J. Boucheron

THE EFFECT OF IONS ON THE
SYMMETRICAL THROBBING BEAM MODE

by

H.G. Hereward, P.L. Morton,* K.H. Schindl

- I. Introduction
- II. Dynamics of the Ions
- III. Dynamics of the Beam
- IV. Numerical Examples
- Appendix

*)

On leave from Stanford Linear Accelerator Center.

I. INTRODUCTION

It has been demonstrated that the presence of ions produced by a beam of particles in an accelerator can cause unstable coherent oscillations ¹⁾ in the same way (apart from a sign difference) that the presence of the vacuum chamber with finite conductivity can produce unstable oscillations. ²⁾ However, for the monopole mode, where the radius of the beam oscillates, there is a great deal of difference between the effects with ions and that with finite wall conductivity.

For a round beam in a round chamber, a change in the radius of a uniform beam does not change the electric or magnetic field at the vacuum chamber wall so that the only effect that the chamber wall has upon the beam is due to the variation of the beam radius with azimuthal position around the accelerator. Therefore, unstable monopole oscillations due to the chamber wall have very slow growth rates. ³⁾ In contrast the motion of an ion inside of the beam is affected by the beam radius and can produce unstable monopole oscillations of the beam by leaving behind a "memory" field. It is the purpose of this report to investigate the stability and growth rates for monopole oscillations with ions present.

It is assumed that the beam has a bunch structure, such that the time between bunches is long compared to the oscillations period of electrons in the potential well of the beam. Then the electrons produced by the beam escape and do not affect the motion of the beam. It is also assumed that the movement of the positive ions during the passage of the beam is slow enough that the beam can be assumed to be continuous for the purpose of calculating the dynamics of the ions.

II. DYNAMICS OF THE IONS

The radial force F_{ri} on an ion with mass M_i due a beam of radius a and linear particle density λ is given by

$$\frac{1}{M_i} F_{ri} = \frac{2 e^2 \lambda}{a^2 M_i} r \quad (\text{cgs units}) \quad (1)$$

for

$$r < a.$$

We now label the protons by their azimuthal position θ_0 at time zero. At other times the azimuthal position is given by

$$\theta = \theta_0 + \Omega t, \quad (2)$$

with Ω the revolution frequency. The protons designated by θ_0 will be inside of a circle of radius a which is assumed to oscillate as

$$a = a_0 + \xi(\theta_0) e^{i\nu\Omega t}. \quad (3)$$

The function $\xi(\theta_0)$ must be periodic and can be written as

$$\xi(\theta_0) = \sum \xi_n e^{in\theta_0}. \quad (4)$$

It is convenient to consider the case where ξ_n is zero for all values except one, so that the radius of the θ_0 particles is given by

$$a = a_0 + \xi_n e^{i(\nu\Omega t + n\theta_0)} \quad (5)$$

From eqns (2) and (5) we see that the radius of the beam oscillates as

$$a = a_0 + \xi_n e^{i[(\nu - n)\Omega t + n\theta_0]} \quad (6)$$

The equation of motion for the ions can be obtained by combining equations (1) and (6). To first order in ξ_n/a_0 we have

$$\ddot{y}_i + \left[-\mu_0^2 + 2\mu_0^2 \frac{\xi_n}{a_0} e^{i(\omega t + n\theta_0)} \right] y_i = 0, \quad (7)$$

where

$$\omega = (\nu - n)\Omega,$$

$$\mu_0^2 = \frac{2e^2\lambda}{M_i a_0^2} \text{ (cgs units)}$$

$$= \frac{2\lambda r_p c^2}{A a_0^2} \text{ (any system)}$$

$$r_p = \text{classical proton radius} = 1.53 \times 10^{-18} \text{ m},$$

$$c = \text{velocity of light} = 3 \times 10^8 \text{ m/sec},$$

$$A = \text{atomic mass of the ion} = 14 \text{ for N},$$

and y_i = either transverse coordinate.

