \( G \) does not depend very sharply on \( \alpha \) and \( \beta \); therefore it will be interesting to make the coil a little larger than the one corresponding to the optimum conditions. This will make the cooling problem easier. Assuming \( \lambda = 0.6, \quad a_1 = 3 \text{ cm}, \quad a_2 = 14.4 \text{ cm}, \quad h = 14.3 \text{ cm}, \quad \rho = 2.00 \times 10^{-16} \Omega \text{ cm (at } 60^\circ \text{ C)}, \quad p = 4.5 \text{ MW}, \quad G = 0.165 \) equation (1) gives \( B_0 \approx 110 \text{ kG} \).

As a matter of fact, in the type of construction we shall use, the coil turns will not be exactly circular concentric loops.

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The conductors will be shaped in form of spiral which will not cross the radii at right angles. This will reduce the field by $\approx 7\%$. However, an external iron yoke, which will be necessary for mechanical and safety reasons, will again raise the field by approximately the same amount.

2. Coil construction and cooling

The main problem in this kind of coil is the cooling.

The problem is rather difficult, when as in our case, the power generator is rated for comparatively high voltage (600 V) and low current (7500 Amps). These generator ratings oblige us to wind the coil with hollow copper conductor and we must exclude Bitter-type coils which would be advantageous from many other points of view.

The coil will consist of 11 double pancakes. Each pancake will have 12 independent parallel water circuits.

Each circuit will spiral from outside towards the coil centre where it will be connected to an identical spiral at a lower level, (see fig. 1).

Twelve of these double spirals will form a complete pancake. Fig. 2 shows a trial model construction with 6 mm copper tubes. Each double pancake will be vacuum impregnated in epoxy resin. With a water pressure of 15 atm, the flow will be 0.18 l/sec per circuit which corresponds to $\Delta t = 45^\circ$.

3. Coil stresses

The equilibrium condition for an element of the coil is (see fig. 3):

$$B J R \, d\alpha \, dR = \sigma_2 \, d\alpha \, dR + (\sigma_1 + d \sigma_1)(R + dR) \, d\alpha - \sigma_1 \, dR$$

It must also be

$$\begin{align*}
E \, \varepsilon &= \sigma_2 + m \sigma_1 \\
E \left( \varepsilon + \frac{d\varepsilon}{dR} R \right) &= - (\sigma_1 + m \sigma_2)
\end{align*}$$

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where :

\( \varepsilon = \) Circumferential strain

\( \mu = \) Poisson's modulus

\( E = \) Young's modulus

\( J = \) Current density

\( B = \) Magnetic field at radius R

Equations (2) and (3) give :

\[ R^2 \frac{d^2 X}{dR^2} + R \frac{dX}{dR} - X = -BJR^2 \frac{1 - \mu^2}{E} \]

where :

\( X = \varepsilon R \)

Assuming \( B = B_0 \frac{R_e - R}{R_e - R_1} \) the solution of (4) is :

\[ X = \varepsilon R = CR^{-1} + DR + AR^3 + BR^2 \]

where :

\[ A = \frac{K}{E} \], \quad \[ B = -\frac{KRe}{J} \], \quad \[ K = \frac{B_0 J}{E} \frac{1 - \mu^2}{R_e - R_1} \]

C and D are constants to be determined by the boundary conditions.

In our case we have assumed :

\( \mu = 0.5 \) \quad \( J = 12000 \) Amp/cm\(^2\), and

\( \sigma_1 = 0 \) for \( R = R_e \) and \( R = R_1 \)

The results i.e. \( \sigma_2 \) and \( \sigma_1 \) versus R are plotted in fig. 4.

Our results are approximate : in fact the situation will be improved by the axial precompression of the pancakes and by the fact that \( \sigma_1 \neq 0 \) for \( R = R_e \).

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4. Iron frame

An iron frame gives a certain contribution to the central field. This contribution can be estimated by substituting the iron with equivalent coils. The coils must be such to produce in air the same field which is expected to be in iron.

The cylindrical external yoke is equivalent to two coaxial coils (see fig. 5) carrying a current:

\[ I_M = \frac{B_M}{\mu_0} h \]

where:

\[ B_M = \frac{B_0}{3} \left( \frac{R_e^2 - R_i^2}{R_e - R_i} \right) \]

\[ B_0 = \text{field in the centre without iron yoke} \]

Therefore the contribution to the central field will be:

\[ \Delta B_M = \frac{\mu_0 I_M}{h} (\cos \alpha' - \cos \alpha") = B_M (\cos \alpha' - \cos \alpha") \]

For the end plates we can proceed in a similar way and we find

\[ \Delta B_p \sim \frac{h' - h}{2 R_M} B_M (\sin^3 \beta" - \sin^3 \beta') \]

In total, assuming \( B_M = 20'000 \) Gauss in iron, we find:

\[ \frac{\Delta B_p + \Delta B_M}{B_0} = 0.083 \]

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