Consider the ions produced at time t_0 . Since the ions are produced uniformly inside of a circle of radius

$$a = a_0 \left[1 + \xi_n / a_0 e^{i(\omega t_0 + n\theta)} \right], \quad (8)$$

with very little transverse energy, we solve (7) first with the initial conditions

$$y_i(t_0) = 1 + \xi_n / a_0 e^{i(\omega t_0 + n\theta)}, \quad (9)$$

and

$$\dot{y}_i(t_0) = 0 \quad (10)$$

To first order in ξ_n / a_0 the following function y_i satisfies the equation of motion as well as the desired initial conditions for an ion produced at time t_0 :

$$\begin{aligned} y_i(t, t_0) = & \cosh \mu_0(t - t_0) + \frac{\xi_n}{a_0} \left\{ e^{i(\omega t_0 + n\theta)} \cosh \mu_0(t - t_0) \right. \\ & + \frac{2\mu_0^2}{(\omega^2 + 4\mu_0^2)} \cosh \mu_0(t - t_0) [e^{i(\omega t + n\theta)} - e^{i(\omega t_0 + n\theta)}] \\ & + i \frac{4\mu_0^3}{\omega(\omega^2 + 4\mu_0^2)} \sinh \mu_0(t - t_0) [e^{i(\omega t + n\theta)} - e^{i(\omega t_0 + n\theta)}] \\ & \left. - i \frac{2\mu_0 \omega}{(\omega^2 + 4\mu_0^2)} \sinh \mu_0(t - t_0) e^{i(\omega t_0 + n\theta)} \right\}. \end{aligned} \quad (11)$$

This ceases to be correct as soon as the ion moves out of the beam, but if ξ_n/a_0 is small the chance of such an ion being inside the beam again later can be neglected, so the density of ions within the beam at time t will be given, to first order in ξ_n/a_0 , by

$$\rho_i = \frac{\eta\lambda}{\pi a_0^2} \int_{-\infty}^t \frac{dt_0}{y_i^2(t, t_0)} \quad (12)$$

with η the ion production rate per beam particle. The integration is rather lengthy and the details are presented in the Appendix. The following result to first order in ξ_n/a_0 is

$$\begin{aligned} \rho_i = \frac{\eta\lambda}{\pi a_0^2 \mu_0} \left\{ 1 - \frac{\xi_n}{a_0} \left(\frac{4}{4 + \omega^2/\mu_0^2} \right) e^{i(\omega t + n\theta)} \right. \\ \left. + i \frac{\xi_n}{a_0} \left(\frac{2\omega/\mu_0}{4 + \omega^2/\mu_0^2} \right) e^{i(\omega t + n\theta)} \right\} \end{aligned} \quad (13)$$

III. DYNAMICS OF THE BEAM

The total force on a beam particle is the sum of : the external force due to magnetic focusing elements, the space-charge force due to the beam particles, and the space-charge force due to the ions. The external force F_{ext} is given by

$$\frac{1}{m_p \gamma} F_{\text{ext}} = - v_{00}^2 \Omega^2 y_p, \quad (14a)$$

where m_p is the proton mass ν_{00} is the single particle betatron oscillation frequency, and y_p is either transverse coordinate of a beam particle. The force F_{sp} due to the space-charge fields of the beam is given by

$$\frac{1}{m_p \gamma} F_{sp} = \left(\frac{2 \lambda r_p c^2}{\gamma^3 a^2 B} \right) y_p \cdot (\text{any system}) \quad (14b)$$

It seems clear that the bunching factor B (< 1) is correctly included in this formula, though not so clear what is the correct average B .

For a beam particle with

$$\theta = \theta_0 + \Omega t,$$

the beam radius is given by

$$a = a_0 \left[1 + \frac{\xi n}{a_0} e^{i(\nu \Omega t + n \theta_0)} \right],$$

so that to first order in $\frac{\xi n}{a_0}$ we have for the self space-charge force

$$\frac{1}{m_p \gamma} F_{sp} = \left(\frac{2 \lambda r_p c^2}{\gamma^3 a_0^2 B} \right) \left[1 - \frac{2 \xi n}{a_0} e^{i(\nu \Omega t + n \theta_0)} \right] y_p. \quad (14c)$$

The force on a beam particle due to the ions is given by

$$\begin{aligned} \frac{1}{m_p \gamma} F_{si} = & \left(\frac{2 n \lambda r_p c^2}{\mu_o \gamma a_o^2} \right) \left\{ 1 - c_1 \left(\frac{\omega}{\mu_o} \right) \frac{\xi_n}{a_o} e^{i(\nu \Omega t + n \theta_o)} \right. \\ & \left. + i c_2 \left(\frac{\omega}{\mu_o} \right) \frac{\xi_n}{a_o} e^{i(\nu \Omega t + n \theta_o)} \right\} y_p, \end{aligned} \quad (14d)$$

with

$$c_1(z) = \frac{4}{4 + z^2}, \text{ and } c_2(z) = \frac{2z}{4 + z^2}.$$

The function c_2 has a broad maximum of value $1/2$ at $z = 2$.

The equation of motion for a beam particle is

$$\begin{aligned} \ddot{y}_p + & \left\{ \nu_{oo}^2 \Omega^2 - (2 \nu_{oo} \Delta \nu \Omega^2 + \alpha) \right. \\ & + [4 \nu_{oo} \Delta \nu \Omega^2 + \alpha c_1(\omega/\mu_o)] \frac{\xi_n}{a_o} e^{i(\nu \Omega t + n \theta_o)} \\ & \left. - i \alpha c_2(\omega/\mu_o) \frac{\xi_n}{a_o} e^{i(\nu \Omega t + n \theta_o)} \right\} y_p = 0, \end{aligned} \quad (15)$$

with
$$\Delta v = \left(\frac{\lambda r_p R^2}{v_{oo} \beta^2 \gamma^3 a_o^2 B} \right), \text{ (any system)} \quad (15a)$$

and
$$\alpha = \frac{2\eta e^2 \lambda}{m_p \gamma a_o^2 \mu_o}, \text{ (cgs units)} \quad (15b)$$

$$= \frac{\eta (2 \lambda A r_p)^{\frac{1}{2}} c}{\gamma a_o} \cdot \text{ (any system)}$$

We take as basis two independant solutions to equation (15), which we write as ⁴⁾

$$y_1 = \Psi \cos \gamma, \quad y_2 = \Psi \sin \gamma \quad (16)$$

so normalised that to zero order in ξ_n/a_o we have

$$\Psi = 1.$$

The value of the Wronskian must be constant so we can put

$$\dot{\gamma} \Psi^2 = v_o \Omega$$

and this defines v_o . Then the function Ψ must satisfy the following equation

$$\ddot{\Psi} - \frac{v_o^2 \Omega^2}{\Psi^3} + \left\{ v_{oo}^2 \Omega^2 - [2v_{oo} \Delta v \Omega^2 + \alpha] \right. \quad (18)$$

$$\left. + \frac{\xi_n}{a_o} [4v_{oo} \Delta v \Omega^2 + \alpha c_1 \left(\frac{\omega}{\mu_o}\right) - i\alpha c_2 \left(\frac{\omega}{\mu_o}\right)] e^{i(v\Omega t + n\theta_o)} \right\} \Psi = 0.$$

We now suppose that the outline of the beam in phase space is given by that family of linear combinations of the two solutions (16) which can be written

$$y_p = a_o \Psi \cos(\gamma + \gamma_o)$$

with γ_o running through 2π . Then in order for the solution to be self consistent to first order in ξ_n/a_o , (6) implies

$$\Psi = [1 + \xi_n/a_o e^{i(v\Omega t + n\theta_o)}] \quad (19)$$

Upon substitution of eqn (19) into eqn (18), first we obtain to zero order in ξ_n/a_o

$$v_o^2 \Omega^2 = v_{oo}^2 \Omega^2 - [2v_{oo} \Delta v \Omega^2 + \alpha], \quad (20)$$

which displays the incoherent shift in the single particle betatron oscillation frequency due to both the beam and ions. Then going to first order in ξ_n/a_0 we obtain

$$(4v_0^2 \Omega^2 - v^2 \Omega^2) = -\left\{4v_{00} \Delta v \Omega^2 + \alpha c_1 \left[\frac{(v - n) \Omega}{\mu_0}\right]\right\} \quad (21)$$

$$+ i \alpha c_2 \left[\frac{(v - n) \Omega}{\mu_0}\right].$$

The quantities $4v_{00} \Delta v$, $\frac{\alpha c_1}{\Omega^2}$ and $\frac{\alpha c_2}{\Omega^2}$ are usually small compared to v_0^2 so that we obtain

$$v\Omega \approx 2v_0 \Omega + \Delta v \Omega + \frac{\alpha}{4v_0 \Omega} \left(\frac{4}{4 + z^2}\right) \quad (22)$$

$$- i \frac{\alpha}{4v_0 \Omega} \left(\frac{2z}{4 + z^2}\right),$$

with

$$z = (2v_0 - n) \left(\frac{\Omega}{\mu_0}\right).$$

From eqn (3) we see that the motion is stable when the imaginary part of v is positive, and unstable when the imaginary part of v is negative. Thus : the oscillations of the beam radius are unstable for $n < 2v_0$ with a growth rate

$$\frac{1}{\tau} = \frac{\alpha}{4v_0 \Omega} \left(\frac{2z}{4 + z^2}\right) \quad (23)$$

where

$$z = (2\nu_o - n) \left(\frac{\Omega}{\mu_o} \right)$$

and

$$\alpha = \frac{\eta (2 \lambda A r_p)^{\frac{1}{2}} c}{\gamma a_o}$$

In order for unstable oscillations to be Landau damped it is necessary for the spread in the quantity $2\nu_o \Omega$ to be larger than the coherent frequency shift for the mode under consideration. If the ion density is sufficiently low compared to the beam density the coherent frequency shift is, from (22), just $\Delta\nu\Omega$, and we require a betatron ν -spread for Landau damping given by

$$\delta \nu_o > \frac{\Delta \nu}{2} = \frac{\lambda r_p R^2}{2\nu_{oo} \beta^2 \gamma^3 a_o^2 B} \quad (24)$$

This is for a ν -spread within each bunch : the effect of having different ν values for different bunches requires a more detailed analysis of the bunch to bunch coupling.

IV. NUMERICAL EXAMPLES

Values of parameters for the CPS Booster are used as a numerical example. The value of the parameters used are

r_p	= classical proton radius = 1.53×10^{-18} m
r_e	= classical electron radius = 2.81×10^{-15} m
N	= number of protons per ring per pulse = 2.5×10^{12}
ν_{oo}	= betatron oscillation frequency without ions = 4.6
a_o	= unperturbed beam radius = 2 cm
β_{inj}	= v/c at injection (50 MeV) = 0.314
γ_{inj}	= $(1 - \beta_{inj}^2)^{-\frac{1}{2}} = 1.053$
R	= average radius = 25 m
Ω	= angular revolution frequency at injection = 3.77×10^6 rad/sec
$-\frac{dE}{dx}$	= energy loss due to ionization (50 MeV) = $11 \frac{\text{MeV cm}^2}{\text{g}}$
A	= atomic mass of ions produced (Nitrogen) = 14
P	= pressure = 10^{-7} mmHg
ρ	= density of N_2 at 10^{-7} mmHg = 1.65×10^{-13} g/cm ³
eW	= energy necessary to produce 1 ion pair = 120 eV*
λ	= average linear particle density = 1.59×10^{10} protons/m
h	= rf harmonic number = 5
B	= bunching factor = 0.6

*A multiplication factor of 4 has been used to account for the absence of secondary ionization processes due to the small size of the vacuum chamber compared to the mean free path of an electron at this pressure.⁵⁾

The ion production rate per beam particle

$$\eta = - \left(\frac{dE}{dx} \right) \frac{\beta c \rho}{eW} = 143 \text{ sec}, \quad (25)$$

at a pressure of 10^{-7} mmHg.

In order to check the assumptions that the electrons escape we note that the time between the end of one bunch and the beginning of the next bunch is in the region of

$$\Delta\tau = 0.3 \frac{2\pi}{h\Omega} = 0.1 \text{ } \mu\text{sec},$$

with h the number of bunches

The electron oscillation frequency*

$$\omega_e = \left(\frac{2\lambda r_e c^2}{a_0^2} \right)^{\frac{1}{2}} = 142 \text{ rad}/\mu\text{sec}, \quad (26)$$

and the ion e-folding rate*

$$\mu_0 = \left(\frac{2\lambda r_p c^2}{A a_0^2} \right)^{\frac{1}{2}} = .887/\mu\text{sec}. \quad (27)$$

* Strictly these are rms values, because we have based them on the average linear proton density.

An electron would undergo 14 radians of oscillation and an ion 0.09 of an e-folding during the time between bunches. Thus the assumptions that the electrons are not captured by the beam while the ions experience only the average electric field from the beam are justified.

Returning to eqn (23) we obtain for

$$\alpha = \frac{\eta(2\lambda A r_p)^{\frac{1}{2}} c}{\gamma a_0} = 1.69 \times 10^9 / \text{sec}^2$$

and the two fastest modes are

$$n = 9,$$

$$\text{with } z = \frac{(2v_0 - n)\Omega}{\mu_0} = .860, \quad c_2 = 0.363 ; \quad (28)$$

$$\text{and } n = 8,$$

$$\text{with } z = \frac{(2v_0 - n)\Omega}{\mu_0} = 5.16, \quad c_2 = 0.337 ; \quad (29)$$

which give as growth rates

$$\frac{1}{\tau} = \begin{array}{ll} 8.82/\text{sec} & \text{for } n = 9 \\ 8.23/\text{sec} & \text{for } n = 8 \end{array} \quad (30)$$

Thus for the Booster we would expect unstable oscillations of the monopole throbbing beam mode to have a growth time of about 0.1 sec.

This result is rather insensitive to most of the assumed values. Quite considerable factors in μ_0 or in the fractional part of ν_0 may shift the emphasis between the modes, but cannot much change the largest value of c_2 . So the growth rate will be largely determined by α where the vacuum (η^{+1}) and beam size (a_0^{-1}) are important factors, while the mass number of the ions and the proton intensity only come in the square root.

In order for unstable oscillations in the Booster to be damped it is necessary that the spread in the betatron oscillation frequency $\delta\nu_0$ satisfy the following condition :

$$\delta\nu_0 > \frac{\Delta\nu}{2} = \frac{\lambda \ r_p \ R^2}{2\nu_{00} \ \beta^2 \ \gamma^3 \ a_0^2 B} = 0.06 \quad (31)$$

As an additional numerical example we consider the present CPS. The values of the parameters used are :

$$\begin{aligned} N &= 10^{12} \\ \nu_{00} &= 6.25 \\ a_0 &= 1.5 \text{ cm} \\ \beta &= .507 \text{ (150 MeV)} \\ \gamma &= 1.16 \\ R &= 100 \text{ m} \\ \Omega &= 1.52 \times 10^6 \text{ rad/sec} \\ -\frac{dE}{dx} &= 5 \text{ MeV cm}^2/\text{g (150 MeV)} \end{aligned}$$

$$\begin{aligned}
 P &= 5 \times 10^{-6} \text{ mmHg} \\
 \rho &= 8.25 \times 10^{-12} \text{ g/cm}^2 \\
 \lambda &= 1.59 \times 10^9 \text{ protons/m} \\
 h &= 20 \\
 B &= 0.3
 \end{aligned}$$

The ion production rate per beam particle

$$\eta = -\left(\frac{dE}{dx}\right) \frac{\beta c \rho}{eW} = 5.23 \times 10^3 / \text{sec} \quad (32)$$

at 5×10^{-6} mmHg.

The time between the end of one bunch and the beginning of the next is in the region of

$$\Delta\tau = 0.5 \frac{2\pi}{h\Omega} = 0.1 \text{ } \mu\text{sec.} \quad (33)$$

The electron oscillation frequency

$$\omega_e = (2\lambda r_e)^{\frac{1}{2}} \frac{c}{a_0} = 59.8 \text{ rad}/\mu\text{sec} \quad (34)$$

and the ion e-folding rate

$$\mu_0 = \left(\frac{2\lambda r_p}{A}\right)^{\frac{1}{2}} \frac{c}{a_0} = .374/\mu\text{sec.} \quad (35)$$

An electron would undergo six radians of oscillation and an ion 0.04 of an e-folding during the time between bunches. Thus for the CPS the assumptions used are valid.

We obtain for

$$\alpha = \frac{\eta(2\lambda A r_p)^{\frac{1}{2}} c}{\gamma a_o} = 2.35 \times 10^{10} / \text{sec}^2 \quad (36)$$

and the fastest mode is $n = 12$, with

$$z = (2v_o - n) \frac{\Omega}{\mu_o} = 2.03, \quad c_2 = 0.500, \quad (37)$$

which gives as a growth rate

$$\frac{1}{\tau} = 310 / \text{sec} \quad \text{for } n = 12. \quad (38)$$

Thus one expects an unstable monopole oscillation in the CPS to have a growth time of about 3 ms. The amount of betatron oscillation frequency spread needed to stabilize this instability is

$$\delta v_o > \frac{\lambda r_p R^2}{2v_{oo} \beta^2 \gamma^3 a_o^2 B} = 0.07.$$

APPENDIX

The following integral is evaluated to first order in ξ_n/a_o .

$$I = \int_{-\infty}^t \frac{d t_o}{y_i^2(t, t_o)} \quad (1)$$

with $y_i(t, t_o)$ given by eqn (11) of the text. First we define $\alpha = (t - t_o)$ and note that

$$e^{i(\omega t_o + n\theta)} = e^{-i\omega\alpha} e^{i(\omega t + n\theta)},$$

to obtain

$$I = \int_0^{\infty} \frac{d \alpha}{y_i^2(t, t-\alpha)}, \quad (2)$$

with

$$\begin{aligned} y_i = & \cosh \mu_o \alpha + \frac{\xi_n}{a_o} e^{i(\omega t + n\theta)} \cosh \mu_o \alpha \left\{ e^{-i\omega t} \right. \\ & + \left(\frac{2\mu_o^2}{\omega^2 + 4\mu_o^2} \right) (1 - e^{-i\omega\alpha}) + i \tanh \mu_o \alpha \left[\frac{4\mu_o^3}{\omega(\omega^2 + 4\mu_o^2)} \right] (1 - e^{-i\omega\alpha}) \\ & \left. - i \tanh \mu_o \alpha \left(\frac{2\mu_o \omega}{\omega^2 + 4\mu_o^2} \right) e^{-i\omega\alpha} \right\}. \end{aligned} \quad (3)$$

Next we expand $\frac{1}{y^2}$ to first order in ξ_n/a_0 and separate the real and imaginary parts to obtain

$$I = \left\{ A_1 - \frac{\xi_n}{a_0} e^{i(\omega t + n\theta)} \left[\frac{4\mu_0^2}{4\mu_0^2 + \omega^2} A_1 + \frac{2(\omega^2 + 2\mu_0^2)}{(\omega^2 + 4\mu_0^2)} A_2 - \frac{4\mu_0(\omega^2 + 2\mu_0^2)}{\omega(\omega^2 + 4\mu_0^2)} A_6 \right] \right. \\ \left. + i \xi_n/a_0 e^{i(\omega t + n\theta)} \left[\frac{2(\omega^2 + 2\mu_0^2)}{(\omega^2 + 4\mu_0^2)} A_3 - \frac{8\mu_0^3}{\omega(\omega^2 + 4\mu_0^2)} A_4 + \frac{4\mu_0(\omega^2 + 2\mu_0^2)}{\omega(\omega^2 + 4\mu_0^2)} A_5 \right] \right\}, \quad (4)$$

with

$$A_1 = \int_0^\infty \frac{d\alpha}{\cosh^2 \mu_0 \alpha}, \quad A_2 = \int_0^\infty \frac{\cos \omega \alpha}{\cosh^2 \mu_0 \alpha} d\alpha,$$

$$A_3 = \int_0^\infty \frac{\sin \omega \alpha}{\cosh^2 \mu_0 \alpha} d\alpha, \quad A_4 = \int_0^\infty \frac{\tanh \mu_0 \alpha}{\cosh^2 \mu_0 \alpha} d\alpha,$$

$$A_5 = \int_0^\infty \frac{\tanh \mu_0 \alpha \cos \omega \alpha}{\cosh^2 \mu_0 \alpha} d\alpha,$$

and

$$A_6 = \int_0^\infty \frac{\tanh \mu_0 \alpha \sin \omega \alpha}{\cosh^2 \mu_0 \alpha} d\alpha.$$

The integrals A_1 and A_4 can be evaluated straight away to give

$$A_1 = \frac{1}{\mu_0}, \quad A_4 = \frac{1}{2\mu_0}. \quad (5)$$

Integration by parts gives

$$A_5 = \frac{1}{2\mu_0} - \frac{\omega}{2\mu_0} A_3, \quad (6)$$

and

$$A_6 = \frac{\omega}{2\mu_0} A_2. \quad (7)$$

When the integrals A_1 through A_6 are substituted into eqn (4) we obtain

$$I = \frac{1}{\mu_0} \left\{ 1 - \frac{\xi n}{a_0} \left(\frac{4}{4 + \omega^2/\mu_0^2} \right) e^{i(\omega t + n\theta)} \right. \\ \left. + i \frac{\xi n}{a_0} \left(\frac{2\omega/\mu_0}{4 + \omega^2/\mu_0^2} \right) e^{i(\omega t + n\theta)} \right\}. \quad (8)$$

Distribution

List MPS-SI/1

K. Hübner	W. Schnell
E. Keil	V. Vaccaro
B. Montague	B. Zotter

REFERENCES

- 1) H.G. Hereward; "The Instability of Radial Betatron Oscillations in the CPS", MPS/Int. DL 64-8, (March 1964).
- 2) L.J. Laslett, V.K. Neil and A.M. Sessler; Rev. Sci. Instr. 36, 436 (1965).
- 3) M.J. Lee, F.E. Mills and P.L. Morton; "Throbbing Beam Transverse Resistive Instabilities in Circular Accelerators and Storage Rings", SLAC-76, (Aug. 1967).
- 4) I. Kapchinskii and V. Vladimirskii; "Limitations on Proton Beam Current in a Strong Focusing Linear Accelerator Associated with the Beam Space Charge", Proceedings of the International Conference on High Energy Accelerators, CERN, Geneva, (1959), p. 274.
- 5) E. Fischer and F. Schneider; (Private Communication).